Bull. Astr. Soc. India (1987) 4, 193–203

An analysis of proper motion and membership probability in open clusters

Ram Sagar *Indian Institute of Astrophysics, Bangalore 560 034*

Received 1987 April 16; accepted 1987 August 16

Abstract. An analysis of the proper motion data and membership probability has been carried out for nine open clusters. The effects of observational errors on the parameters characterizing the frequency distributions of cluster and field stars have been discussed with a view to studying their combined effect on membership assignment of a cluster star. The proper motion estimates of stars in various studies of a clusters are generally in fair agreement.

Key words: proper motion—open cluster—membership

1. Introduction

Recently proper motion studies have been carried out in several open cluster regions (see van Leeuwen 1985; Zhao *et al.* 1985 and the references therein). Kinematic criteria based on accurate data segregate cluster members from the field stars in a better way than other criteria. For example, statistical criterion is not so effective in open clusters as it is in globular clusters because in contrast to globular clusters, open clusters are usually projected against rich stellar backgrounds of the Galactic disc. Also, a comparison of photometric membership criterion with the proper motion one for IC 4756 indicates nearly 50% uncertainty in the former (Sanders 1976). However, the extent of separation of cluster members from field stars in proper motion studies depends upon the extent of differentiation between their motions (Vasilevskis 1962, Mathieu 1983).

Sanders' (1972) analysis for IC 1805 showed the effect of different reduction techniques on the assignment of proper motion membership probability p of a star. McNamara & Schneeberger (1978) have studied parameters characterizing the frequency distributions of cluster and field stars and their effect on p in the case of stars in M 11. Zhao *et al.* (1985) calculated p of stars in 42 open clusters. Because of homogeneity in membership assignment, this work provides a rather rare opportunity to examine the effects of measurement errors, etc., on the cluster membership estimation, which we attempt in this paper. We have also compared the components of proper motion vectors of the stars obtained by different investigators and analysed the differences.

2. Data

The data used in the present analysis are taken from Zhao *et al.* (1985), who compiled the relative proper motion data for 42 clusters from the literature and estimated membership probabilities for the stars in each cluster region using the improved Sanders' (1971) maximum likelihood method and the discriminatory analysis described by Zhao *et al.* (1982b). It is observed that members according to either discriminatory analysis or maximum likelihood principle generally have $p \geqslant 0.50$. Therefore, in the following, stars with $p \geqslant 0.50$ have been considered as cluster members. Out of 42 open clusters, nine have proper motion data from two or more independent sources. We here discuss proper motion data for these 9 clusters, NGC 129, 581, 2099, 2168, 2682, 7092, 7788, 7790, and Tr 1, by taking various studies pairwise (see table 1). In these open clusters p estimates are homogeneous,

Table 1. General information and proper motion details of the clusters under study

Cluster name			Information about proper motion studies					
IAU	OCL	Other	Investigated area (arcmin²)	Limiting V mag	Epoch difference in yr	Accuracy (arcsec/ century)	No. of stars	Ref.
C0027+599	294	NGC 129	40×40	12.4	42	0.11	70	1
			150×150	14.7	72	0.21	326	2
			80×80	14.6	50	0.20–0.29	1025	3
C0129+604	326	NGC 581	80×80	14.0	56	0.18–0.33	791	3
		≡ M103	80×80	15.5	49	0.14–0.37	997	4
C0132+610	328	Tr 1	80×80	14.0	56	0.18–0.33	555	3
			80×80	15.5	49	0.14–0.37	718	4
C0549+325	451	NGC 2099	40×40	—	50	0.3	243	5
		≡ M37	30×30	13.5	12	—	292	6
			20×20	14.0	60	0.1	489	7
C0605+243	466	NGC 2168	80×80	15.5	60	0.11–0.29	2784	3
		≡ M35	40×40	14.5	41	0.11	763	8
C0847+120	549	NGC 2682	30×30	13.0	35	0.12	158	9
		≡ M67	20×20	15.0	21	0.08	343	10
			90×90	17.0	67	0.10	1866	11
			100×100	15.5	69	0.27	1067	12
C2130+482	211	NGC 7092	30×30	11.2	23	0.13	50	13
		≡ M39	80×80	14.8	53	0.13–0.20	1079	3
			70×65	14.5	52	0.18	1710	14
C2354+611	275	NGC 7788	12×12	16.5	49	0.3	59	15
			60×60	14.0	42	—	520	16
C2355+609	276	NGC 7790	14×14	16.5	49	0.3	85	15
			60×60	14.0	42	—	638	16

