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Tidal interaction between a disc galaxy and a spherical galaxy

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Abstract. Tidal interaction between a disc and a Plummer model spherical galaxy is considered. Fractional change in the internal energy and merger velocities for the disc-sphere pair is derived under impulsive approximation for various orientations of the disc. It is found that the ratio of the actual velocity of escape taking the tidal effects into account and the parabolic velocity of escape of the two galaxies at zero separation for head-on collision is practically independent of density model and inclination of the disc. It is shown that in the model considered the chance of two spherical galaxies merging in a head-on collision due to tidal capture is greater than that of a disc-sphere pair.

Key words : Galaxies, interaction—stellar dynamics

1. Introduction

The study of interacting galaxies has attracted considerable attention in recent years (*see e.g.* Tremaine 1981; Alladin & Narasimhan 1982; and White 1983). An important aspect in the study of galaxy interactions has been the determination of merger velocities, *i.e.*, the maximum relative velocity with which the two galaxies should approach each other from a large distance in order to form a double system, as a result of encounter. Following Rood (1965), we shall call this the capture velocity V_{cap}. Alternatively one may enquire what is the minimum relative velocity at closest approach needed for two galaxies to become unbound. We refer to this velocity as critical velocity of escape, V_{crit}. In the case of two mass points, V_{crit} would be identical with parabolic velocity. But for extended bodies the critical velocity is greater than parabolic velocity on account of inelastic nature of galactic encounters. Our present discussion of merger velocity includes both V_{cap} and V_{crit} for sphere-disc pair of galaxies. Galaxy interaction and merger velocities have been studied using impulsive approximation and restricted three body problem approach and N-body simulations for pairs of galaxies comprising spherical or disc galaxies.

The impulsive approximation, where we neglect the motion of the stars in the galaxies in comparison with relative orbital motion of the galaxies has been used by Spitzer (1958), Alladin (1965) and several others to study tidal effects in galaxies. Miller & Smith (1980) observe that this approximation is not valid for relative velocities less than 1000 km s^{-1} for typical galaxies. Nevertheless studies with impulsive approximation seem to give surprisingly accurate results even for slow hyperbolic collision (Toomre 1977; Dekel, Lecar & Shaham 1980). Aguilar & White's (1985) study of the tidal interaction between spherical galaxies using impulsive approximation and N-body experiments showed that the impulsive approximation gives fairly accurate results for changes in total mass and binding energy even in slow collision ($V_\infty \approx V_{\rm rms}$).

Merger velocities have been determined by this method for spherical galaxies by Sastry (1972); Alladin, Potdar & Sastry (1975); Toomre (1977); and Sastry & Alladin (1983). Chatterjee (1984) studied the problem of formation of ring galaxies in a head-on collision between a disc galaxy and a spherical galaxy using impulsive approximation.

Yabushita (1977) has studied the possibility of formation of galactic binary system by tidal interaction between a spiral galaxy and a disc galaxy of identical mass using the method and restricted three-body problem. Palmer & Papaloizou (1982) derived analytical expressions for transfer of energy and angular momentum in an encounter between a disc galaxy (cold and hot) and a point mass perturber. Palmer (1982) finds that the impulsive approximation breaks down only when the velocity of perturbing galaxy is of the order of the rotational velocity of the disc galaxy.

Roos & Norman (1979) give analytical expressions for both capture velocity and critical velocity of escape based on the results from N-body simulations for spherical galaxies. Aarseth & Fall (1980) have summarized the results of merger velocities obtained from N-body simulations. Farouki & Shapiro (1982) find that the head-on collision between two identical disc-halo galaxies leads to a merger if the relative velocity at closest approach does not exceed 1.10 times the escape velocity when the discs are co-planar with the orbit.

The impulsive approximation has so far been applied to spherical galaxies. The aim of this paper is to extend impulsive approximation to disc galaxies with a view to determine the model dependence of the earlier results. For this purpose we develop the necessary potential theory in section 2. New analytical expressions for stellar velocity perturbations are given in section 3 (Zafarullah, Narasimhan & Sastry 1983). Finally in section 4 we derive merger velocities and compare our results with the earlier ones.

