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Lower bounds on axion rest mass in a general cosmological scenario*

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Abstract. On the basis of general properties and the large scale structure of spacetime, we derive general lower limits on the rest mass of the axion, assuming that axions make up the dark matter in the universe. These limits on mass are derived in terms of the possible age of the universe.

Key words : axion—dark matter—general spacetime

1. Introduction

A most remarkable development in contemporary astrophysics is the consensus that nonluminous or dark matter surrounds bright stars and galaxies, and makes up the dominant material content of our universe (Primack 1984). Evidence for this derives from the (Newtonian) gravitational effect that such matter seems to exert on visible stellar systems. Furthermore, dark matter seems to be present on all distance scales—from the local neighbourhood of our sun and the Milky Way, to clusters and superclusters of galaxies and the expansion scale of the universe itself.

For the local solar neighbourhood, explanation of dark matter can be had in terms of brown dwarfs (Jupiter-like objects). However, on larger scales no satisfactory explanation exists in terms of conventional, ordinary matter (namely, baryons and leptons). The search for an answer to this has covered a wide span of possibilities. At the present time, the most likely candidates for dark matter seem to be (a) low mass, faint stars; (b) massive blackholes; and (c) massive neutrinos, axions or particles predicted by supersymmetry (Blumenthal *et al.* 1984). Requirement of consistency with the observed abundance of primordial helium discounts the possibilities (a) and (b), implying that the possibility (c) merits a serious study,

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although, in fairness, it must be stated that the actual existence of these particles is yet to be established. Of these, axion is the most popular candidate as it seems to fit best the astrophysical requirements. Axions could have been produced in the early universe, and if the axion has a small, nonzero rest mass, then they would be gravitationally dominant today, and hence would acquire cosmological significance in relation to the dark matter problem.

Existence of axions was originally invoked (Peccei & Quinn 1977) in order to explain the property of charge-parity conservation of strong interactions. The axion rest mass m_a is not exactly specified in these theories; it depends on a parameter which has a wide range of possible values. In this regard, more useful information can be derived from astrophysical considerations. For instance, the requirement that the axion density be less than the critical density for the closure of the universe implies on the basis of the standard Friedmann model that $m_a \gtrsim 10^{-5}$ eV (Preskill, Wise & Wilczek 1983; Abbott & Sikivie 1983; Dine & Fischler 1983).

Now, the Friedmann model rests on the premises that the universe is isotropic and homogeneous. Although, this leads to a picture that is in fair consistency with local observations, there is no fundamental physical justification that isotropy and homogeneity (and a spherically symmetric expansion scheme) are strictly obeyed in all regions of space and at all epochs of time. In reality, it is known that the universe is not homogeneous at arbitrary spatial scales (although isotropy has good observational support); hence, studies of relativistic cosmological models not obeying homogeneity and isotropy have gained importance in recent times (Raychaudhuri 1979; Barrow 1984). This apart, any conclusion inferred within the framework of Friedmann models would normally require the knowledge of the Hubble parameter H_0 and the deceleration parameter q_0. Keeping in mind the present uncertainties that prevail in the cosmological observations to ascertain the values of these quantities (as well as the inherent restrictive nature of the basic assumptions of the Friedmann models), it would be both interesting and desirable to have some model independent conclusions regarding the constituents of dark matter, as this is a major problem in cosmology today.

It is the purpose of this essay to address to the above issue. We demonstrate that, if axions make up the dark matter in the universe, general lower limits on m_a can be set on the basis of a consideration of general spacetimes without recourse to specific assumptions of isotropy and homogeneity, characteristic of the Friedmann model of the universe. The limits on m_a are derived in terms of the age of the universe, the only parameter that is involved in such an approach.

2. Theoretical formalism

We begin by modelling the cosmological universe by retaining only very general features of the Friedmann models. A general anisotropic and inhomogeneous universe can be modelled by means of a globally hyperbolic spacetime. Such a spacetime can be covered by a one-parameter family (S_t) of spacelike Cauchy surfaces (Hawking & Ellis 1983), which makes it possible to refer to the state of the universe at any epoch in terms of surfaces of cosmic simultaneity $t =$ const.

Wellknown cosmological spacetimes such as described by Robertson-Walker line elements, Einstein-de Sitter universe, Bianchi models, the steady state model, *etc.* are all globally hyperbolic.