**Reference :* (1) Lenham & Franz (1961), (2) Frolov (1975), (3) Lavdovskij (1961), (4) Oja (1966), (5) Jefferys (1962), (6) Joy (1916), (7) Upgren (1966), (8) Cudworth (1971), (9) Ebbighausen (1940a), (10) Van Maanen (1942), (11) Sanders (1977), (12) Tian *et al.* (1982), (13) Ebbighausen (1940b), (14) McNamara & Sanders (1977), (15) Frolov (1979), (16) Ishmukhamedov (1966).

so that any difference in the p values of stars common in the two proper motion studies of a cluster should depend upon factors like quality of the proper motion data and parameters characterizing the frequency distributions of cluster and field stars.

3. Stars common in the two proper motion studies

The (x, y) coordinate of a star in the two studies called A and B are interrelated:

$$X_B = A_0 + A_1 X_A + A_2 y_A, \qquad ...(1)$$

$$Y_B = B_0 + B_1 X_A + B_2 y_A. \qquad ...(2)$$

Here A_0, A_1, A_2, B_0, B_1 and B_2 are their transformation coefficients, estimated using the least square methods and nearly 10–30 visually identified common stars. These coefficients indicate that most of the coordinate axes of the compared areas are linearly transferred; only in a few cases are they magnified and in none of the cases are they rotated.

If the coordinate set (x_A, y_A) after conversion agrees with (x_B, y_B) within three sigma error of the transformations, and if also the brightness of the star in the two studies if available does not differ significantly, the star is considered common. If more than one star satisfies these conditions, then the star closest to the converted (x, y) coordinate is considered as common. For some clusters, star numbers with known (x, y) coordinates are also marked on the photograph of the cluster region in both studies. In such cases, their visual comparison confirms that our above procedure of identifying the common stars has done the job quite satisfactorily. Wherever (x, y) coordinates are not available, identification of the stars has been done visually.

Number of stars common in the two proper motion studies of a cluster are listed in column 4 of table 2. A comparison with table 1 shows that a significant fraction of stars of one study are not present in the other. This may be due to the fact that in the two studies only a small part of investigated area is common and/or the limiting magnitudes are not the same. However, where these factors are absent, relatively small fraction but good number of stars of one study are still not present in the other.

An inspection of columns 5–8 of table 2 indicates that except for NGC 7788, 7790, 129, and 7092, generally $\geqslant 70\%$ members of one study are also members of the other. However, p values of stars common in the two studies differ significantly in some cases. Before discussing the possible causes for such differences, a discussion on the procedure used for estimating p seems necessary.

4. Proper motion membership probability and effect of parameters

It is assumed that the proper motions of stars in the vicinity of an open cluster follow a bivariate frequency distribution; normal circular distribution of cluster members is superposed over a normal elliptical distribution of the field stars, (Vasilevskis, Klemola & Preston 1958). The p of a star having (μ_{x1}, μ_{y1}) as proper

Table 2. Comparison of membership of stars common in the two proper motion studies (i.e. Reference A and B) of a cluster. Reference numbers are the same as in table 1. *N* is the number of stars common in the two studies

Cluster name	Reference		N	% of N having p ⩾ 50% in ref.		% of stars having p ⩾ 50% in ref.	
	A	B		A	B	A with p ⩾ 50% in ref. B	B with p ⩾ 50% in ref. A
NGC 129	2	1	45	24	33	73	53
	3	1	54	30	31	50	47
	3	2	248	55	25	32	69
NGC 581	4	3	483	70	78	92	82
Tr 1	4	3	381	66	62	80	86
NGC 2099	7	6	234	90	69	70	92
	7	5	203	91	94	98	95
	6	5	219	61	80	94	73
NGC 2168	3	8	528	85	81	88	93
NGC 2682	10	9	119	81	90	91	81
	11	9	158	78	83	82	77
	11	10	209	86	88	96	95
	12	10	229	77	90	97	86
	11	12	268	43	45	83	79
NGC 7092	14	3	819	56	59	88	84
	14	13	45	18	11	25	40
	3	13	44	16	11	0	0
NGC 7788	16	15	33	76	09	08	67
NGC 7790	16	15	50	74	08	03	25

motion vector components along the principal axes of elliptical distribution is defined as

$$p_i = \frac{\phi_c(\mu_{xi}, \mu_{yi})}{\phi_c(\mu_{xi}, \mu_{yi}) + \phi_f(\mu_{xi}, \mu_{yi})},$$