2. Potential Theory

The spherical galaxy is represented by a Plummer model with mass distribution

$$M_{\rm S}(r) = M_{\rm S}\left(1 + \frac{\epsilon^2}{r^2}\right)^{-3/2} \qquad \ldots(1)$$

truncated at the radius R_S. The scale length is chosen $\epsilon = 0.107\ R_s$ so that the potential

$$V_S(r) = -GM_S(r^2+\epsilon^2)^{-1/2} \qquad ...(2)$$

closely resembles that of a polytrope of index $n = 4$ (Zafarullah *et al.* 1983). The total internal energy of the Plummer model galaxy is (Ahmed 1979)

$$|U| = \frac{3\pi}{64}\frac{GM_s^2}{\epsilon}. \qquad ...(3)$$

The disc galaxy is represented by a thin exponential disc, with surface density distribution (Chatterjee 1984)

$$\sigma(\omega) = \sigma_C \exp(-4\omega/R_D), \qquad ...(4)$$

where ω is the distance measured from the centre along the plane of the disc; and σ_C and R_D are respectively the central density and radius of the disc. An exponential disc is most suitable for spiral galaxies (Freeman 1975). However, computer models of exponential discs have shown that these are also unstable unless they are surrounded by massive halos. Since we are concerned with stability against tidal forces only, we shall assume the disc to be thin. Following Ballabh (1973) and Chatterjee (1984), the potential V_D at any point for an exponential model distribution represented by equation (4) is

$$V_D = -\frac{GM_D}{R_D}\Phi(\alpha, \theta), \qquad ...(5)$$

where $\alpha = r/R_D$ and (r, θ, φ) are the spherical polar coordinates of the point with respect to the centre of the disc. The function Φ takes into account the extended nature of the disc galaxy. The expressions for computing $\Phi(\alpha, \theta)$ for a particular density distribution are given by the equations (13) and (14) in Ballabh (1973). The function $\Phi(\alpha, \theta)$ is tabulated in Zafarullah (1984). By taking the gradient of the equation (5) we obtain the following expressions for the force parallel F_ω and perpendicular F_z to the plane of the disc :

$$F_\omega = -\frac{GM_D}{R_D^2}\Psi_\omega(\alpha, \theta),$$

$$F_z = -\frac{GM_D}{R_D^2}\Psi_\omega(\alpha, \theta), \qquad ...(6)$$

where

$$\Psi_\omega = -\frac{1}{f}\left[\sin\theta\sum_{j=1}^{5} a_j M_j + \frac{\cos\theta}{\alpha}\sum_{j=1}^{5} a_j N_j\right],$$

$$\Psi_z = \frac{-1}{f}\left[\cos\theta\sum_{j=1}^{5} a_j M_j - \frac{\sin\theta}{\alpha}\sum_{j=1}^{5} a_j N_j\right]. \qquad ...(7)$$

when $\alpha \leqslant 1$ $(r \leqslant R_D)$

$$M_j(\alpha, \theta) = j\Big[\frac{j+3}{2(j+2)}\alpha^j + \frac{1-(j+1)\alpha^j}{j} P_1(\cos\theta) + \sum_{\substack{n=1 \\ n\neq (j+1)/2}}^{\infty} \Big\{\frac{(j+1)\alpha^j - 2n\alpha^{2n-1}}{2n-j-1} A_n\Big\} - \alpha^j \{(j+1)\ln(\alpha)+1\} \times \underset{[n\neq(j+1)/2]}{A_n} - (j+1)\alpha^j \sum_{n=1}^{\infty} \frac{B_n}{2n+j+2}\Big] \qquad \ldots(8)$$

$$N_j(\alpha, \theta) = j\Big[\frac{\alpha(1-\alpha^j)}{j} P_1' + \sum_{\substack{n=1 \\ n\neq (j+1)/2}}^{\infty} A_n' \Big(\frac{\alpha^{j+1} - \alpha^{2n}}{2n-j-1}\Big) - \alpha^{j+1}\ln(\alpha)\, A'_{n\,[=(j+1)/2]} - \alpha^{j+1}\sum_{n=1}^{\infty}\frac{B_n'}{2n+j+2}\Big](-\sin\theta). \qquad \ldots(9)$$

When $\alpha > 1$ $(r > R_D)$

$$M_j(\alpha, \theta) = j\Big[\frac{1}{2(j+2)\alpha^2} + \sum_{n=1}^{\infty}\frac{B_n(2n+1)}{(2n+j+2)\alpha^{2n+2}}\Big] \qquad \ldots(10)$$

$$N_j(\alpha, \theta) = j\Big[\sum_{n=1}^{\infty}\frac{B'}{(2n+j+2)\alpha^{2n+1}}\Big]\sin\theta, \qquad \ldots(11)$$

wherein the functions $P_n(\theta)$, $A_n(\theta)$, and $B_n(\theta)$ and the constant f are defined in Ballabh (1973). Prime denotes derivative with respect to cos θ. The functions Ψ_ω and Ψ_z, for a few values of θ, are plotted in figures 1 and 2 and tabulated in Zafarullah (1984). The internal energy of an exponential galaxy U_D is

$$|U_D| = \frac{GM_D^2}{2R_D} U_{het}, \qquad \ldots(12)$$

where U_{het} is a constant depending upon the density distribution of the disc galaxy and is tabulated in Ballabh (1973).

