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Tidally affected galaxies modelled as Maclaurin spheroids

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Abstract. We have used tensor virial equations and impulse approximation to study the tidal transfer of energy and angular momentum to a test galaxy, modelled as a Maclaurin spheroid. It is shown that the fractional increase in kinetic energy falls off with increasing eccentricity, whereas the fractional increase in angular momentum squared rises till $e = 0.83$ and then falls off. The tidally transferred angular momentum is about an axis lying in the equatorial plane of the spheroidal test galaxy.

Key words : galaxies—tidal interaction

1. Introduction

Following Spitzer (1958), many authors have studied the results of tidal encounter between spherical galaxies, on the assumption that the stars in the test galaxy remain stationary during the encounter (impulse approximation). Som Sunder, Kochhar & Alladin (1990) have analytically investigated the problem of transfer of energy and angular momentum to a spheroidal galaxy. (See Namboodiri & Kochhar 1990 for numerical simulations.) In this paper we discuss the results applicable to a test galaxy that can be modelled as a Maclaurin spheroids.

In a study of shape-velocity relations, Maclaurin spheroids are a good approximation to numerical models, and the uniform density models can be easily scaled to centrally condensed real elliptical galaxies (Gott & Thuan 1976.) In studies of tidal perturbation, homogeneous density models are of limited use, but are nevertheless attractive because of their tractability.

2. Mathematical formulation

Consider the test galaxy to be a homogeneous oblate spheroid of mass M and semi-axes $a_1 = a_2 > a_3$. It may be made up of gas or purely of stars depending upon whether the 'test galaxy' is a protogalaxy or a galaxy. Its gravitational energy is given by (Chandrasekhar 1969)

$$W = -\frac{3}{5}\frac{GM^2}{a_1}\frac{\sin^{-1} e}{e} \qquad ...(1)$$

where

$$a_3 = a_1(1 - e^2)^{1/2}. \qquad ...(2)$$

If the spheroid is in rotational equilibrium, its angular momentum J is given by

$$J^2 = -\frac{3}{25} GM^3 a_1 \frac{\Omega^2(e)}{(1-e^2)^{1/2}}, \qquad ...(3)$$

where $\Omega(e)$ is the angular velocity in units of $(\pi G\rho)^{1/2}$:

$$\Omega^2(e) = \frac{2(1-e^2)^{1/2}}{e^3}[(3-2e^2)\sin^{-1} e - 3e(1-e^2)^{1/2}]. \qquad ...(4)$$

The kinetic energy of the spheroid is given by

$$T = 2T_{11} = I_{11}\,\Omega^2(e)\,\pi G\rho. \qquad ...(5)$$

The Maclaurin spheroid satisfies the equilibrium condition $2T + W = 0$ so that its binding energy is $E = T + W = W/2$.

The spheroid is now tidally perturbed by a passing mass point which is moving with a large velocity v at a pericentric distance p (figure 1). Then the increase in the internal energy is given by (Som Sunder, Kochhar & Alladin 1990)

$$\Delta K = \eta^2 I_{11}\left(1 - \frac{e^2}{4}\right), \qquad ...(6)$$

where we have got rid of angles by puting $\theta = \frac{\pi}{4} = \varphi$.

In equation (6)

$$\eta = \frac{2GM}{p^2 v}. \qquad ...(7)$$

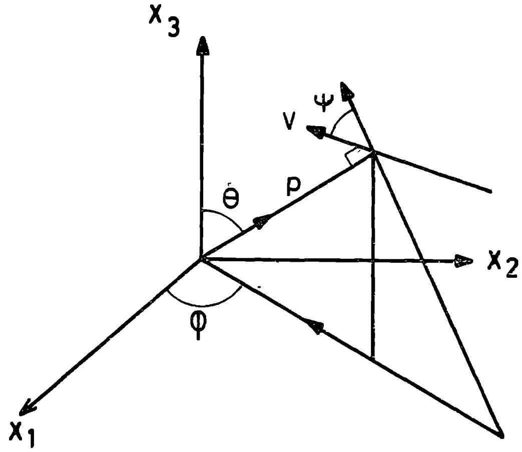

Figure 1

Equation (6) can be rewritten as

$$\Delta K = \frac{4}{5}\frac{GM^2}{a_1}\left\{\frac{GM'}{pv^2}\right\}\left\{\frac{M'}{M}\right\}\left\{\frac{a_1}{p}\right\}^3\left\{1-\frac{e^2}{4}\right\}. \qquad ...(8)$$

The tidally-transferred angular momentum is given by (Som Sunder, Kochhar & Alladin 1990)

$$\Delta J = \tfrac{1}{2}\,\eta\, I_{11}\, e^2 \qquad ...(9)$$

or

$$\Delta J^2 = \frac{2}{25}\, GM^3\, a_1\left\{\frac{GM'}{pv^2}\right\}\left\{\frac{M'}{M}\right\}\left\{\frac{a_1}{p}\right\}^3 e^4. \qquad ...(10)$$

This angular momentum is about an axis in the equatorial plane that is, perpendicular to the axis of the equilibrium rotation. (This equation has been used by Kochhar (1989) to explain the peculiar rotation of Uranus).