where

$$\phi_c(\mu_{xi}, \mu_{yi}) = \frac{n_c}{2\pi\sigma^2} \exp\left\{-\frac{1}{2}\left[\left(\frac{\mu_{xi} - \mu_{xc}}{\sigma}\right)^2 + \left(\frac{\mu_{yi} - \mu_{yc}}{\sigma}\right)^2\right]\right\}$$

and

$$\phi_f(\mu_{xi}, \mu_{yi}) = \frac{n_f}{2\pi\sigma_x\sigma_y} \exp\left\{-\frac{1}{2}\left[\left(\frac{\mu_{xi} - \mu_{xf}}{\sigma_x}\right)^2 + \left(\frac{\mu_{yi} - \mu_{yf}}{\sigma_y}\right)^2\right]\right\}.$$

Here n_f and n_c are the number of field and cluster stars respectively; σ and (σ_x, σ_y) are the standard deviations in x and y components of proper motions of cluster members and field stars respectively; (μ_{xf}, μ_{yf}) and (μ_{xc}, μ_{yc}) are respectively the centroid of field star and cluster member distributions. The value of p depends not only upon proper motion components of a star but also upon parameters n_f, n_c, μ_{xf}, μ_{yf}, μ_{xc}, μ_{yc}, σ_x, σ_y and σ characterizing the distribution of field and cluster stars.

These parameters for various proper motion studies of the clusters under study taken from Zhao *et al.* (1985) are listed in table 3 and as can be seen vary from one study to another. The values of parameters depend mainly upon the precision of the proper motion data, procedure of pruning the vector point diagram and the number of stars used (*cf.* McNamara & Schneeberger 1978, Zhao *et al.* 1982a, b). If (μ_{xc}, μ_{yc}, and (μ_{xf}, μ_{yf}) significantly differ, cluster members are relatively easily segregated from field stars. The parameters which significantly affect the value of p are σ_x, σ_y, and σ. The main factor responsible for producing a dispersion in (μ_{xi}, μ_{yi}) of cluster members is the observational errors.

On the other hand, the observed motions of field stars depend upon the dispersion in secular parallax, the peculiar motion of stars, the differential galactic rotation as well as the errors of observations. It may be because of these factors that σ_x and σ_y are always greater than σ in table 3. Actually, the larger the difference between standard deviations of field stars and cluster members, the better the separation between them; separation becomes impossible if σ_x and σ_y are of the order of σ (*cf.* Vasilevskis 1962). If $n_c < n_f$, the separation of cluster

Table 3. Parameters characterizing the frequency distribution of cluster and field stars from Zhao *et al.* (1985). n_c, n_f are number of cluster and field stars; (μ_{xf}, μ_{yf}) and (μ_{xc}, μ_{yc}) are the centroid of field and cluster stars respectively in the vector point diagram. (σ_x, σ_y) and σ are respectively the standard deviations in x and y components of proper motion of field and cluster stars. τ_m is the relative number of spurious members. Reference numbers are the same as in table 1. μ_x, μ_y are in units arcsec per century.