3. Energy transfer

We now derive expressions for increments, ΔV, in the velocity of representative stars in a galaxy due to tidal effects of perturber and use these to obtain the increase

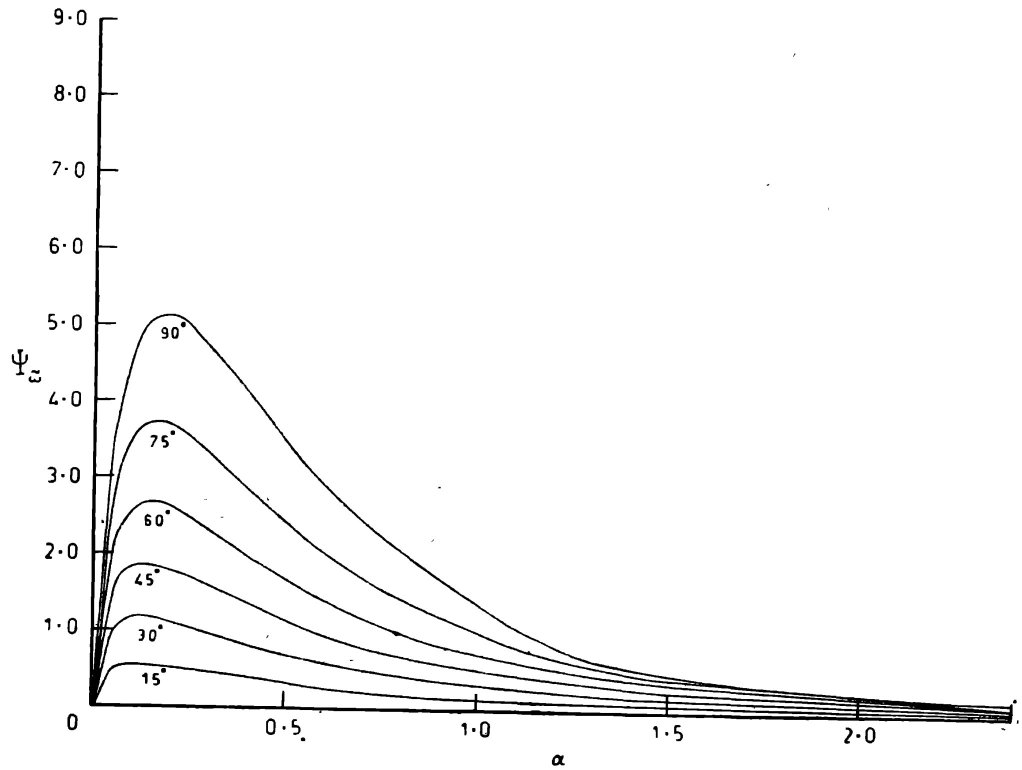


Figure 1. Variation of Ψ_ω (α, θ).

in the binding energy, ΔU_1 and ΔU_2, of the two galaxies (*cf.* Zafarullah *et al.* 1983). Finally, we derive the merger velocities. We first consider the effects on the disc galaxy of an encounter with a spherical galaxy. The tidal acceleration of a star in the disc galaxy is

$$\mathbf{f}_T = \mathbf{f}_X - \mathbf{f}_G, \quad \text{...(13)}$$

where $\mathbf{f}_X$ and $\mathbf{f}_G$ are gravitational force per unit mass acting on the star and the galaxy as a whole due to perturber. The change in the velocity of the representative star as a result of the encounter is

$$\Delta \mathbf{V} = \int_{-\infty}^{+\infty} \mathbf{f}_T \, dt. \quad \text{...(14)}$$