All these formulae can be applied to a disc galaxy by formally setting $e = 1$.

The fractional increases in the total kinetic energy K and angular momentum J are given by

$$\frac{\Delta K}{|E|} = \frac{4}{3}\left(1-\frac{e^2}{4}\right)\frac{e}{\sin^{-1} e}\,\frac{GM'}{pv^2}\left(\frac{a_1}{p}\right)^3, \qquad ...(11)$$

$$\frac{\Delta J}{J^2} = \frac{2}{3}\, e^4\,\frac{(1-e^2)^{1/2}}{\Omega^2(e)}\,\frac{GM'}{pv^2}\,\frac{M'}{M}\left(\frac{a_1}{p}\right)^3, \qquad ...(12)$$

where $\Omega(e)$ is given by equation (4).

Figure 2 shows the plot of $\Delta K/|E|$ and $\Delta J^2/J^2$ against eccentricity e, with

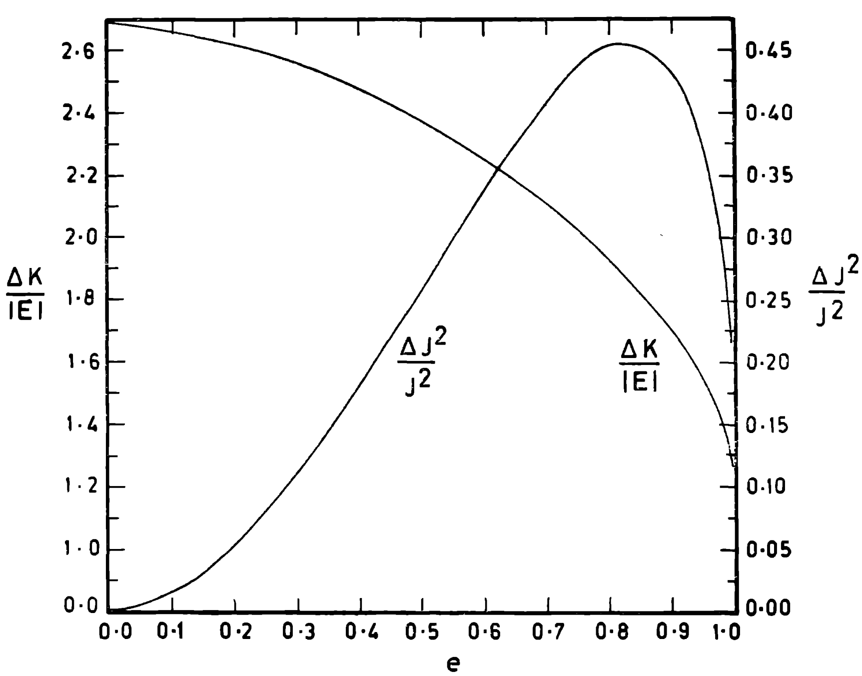

Figure 2

$\frac{GM'}{pv^2}\frac{M'}{M}\left(\frac{a_1}{p}\right)^3$ suppressed. We note that $\Delta K/|E|$ falls off with increasing e, whereas $\Delta J^2/J^2$ rises till $e = 0.83$ and then falls off. This behaviour is similar to that of Ω^2 except that Ω^2 reaches its peak value at $e = 0.93$ (Chandrasekhar 1969).

Considering the dimensionless combinations on the right-hand sides of equations (11) and (2), we note that $GM'/(pv^2) \sim 1$ in any realistic situation; and we can set $M' \approx M$. p is at least about $2a_1$ so that $(a/p)^3$ is at least about 1/10.

We can use figure 2 to study the response of a nearly spherical galaxy and of a flattened galaxy to a tidal encounter.

In the case of an $E1$ galaxy ($e = 0.44$), the internal energy would increase by 5% with respect to its binding energy, whereas an $E5$ galaxy ($e = 0.87$) would gain about 18%. On the other hand the increase in the tidal rotation energy would be about zero for an $E1$ galaxy and about 5% for an $E5$ galaxy.

Thus the tidally-introduced angular momentum is very small, but recall that it is along a direction prependicular to the symmetry axis.

3. Conclusions

We have discussed the tidal transfer of energy and angular momentum to a test galaxy modelled as a homogeneous Maclaurin spheroid. The fractional increase in the kinetic energy falls off with increasing eccentricity e, whereas fractional energy is angular momentum squared rises till $e = 0.83$ and then falls off. The transferred angular momentum is about an axis in the equatorial plane of the spheroidal galaxy,

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