Cluster	Ref.	n_f	n_c	μ_{xc}	μ_{yc}	μ_{xf}	μ_{yf}	σ_x	σ_y	σ	τ_m
NGC 129	1	52	18	−0.09	0.02	0.04	0.01	0.37	0.27	0.11	0.16
	2	239	87	−0.16	0.00	−0.17	0.06	0.53	0.34	0.22	0.24
	3	655	370	−0.11	0.10	0.15	0.06	0.64	0.48	0.26	0.21
NGC 581	3	311	480	−0.12	0.09	0.03	−0.02	0.84	0.57	0.31	0.18
	4	493	504	−0.20	−0.08	−0.08	−0.11	0.70	0.43	0.21	0.19
Tr 1	4	352	366	−0.14	−0.03	0.04	−0.08	0.73	0.47	0.22	0.17
	3	290	265	−0.20	0.08	0.07	0.02	0.70	0.49	0.29	0.21
NGC 2099	5	27	216	0.01	0.01	0.28	0.28	0.46	0.27	0.15	0.05
	6	131	161	0.04	0.26	0.02	0.00	0.33	0.31	0.22	0.27
	7	140	349	0.01	−0.02	−0.09	−0.11	0.40	0.27	0.13	0.14
NGC 2168	8	218	545	0.03	0.01	−0.14	0.16	0.51	0.38	0.13	0.10
	3	1287	1497	0.02	−0.05	−0.07	−0.07	0.67	0.53	0.26	0.22
NGC 2682	9	44	114	0.05	−0.13	0.14	−0.19	0.60	0.57	0.19	0.11
	10	69	274	0.00	−0.01	0.30	0.27	0.58	0.53	0.12	0.04
	12	690	377	−0.10	−0.03	0.10	0.07	0.80	0.66	0.16	0.09
	11	1352	514	−0.01	0.00	0.23	−0.09	0.76	0.64	0.09	0.04
NGC 7092	13	44	6	0.65	0.08	−0.08	−0.01	0.68	0.26	0.09	0.07
	3	545	534	−0.03	0.12	0.02	−0.17	0.96	0.52	0.26	0.15
	14	841	869	0.01	−0.03	0.02	−0.16	0.79	0.42	0.19	0.15
NGC 7788	15	54	5	0.16	0.64	0.33	0.22	0.36	0.27	0.04	0.02
	16	288	232	−0.15	0.01	0.00	0.04	0.64	0.58	0.37	0.31
NGC 7790	15	76	9	−0.31	−0.19	0.24	0.24	0.34	0.31	0.11	0.03
	16	355	283	0.02	0.02	0.09	−0.09	0.75	0.59	0.35	0.27

members from field stars is poor. In conclusion, one may say that for the reliable separation of cluster members from field stars, we should have $\Phi_c \gg \Phi_f$ for cluster stars and $\Phi_c \ll \Phi_f$ for field stars.

We estimate p for a given (μ_x, μ_y) using the different parameters of a cluster listed in table 3. Table 4 lists such estimates for NGC 129 and 581. This indicates that the effect is negligible if $p \geqslant 0.80$ and is significant if p is in the middle range. McNamara & Schneeberger (1978) have also observed a similar effect for M 11. High and middle-range membership probability stars are located respectively near the peak and the wings of the assumed circular Gaussian frequency distribution of cluster stars. Therefore, in contrast to high probability stars, the middle-range ones are very much more sensitive to reduction techniques and measurement errors. This is further supported by the following two observations :

(i) Some stars common in proper motion investigations of NGC 581 and Tr 1 have $p \geqslant 0.50$ for both the clusters. But a star cannot be member of more than

Table 4. Effect of different cluster parameters on the assignment of membership probability (p) of a star. Reference numbers are the same as in table 1. μ_x and μ_y are in unit of arcsec per century

Cluster name	μ_x	μ_y	p (%) based on parameters from		
			Ref. 1	Ref. 2	Ref. 3
NGC 129	0.00	0.20	33	40	71
	−0.05	0.15	56	52	73
	−0.10	0.10	69	56	74
	−0.10	0.05	74	57	73
	−0.10	0.00	75	57	72
	−0.15	0.10	67	56	74
	−0.15	0.05	72	58	73
	−0.15	0.00	74	58	72
	−0.20	0.10	61	56	74
	−0.20	0.05	67	57	73
	−0.20	0.00	69	58	72
	−0.25	0.15	40	53	73
	−0.25	0.10	02	24	52
	−0.25	0.00	03	25	49
	−0.25	−0.05	02	24	47
NGC 581			Ref. 3	Ref. 4	
	0.00	0.00	89	81	
	−0.05	−0.05	89	84	
	−0.05	0.05	88	83	
	−0.05	−0.10	89	83	
	−0.10	−0.05	88	86	
	−0.10	0.05	87	85	
	−0.10	−0.10	88	85	
	−0.15	0.10	86	85	
	−0.15	0.00	87	87	
	−0.15	−0.10	87	87	
	−0.21	−0.10	86	87	
	−0.21	0.00	86	88	
	−0.21	0.10	84	86	
	−0.40	−0.20	77	80	
	−0.40	−0.10	78	78	
	−0.40	0.10	75	81	

one cluster. (ii) Using Frolov's (1979) proper motion study of NGC 7788 and 7790, Zhao *et al.* (1985) estimated parameters only on the basis of stars considered members by Frolov (1979) instead of using all stars and found only 6 and 9 members respectively in NGC 7788 and 7790 in comparison to 59 and 85 members found by Frolov (1979).

Both these points underline the effect of parameters on determining the membership probability.