Choosing a coordinate system as shown in figure 3, we obtain the following expressions for velocity increments of stars in the disc galaxy due to spherical galaxy of mass M_S, radius R_S, (Zafarullah *et al.* 1983)

$$\Delta V_x = -\frac{2GM_S}{VR_S}\left[\frac{x'}{(p-y')^2 + x'^2 + \epsilon_S^2}\right],$$

$$\Delta V_y = -\frac{2GM_S}{VR_S}\left[\frac{p-y'}{(p-y')^2 + x'^2 + \epsilon_S^2} - \frac{p}{p^2 + \epsilon_{SD}^2}\right],$$

$$\Delta V_z = 0, \quad \text{...(15)}$$

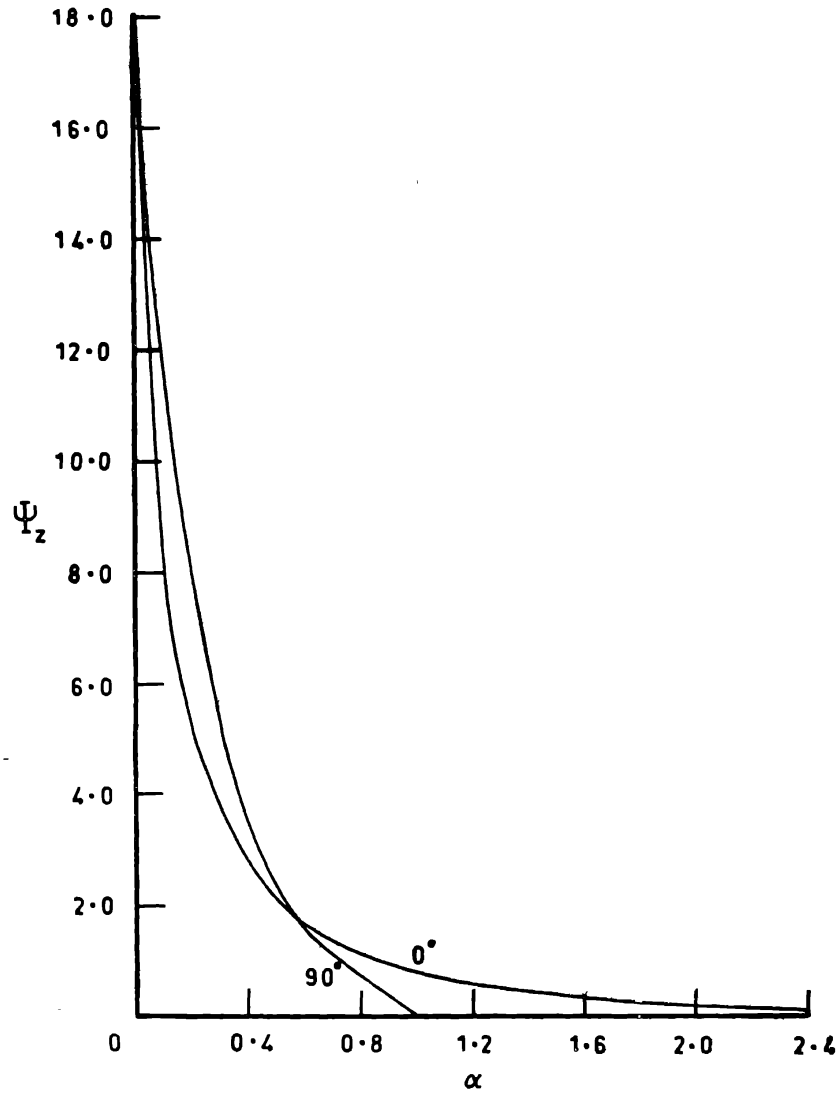


Figure 2. Variation of $\Psi'_z(\alpha, \theta)$.

where p is the distance of closest approach; V the relative velocity of the galaxies at p; x', y', z' are the coordinates of the representative star; $\epsilon_S = 0.107\ R_S$; and ϵ_{SD} the softening parameter for sphere-disc pair, is obtained from

$$\epsilon_{SD} = \left[\left(\frac{\Psi'_{SD}}{r}\right)_{r=0}\right]^{-1} \qquad \text{...(16)}$$

where Ψ'_{SD} is the function required for obtaining potential energy of galaxies (*also see* Ballabh 1973). ΔV_z is always zero since we assume that the perturbing galaxy is moving parallel to z-axis with uniform speed from $-\infty$ to $+\infty$. The increase in the internal energy, ΔU_D, of the disc galaxy is

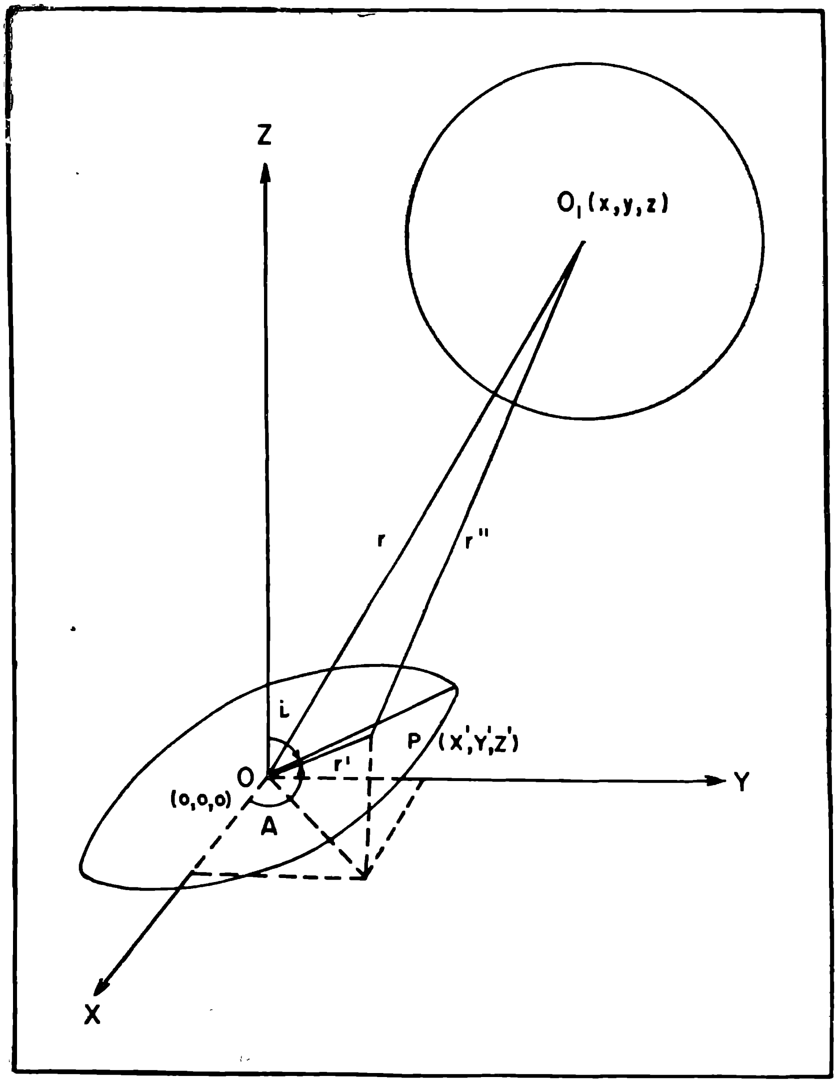


Figure 3. Position of a representative star in the disc galaxy.

$$\Delta U_D = \int_0^1 \langle \Delta U_D(s') \rangle \frac{dM_D(s')}{ds'} ds'$$

$$= \frac{M_D}{2f} \int_0^1 \langle (\Delta V(s')^2 \rangle s' e^{-4s'} ds', \qquad \ldots(17)$$

where $\Delta U_D(s')$ is the increase in the internal energy of the star per unit mass, at distance r' from the centre of the disc galaxy; $s' = r'/R_D$; and $M_D(s')$ is the mass of the disc within s'. The fractional change in the internal energy of the disc galaxy is

$$\frac{\Delta U_D}{|U_D|} = \frac{R_D \int_0^1 \langle (\Delta V(s'))^2 \rangle s' e^{-4s'} ds'}{G M_D^2 U_{het}}. \qquad \ldots(18)$$

We consider a head-on collision ($p = 0$) between a Plummer model spherical galaxy of mass $10^{11} M_\odot$ and radius 20 kpc ($\epsilon_S = 0.107\ R_S$ and half mass radius $R_h = 0.103\ R_S$) and a thin exponential disc galaxy of mass $10^{11}\ M_\odot$, radius 20 kpc ($\epsilon_D = 0.4\ R_D$ and $R_h = 0\ 378\ R_D$). We compute ΔU_D for various inclinations of the disc with respect to the relative motion of the galaxies ($i = 0°$, 45° and 90°). To obtain the integral in equation (18) we divide the disc into sets of stars, each set being characterized by the common distance s' from the centre. Each set consists of 16 stars. These stars are chosen at intervals of azimuthal angle $A = n\pi/8$ where $n = 1, 2, \ldots, 16$. The stars are chosen at distances $s' = 0.05, 0.10, \ldots, 1.0$. The coordinates ($x'$, y', z') of a representative star in the disc are obtained from (see figure 3)

$$x' = s' \cos A,$$
$$y' = s' \sin A \sin i,$$
$$z' = s' \sin A \cos i.$$

Turning our attention to the effects on the spherical galaxy, we note that the acceleration of a star is given by equation (13) wherein $\mathbf{f}_X$ and $\mathbf{f}_G$ are in this case the acceleration of the star and spherical galaxy caused by the disc. The stellar velocity increments are derived using the force components F_ω and F_z (equation 6).