5. Comparison of membership probability and relative proper motion

The minimum, maximum, mean and sigma of the differences in p, μ_x and μ_y of the stars common in two proper motion studies of a cluster are given in table 5. The differences are always in the sense reference A minus reference B. The histograms and the Gaussion fits are shown in figure 1 for NGC 129, 581 and 7092. The Gaussian fits to the histograms of $\Delta\mu_x$ and $\Delta\mu_y$ are always better than that of Δp. The distribution of Δp peaks strongly near $-0.10 < \Delta p < 0.10$ and has fewer values with $| \Delta p | \geqslant 0.5$. The values of p assigned in the two studies are generally in fair agreement, except for NGC 7788 and NGC 7790 (*see* section 4). However, in other clusters also for a few stars, the agreement is far from satisfactory i.e. $| \Delta p | \geqslant 0.70$. For these stars proper motion components in the two studies differ significantly but lie closer to the peak of Gaussian frequency distribution of cluster stars in one study and far from that in the other study.

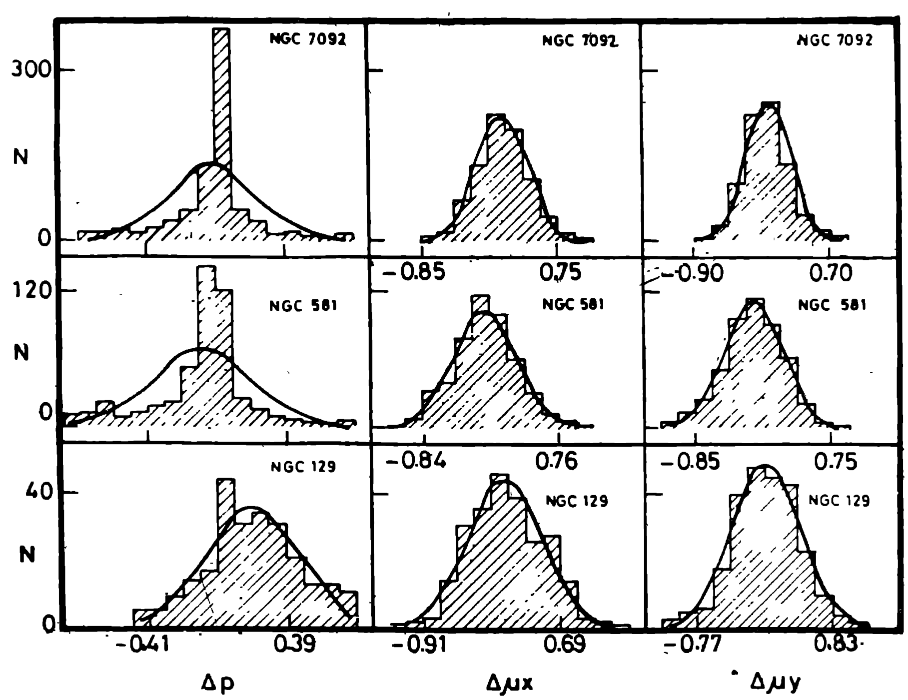

Figure 1. The histograms of differences in p, μ_x and μ_y of stars common in the two proper motion studies of a cluster. Continuous curve is the fitted Gaussian taking mean and sigma values from table 5. Data sources are as follows : NGC 7092 : McNamara & Sanders (1977) and Lavdovskij (1961); NGC 581 : Oja (1966) and Lavdovskij (1961); NGC 129 : Lavdovskij (1961) and Frolov (1975).

Table 5. Statistical results of a comparison of the two proper motion studies. Reference numbers are the same as in table 1. Differences are in the sense ref. A minus ref. B. Membership probability is p; x and y components of proper motion are μ_x and μ_y respectively.

Cluster name	Ref. A	Ref. B	Difference in p Min	Mean ± Sigma	Max	Difference in μ_x arcsec century^{-1} Min	Mean ± Sigma	Max	Difference in μ_y arcsec century^{-1} Min	Mean ± Sigma	Max
NGC 129	2	3	−0.67	−0.07±0.28	0.58	−1.43	−0.09±0.39	2.31	−1.30	0.11±0.43	1.01
	3	1	−0.71	−0.01±0.32	0.77	−1.46	0.22±0.37	1.19	−1.41	−0.11±0.31	0.50
	3	2	−0.51	−0.19±0.29	0.77	−1.31	0.07±0.46	2.06	−1.17	0.50±0.40	1.15
NGC 581	4	3	−0.91	−0.09±0.28	0.87	−1.60	−0.10±0.38	1.30	−1.50	−0.17±0.36	1.10
Tr 1	4	3	−0.84	0.03±0.29	0.85	−1.50	0.05±0.41	1.60	−1.20	−0.08±0.42	1.60
NGC 2099	7	6	−0.78	0.27±0.32	0.92	−0.87	−0.12±0.31	1.02	−1.85	−0.06±0.33	0.88
	7	5	−0.94	−0.08±0.19	0.85	−0.45	−0.01±0.17	0.53	−1.77	0.02±0.21	0.49
	6	5	−0.99	−0.32±0.27	0.77	−0.86	0.14±0.29	0.94	−1.92	0.13±0.32	0.96
NGC 2168	3	8	−0.95	−0.07±0.29	0.85	−0.90	0.09±0.37	0.88	−1.04	−0.07±0.31	3.10
NGC 2682	10	9	−0.99	−0 13±0.43	0.96	−1.72	0.14±0.37	2.10	−5.09	−0.06±0.49	1.58
	11	9	−0.97	−0.05±0.51	0.97	−1.91	0.10±0.48	2.06	−5.03	−0.05±0.51	2.47
	11	10	−0.99	−0.04±0.13	0.93	−0.91	0.01±0.15	1.52	−2.19	0.02±0.13	1.66
	12	10	−0.99	−0.21±0.30	0.92	−1.30	−0.05±0.29	1.20	−0.85	−0.01±0.28	1.68
	11	12	−0.93	0.00±0.34	0.97	−3.53	0.06±0.37	5.01	−4.08	0.00±0.44	1.86
NGC 7092	14	3	−0.89	−0.03±0.26	0.89	−2.52	0.10±0.32	1.16	−0.91	−0.03±0.29	1.43
	14	13	−0.63	0.07±0.35	0.88	−0.78	−0 03±0.31	0.86	−3.04	−0.65±0.78	0.74
	3	13	−0.64	0.08±0.41	0.88	−1.03	−0.06±0.36	0.56	−2.54	−1.01±0.66	0.13
NGC 7788	16	15	−0.49	0.49±0.29	0.71	−0.62	0.13±0.44	1.12	−2.04	−0.18±0.50	0.47
NGC 7790	16	15	−0.44	0.50±0.32	0.75	−0.96	−0.09±0.54	1.71	−2.30	−0.43±0.50	0.27

The difference in the proper motion components of a star in the two studies is either due to systematic or random errors in the observations or both. Systematic errors may arise from errors in the reduction of data, whereas random errors depend upon the accuracy of the data. Presence of systematic errors is reflected in the plots of Δp, $\Delta\mu_x$ and $\Delta\mu_y$ against spatial position and brightness. For this, we have fitted the linear relations to the $(\Delta\mu_x, x)$, $(\Delta\mu_x, y)$, $(\Delta\mu_x, V)$, $(\Delta\mu_y, x)$, $(\Delta\mu_y, y)$, $(\Delta\mu_y, V)$, $(\Delta p, x)$, $(\Delta p, y)$ and $(\Delta p, V)$ plots using least square techniques and also estimated the correlation coefficients (*see* table 6). For illustration, we plot the cases of correlation coefficient (r) equal to 0.01, 0.62 and 0.87 in figure 2. Except for the two plots of $(\Delta\mu_y, y)$ in NGC 7092; $r \leqslant 0.6$ always, indicating that assumed linear relation has a statistical significance level generally $\leqslant 36\%$. In NGC 7092, it appears that systematic error is present only in the μ_y component of the proper motion data from Ebbighausen (1940b) (*see* table 1) because only its comparison with other two proper motion studies of the cluster shows the systematic difference.

The reasonable Gaussian fitting to most of the histograms of $\Delta\mu_x$ and $\Delta\mu_y$ indicates that differences are mainly due to random errors. This is further supported by the fact that except for the two cases of μ_y components in NGC 7092, mean of $\Delta\mu_x$ and $\Delta\mu_y$ (*see* table 5) are always less than their error σ expected

Table 6. The correlation coefficients of the assumed linear relations in the plot of the differences $\Delta\mu_x$, $\Delta\mu_y$ and Δp of stars common in the two proper motion studies of a cluster against their spatial position and brightness for the clusters under discussion. Reference numbers are the same as in table 1