To obtain velocity increments $\Delta \mathbf{V}$ of the stars, we divide the sphere into shells of 14 stars each as shown in table 1. $\Delta V'_s$ are obtained for three orientations of the disc galaxy ($i = 0°$, 45°, and 90°) in head-on collision ($p = 0$). We obtain the fractional change in the binding energy of spherical galaxy $\Delta U_S / \mid U_S \mid$ from $\Delta N'^s$ as explained in Zafarullah *et al.* (1983).

Table 1. Positions of the representative stars in the spherical galaxy

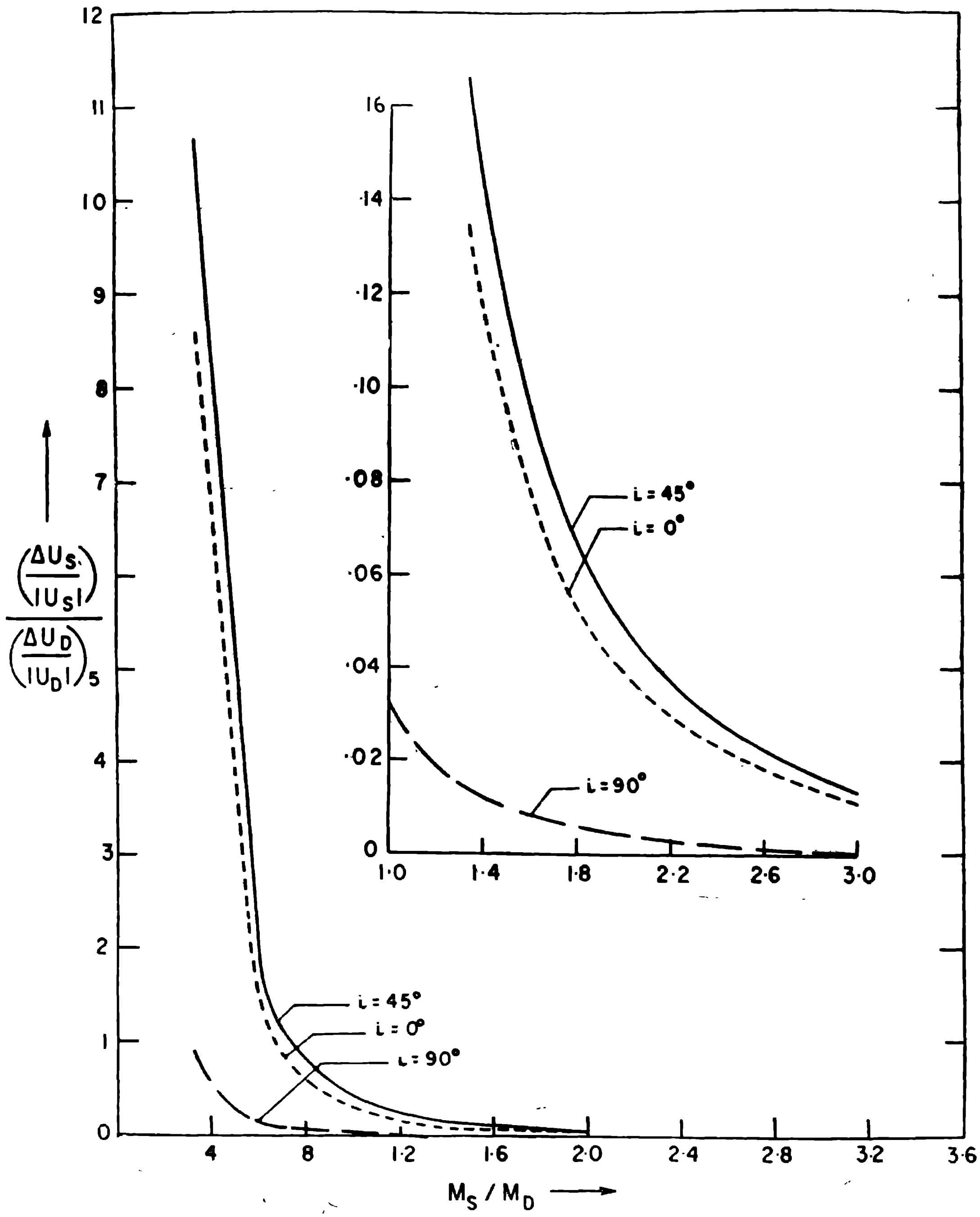
Star No.	x'/R_S	y'/R_S	z'/R_S
1	$s'/\sqrt{3}$	$s'/\sqrt{3}$	$s'/\sqrt{3}$
2	$s'/\sqrt{3}$	$-s'/\sqrt{3}$	$s'/\sqrt{3}$
3	$-s'/\sqrt{3}$	$s'/\sqrt{3}$	$s'/\sqrt{3}$
4	$-s'/\sqrt{3}$	$-s'/\sqrt{3}$	$s'/\sqrt{3}$
5	$-s'/\sqrt{3}$	$-s'/\sqrt{3}$	$s'/\sqrt{3}$
6	$-s'/\sqrt{3}$	$s'/\sqrt{3}$	$-s'/\sqrt{3}$
7	$s'/\sqrt{3}$	$s'/\sqrt{3}$	$-s'/\sqrt{3}$
8	$s'/\sqrt{3}$	$s'/\sqrt{3}$	$-s'/\sqrt{3}$
9	s'	0	0
10	$-s'$	0	0
11	0	s'	0
12	0	$-s'$	0
13	0	0	s'
14	0	0	$-s'$