Cluster name	Ref.		Correlation coefficients of the assumed linear relations between								
	A	B	$(\Delta\mu_x, x)$	$(\Delta\mu_y, y)$	$(\Delta\mu_x, m)$	$(\Delta\mu_y, x)$	$(\Delta\mu_y, y)$	$(\Delta\mu_y, m)$	$(\Delta p, x)$	$(\Delta p, y)$	$(\Delta p, m)$
NGC 129	3	1	0.32	0.17	0.23	0.16	0.13	0.21	0.06	0.13	0.08
	2	1	0.10	0.27	0.15	0.53	0.23	0.62	0.21	0.20	0.25
	3	2	0.29	0.35	0.36	0.24	0.03	0.31	0.10	0.04	0.10
NGC 581	4	3	0.12	0.01	0.20	0.06	0.05	0.24	0.09	0.03	0.09
Tr 1	4	3	0.08	0.05	0.06	0.05	0.14	0.21	0 06	0.00	0.04
NGC 2099	7	6	0.14	0.16	0.36	0.08	0.21	0.43	0.01	0.04	0.35
	7	5	0,38	0.23	—	0.18	0.09	—	0.09	0.15	—
	6	5	0.00	0.03	0.22	0.08	0.04	0.33	0.00	0.14	0.37
NGC 2168	3	8	0.05	0.01	0.06	0.01	0.22	0.01	0.03	0.01	0.02
NGC 2682	10	9	0.00	0.18	—	0.10	0.11	—	0.07	0.04	—
	11	9	0.04	0.04	0.05	0.01	0.06	0.05	0.01	0.01	0.06
	11	10	0.13	0.03	0.18	0.08	0.05	0.08	0.01	0.22	0.17
	12	10	0.01	0.06	0.19	0.09	0.14	0.40	0.01	0.17	0.32
	11	12	0.18	0.04	0.18	0.05	0.02	0.13	0.14	0.04	0.07
NGC 7092	14	3	0.26	0.17	0.27	0.13	0.18	0.20	0.02	0.08	0.05
	14	13	0.56	0.62	0.04	0.39	0.82	0.25	0.16	0.15	0.43
	3	13	0.01	0.01	0.24	0.12	0.87	014	0.19	0.38	0.51
NGC 7788	16	15	0.09	0.26	0.14	0.04	0.17	0.16	0.00	0.04	0.13
NGC 7790	16	15	0.06	0.12	0.06	0.06	-0.21	0.38	0.09	0.36	0.14

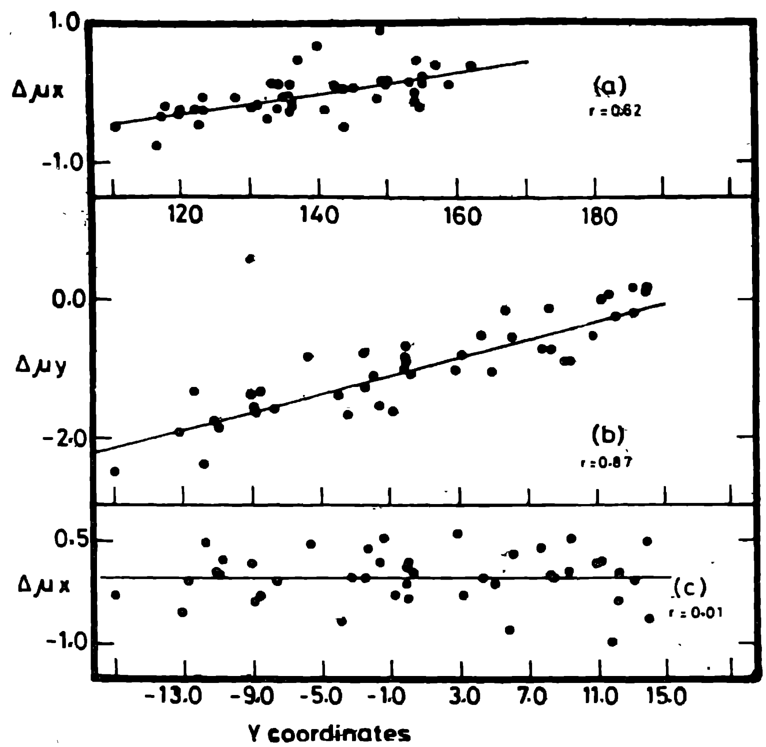

Figure 2. $\Delta\mu_x$ and $\Delta\mu_y$ of stars common in the two proper motion studies of NGC 7092 plotted against y spatial coordinate taken from McNamara & Sanders (1977) in (a) and from Ebbighausen (1940b) in (b) and (c). The line is the fitted least square linear relation and r is the correlation coefficient. Data sources are as follows : (a) McNamara & Sanders (1977) and Ebbighausen (1940b); (b) and (c) Lavdovskij (1961) and Ebbighausen (1940b).

from the errors in the values of μ_x and μ_y i.e. $\sigma^2 = \sigma_A^2 + \sigma_B^2$, where σ_A and σ_B are the observational errors in the reference A and B respectively.