From the numerical work (Zafarullah 1984) we draw the following conclusions:

(i) Stars in the inner regions of the disc galaxy ($r' < R_h$) are affected more than those in the outer region ($r' > R_h$). This is in agreement with the results of Yabushita (1977) and Farouki & Shapiro (1981).

(ii) For $i = 0$, the stars in the YZ plane ($A = 90°$, $270°$) are hardly affected, while for $i = 45°$ these stars are affected the most. For $i = 90°$ all the stars are equally affected, and since inner stars are accelerated more than the outer ones, this leads to formation of ring galaxy (Chatterjee 1984).

It thus follows that a face-on collision ($i = 90°$) leads to the formation of rings while edge-on collision ($i = 0°$) leads to a drastic overhaul in the distribution of



Figuer 4. Variation of $\frac{\Delta U_S}{|U_S|} \Big/ \frac{\Delta U_D}{|U_D|}$ with $\frac{M_s}{M_D}$.

stars. After such a collision ($i = 0°$) there would be enormous mixing of stars between the inner and outer parts of the galaxy. In figure 4 we plot the ratio of fractional change in the binding energy of the two galaxies,

$$\mu_{SD} = \frac{\Delta U_S}{|U_S|} \Big/ \frac{\Delta U_D}{|U_D|}$$

as a function of ratio of mass M_S/M_D of the galaxies for three inclinations of the disc galaxy. It is seen from this figure that $\mu_{SD} = 1$ if $M_S/M_D = 0.6, 0.8$, and 0.3 for the above case. For a pair of sphere and disc galaxies of identical mass and radius, the disc galaxy is more affected than the spherical galaxy as the latter is more centrally concentrated. The effect is maximum for $i = 45°$ and minimum for $i = 90°$. We find that the fractional change in the binding energy $\Delta U/ \mid U \mid$ for sphere-sphere collision lies between sphere-disc and disc-sphere collisions for equal mass and radius of the galaxies, *i.e.*,

$$[\Delta U/ \mid U \mid]_{SD} > [\Delta U/ \mid U \mid]_{SS} > [\Delta U/ \mid U \mid]_{DS}$$

where $[\Delta U/ \mid U \mid]_{SD}$ is the value of $\Delta U/ \mid U \mid$ for disc due to tidal effects of sphere.

In a typical case of head-on collisions between two identical sphere-sphere and sphere-disc pairs of galaxies with $M_S = M_D = 10^{11} M_\odot$ and radii $R_S = R_D = 20$ kpc and $V = 1000$ km s^{-1} we obtain $(\Delta U/ \mid U \mid)_{SD} = 0.6$, $(\Delta U/ \mid U \mid)_{SS} = 0.4$ and $(\Delta U/ \mid U \mid)_{DS} = 0.2$ for $i = 0°$. $\Delta U/ \mid U \mid$ for any other M_1, M_2, R, and V can be obtained by scalling as

$$\left(\frac{\Delta U}{|U|}\right)_2 \propto \frac{M_1^2}{M_2 V^2 R}$$

where M_1 and M_2 represent the masses of perturbing and perturbed galaxies of identical radius R. The values of internal energy of sphere, U_S, and disc, U_D, are -5.95×10^{58} and -2.96×10^{58} C.G.S. units respectively.

In passing we make a comment on the angular momentum of the disc galaxy. Velocity increments perpendicular to the plane of the disc due to collision result in an angular momentum about x-axis. We find that this angular momentum is maximum for $i = 45°$ and is zero for $i = 0°$ and 90°.