6. Conclusion

An analysis of the data on proper motion and membership probabilities of stars in the open clusters under study indicates the following :

(i) A difference in the observational errors, the procedure of pruning the vector point diagram, etc., in two proper motion studies of a cluster may give different values of the parameters describing the frequency distribution of cluster and field stars which can strongly affect the middle-range membership probabilities i.e. p between 0.25–0.75 in contrast to higher membership i.e. $p \geqslant 0.8$, where the effects are very little. Therefore, one should consider these facts in interpreting the membeaship probability in cluster studies because owing to strong dependence of membership probability on parameters marginal members in one study can be firm members in the other study.

(ii) The differences in the proper motion components (μ_x, μ_y) of a star in the two studies of a cluster are generally understood in terms of random errors in observations except for NGC 7092 where the data from Ebbighausen (1940b) (*see* table 1) seem to be in systematic error. It means that (μ_x, μ_y) data obtained by various investigators of a cluster can be combined. In the case of NGC 7092, presence of systematic error should be properly accounted for. Stars present in more than one study will have relatively accurate proper motion estimates in the combined data and hence such data will ultimately improve the segregation of cluster members from field stars. The combined data will also reduce the problem of incompleteness present in the individual proper motion study of a cluster, as one study differs from another either in area investigated or in limiting magnitude of the observations or in both. This will ultimately help in tackling the cluster studies like initial mass function, cluster dynamics, etc., requiring information about all cluster members upto a certain brightness. However, it may not have noticeable effect on the global properties of a cluster, i.e. on the estimation of cluster distance, reddening, age, etc.

Acknowledgement

I thank an anonymous referee for useful comments.

References

Cudworth, K. M. (1971) *Astr. J.* **76**, 475.
Ebbighausen, E. G. (1940a, b) *Ap. J.* **91**, 244; **92**, 434
Frolov, V. N. (1975, 1979) *Bull. Pulkovo Astr. Obs. No. 193*, 80; *196*, 69.
Ishmukhamedov, K. (1966) *Circ. Astr. Obs. Tashkent No. 345*, 1.
Jefferys, W. H. (1962) *Astr. J.* **67**, 532.
Joy, A. H. (1916) *Astr. J.* **29**, 101.
Lavdovskij, V. V. (1961) *Trans. Pulkovo Astr. Obs.* **73**, 5.
Lenham, A. P. & Franz, O. G. (1961) *Astr. J.* **66**, 16.
Mathieu, R. D. (1983) *Ph.D. Thesis*, Univ. of California, Berkeley.
McNamara, B. J. & Sanders, W. L. (1977) *Astr. Ap. Suppl.* **30**, 45
McNamara, B. J. & Schneeberger, T. J. (1978) *Astr. Ap.* **62**, 449.
Oja, T. (1966) *Ark. Astr.* **4**, 15.
Sanders, W. L. (1971, 1972) *Astr. Ap.* **14**, 226; **16**, 58.
Sanders, W. L. (1976, 1977) *Astr. Ap.* **46**, 131: *Astr. Ap. Suppl.* **27**, 89.
Tian, K. P., Yin, M. G., Xu, Z. H. & Zhao, J. L. (1982) *Ann. Shanghai Obs.*, Acad. Sinica **4**, 17.
Upgren, A. R. (1966) *Astr. J.* **71**, 736.
Van Leeuwen, F. (1985) *IAU Symp. No. 113*, p. 579.
Van Maanen, A. (1952) *Ap. J.* **96**, 382.
Vasilevskis, S. (1962) *Astr. J.* **67**, 699.
Vasilevskis, S., Klemola, A. & Preston, G. (1958) *Astr. J.* **63**, 387.
Zhao, J. L., Tian, K. P., Xu, Z. H. & Yin, M. G. (1982a, b) *Chin. Astr. Ap.* **6**, 293; **6**, 344.
Zhao, J. L., Tian, K. P., Jing, J. Y. & Yin, M. G. (1985) *Tables of Membership of 42 Open Clusters*, Zi-Ka-Wei Section, Shanghai Obs., Acad. Sinica, Shanghai.