4. Merger velocities

In this section, we calculate the merger velocities of a disc-sphere pair of galaxies for different orientations of the disc with respect to the direction of relative motion ($i = 0°$, 45°, and 90°).

Following Sastry & Alladin (1983), who have computed the merger velocities for spherical galaxies treated as polytropes, we assume that the change in the internal energy of galaxies during the encounter is symmetric with respect to the position of closest approach p of the galaxies. We obtain V_{crit}, from the relation (*cf.* Sastry 1972)

$$\frac{1}{2}\frac{M_S M_D}{M_S + M_D} V^2_{crit} - \frac{1}{2}\frac{M_S M_D}{M_S + M_D} (V_e^{(1)}(p))^2 = \mid \Delta U_S + \Delta U_D \mid \quad \ldots(20)$$

The parabolic velocity $V_e^{(1)}$ is obtained from

$$V_e^{(1)} = \left[2G \frac{M_S + M_D}{(r^2 + \epsilon_{SD}^2)^{1/2}}\right]^{1/2} \qquad \ldots(21)$$

where the softening parameter, ϵ_{SD}, for sphere-disc pair is obtained from equation (16).

Following Rood (1965) we define the capture velocity V_{cap}, as the initial velocity of the galaxies at infinite separation for which $| \Delta U_S + \Delta U_D | = E$, where E is the energy due to orbital motion of the galaxies. The two galaxies will merger if $V_p < V_{crit}$ or $V_\infty < V_{cap}$ where V_p and V_∞ are the relative velocities of the galaxies at closest approach and at initial separation respectively. V_{cap} is obtained from

$$\frac{1}{2} \frac{M_S M_D}{M_S + M_D} V_{cap}^2 = | \Delta U_1 + \Delta U_2 |_{V_p = V_{crit}}$$

The merger velocities obtained by us for sphere-disc pair with identical mass and radius are given in Table 2.

Table 2. Merger velocities of disc-sphere pair of galaxies in head-on collision ($V_e^{(1)}(0) = 520$ km s^{-1})

i	$\frac{V_{cap}}{V_e^{(1)}(0)}$	$\frac{V_{crit}}{V_e^{(1)}(0)}$
0°	0.79	1.15
45°	0.81	1.15
90°	0.62	1.10

Our value of $\frac{V_{crit}}{V_e^{(1)}(0)} = 1.15$ is in close agreement with that obtained by different workers using different models for galaxies and different methods [*e.g.* Toomre 1977 : Plummer model, impulsive approximation; van Albada and van Gorkom 1977 : Polytropic model with $n = 3$; Farouki & Shapiro 1982 : Disc halo galaxies, n-body simulations for $i = 90°$; Sastry & Alladin 1983 : polytropic model $n = 4$, impulsive approximation]. Thus it appears that the ratio $V_{crit}/V_e^{(1)}(0)$ is practically independent of the density model of the colliding galaxies.

For a typical sphere-sphere collision with $M_1 = M_2 = 10^{11} M_\odot$ and $\epsilon_1 = \epsilon_2 =$ 2 kpc we get $V_e^{(1)}(0) = 720$ km s^{-1}. For a sphere-disc pair of identical mass 10^{11} $M_\odot$ and scale length $\epsilon_S = 2$ kpc, $\epsilon_D = 8$ kpc, we get $V_e^{(1)}(0) = 520$ km s^{-1}. Since $V_{crit}/V_e^{(1)}(0)$ is the same for both sphere-sphere and sphere-disc cases and $V_e^{(1)}(0)$ is greater for sphere-sphere pair it follows that the chance of two spherical galaxies merging in a head-on collision due to tidal capture is greater than that of a disc and sphere pair. The same conclusion can be drawn by considering V_{cap} if $V = V_{\text{cap}}$ (i.e. $V_{\text{p}} = V_{\text{crit}}$). $\Delta U/ \mid U \mid < 1$ for sphere-sphere pair while $\Delta U/ \mid U \mid > 1$ for disc; and $\Delta U/ \mid U \mid < 1$ for sphere in sphere-disc interaction. This means that in the case of two spheres approaching each other with velocity slightly greater than V_{cap}, there is little disruption of the spheres while in the case of disc-sphere collision the disc undergoes considerable change in structure but the sphere does not. This result is in agreement with that of Farouki & Shapiro (1981). They consider slow collision (relative velocity $V = 500$ km s^{-1}) between Plummer model spherical galaxy and an exponential disc halo galaxy of equal mass and radii. They find that in the case of head-on ($p = 0$) collision the disc galaxy suffers violent and chaotic disruption after the closest approach.

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