Our method depends on analysing the gravitational focusing effect on matter in a spacetime which can be characterized by the concept of a point conjugate to a spacelike hypersurface S_t along a timelike geodesic Γ. For a congruence of timelike geodesics orthogonal to S_t, a point along a timelike geodesic is said to be conjugate to S_t if neighbouring timelike geodesics orthogonal to S_t intersect at q. Such a situation arises when the expansion of the congruence (denoted by θ) becomes infinite at q. The location of a point conjugate to a given Cauchy surface reduces to finding a zero in the solution of the differential equation (Tipler 1976)

$$\frac{d^2x}{dt^2} + F(t)\, x = 0, \qquad \ldots(1)$$

where

$$F(t) = \tfrac{1}{3}\,(R_{\mu\nu}\, V^{\mu}V^{\nu} + 2\sigma^2). \qquad \ldots(2)$$

Here $R_{\mu\nu}$ is the contracted Ricci tensor; V^{μ} a unit timelike vector; and σ the shear term. The location of the conjugate point can be done using the Sturm comparison theorem on the solutions of second order differential equations (Hille 1969), and the following conclusion can be derived (Joshi 1980; Joshi & Chitre 1981; Joshi 1986) : If $\Gamma\,(t)$ is any timelike geodesic orthogonal to S_0, then there must be a point q on $\Gamma\,(t)$ conjugate to S_0 within the interval $0 < t \leqslant \pi/2A$ where

$$A^2 = \min F(t) > 0. \qquad \ldots(3)$$

The past extensions of arbitrary timelike trajectories γ from the present epoch S_0 can now be derived. Let p denote an event on S_0 and γ be a past-directed, endless timelike curve from $q = \gamma(\pi/2\mathrm{A})$. Suppose $q = \gamma(\pi/2A)$ be an event on γ. Then, by a well-known property of globally hyperbolic spacetimes, there exists a timelike geodesic λ from q orthogonal to S_0 along which the proper time lengths of all non-spacelike curves from q to S_0 are maximized, and further λ does not contain any conjugate point between q and S_0. However, as shown above, any timelike geodesic $\lambda(t)$ orthogonal to S_0 which is as long as $\pi/2A$ in the past must contain a point conjugate to S_0 within that interval, which is not possible. Hence, no timelike curve from S_0 can be extended into the past beyond the proper time length $\pi/2A$.

The matter distribution on S_0 will be given by the stress tensor $T_{\mu\nu}$ which satisfies the usual energy condition that there are no negative energy fields in the spacetime :

$$(T_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu}T)\, V^{\mu}V^{\nu} \geqslant 0. \qquad \ldots(4)$$

If the dynamics of the universe is assumed to be given by Einstein's equations

$$R_{\mu\nu} = 8\pi G(T_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}\, T) \qquad \ldots(5)$$

then equation (4) implies

$$R_{\mu\nu}V^{\mu}V^{\nu} \geqslant 0, \qquad \ldots(6)$$

for all matter fields. Though we have not required the isotropy or homogeneity of matter distribution on S_0, we shall assume that there exists a minimum for density distribution on S_0, which, in view of the observed expansion of the universe, should exhibit a non-decreasing behaviour in the past. This means that there exists some $A > 0$ such that

$$R_{\mu\nu}V^{\mu}V^{\nu} \geqslant A > 0, \qquad ...(7)$$

at the present and all past epochs.

For the usual form of $T_{\mu\nu}$ $(c = 1)$:

$$T_{\mu\nu} = (P + \rho)\, u_{\mu}u_{\nu} + Pg_{\mu\nu}, \qquad ...(8)$$

where P and ρ are pressure and matter energy density; and u_{μ} is the four-velocity of the comoving volume, equation (5) gives

$$R_{\mu\nu}V^{\mu}V^{\nu} = 8\pi G(P + \rho)\,(V^4)^2 + 4\pi G(P - 2\rho), \qquad ...(9)$$

i.e. $$R_{\mu\nu}V^{\mu}V^{\nu} \geqslant 4\pi G\,(3P + \rho). \qquad ...(10)$$

For matter-dominated era, as at the present epoch, $p \ll \rho$, and we can set

$$A^2 = \min \tfrac{1}{3}(R_{\mu\nu}V^{\mu}V^{\nu} + 2\sigma^2) \geqslant \tfrac{4}{3}\pi G\rho_0, \qquad ...(11)$$

where ρ_0 is the present matter density. Therefore, from what has been discussed earlier, the maximal possible extension for any timelike worldline from the present epoch into the past, or the maximal possible age of the universe will be given by

$$t_{\max} = \frac{\pi}{2}\left(\frac{3}{4\pi G\rho_0}\right)^{1/2}. \qquad ...(12)$$

This is a general result, irrespective of whether or not the distribution of matter on S_0 is isotropic and homogeneous, which does not assume any exact symmetries for the spacetime.

3. Axion rest mass

In a general cosmological scenario where densities may vary on S_0, the only handle on $t_{\max}$ comes from observations. Thus if $t_{\rm ob}$ denotes observed ages, we can write $t_{\rm ob} \leqslant t_{\max}$ and equation (12) gives

$$\rho_0 \leqslant \frac{3\pi}{16G}\,\frac{1}{t_{\rm ob}^2}. \qquad ...(13)$$

Equation (13) implies that a prescribed lower limit on the observed age of the universe will provide an upper limit to the matter density. Since equation (13) is true in general for any globally hyperbolic spacetime, if $\rho_0^{\rm F}$ denotes the Friedmann density parameter we can write without loss of generality

$$\rho_0^{\rm F} \leqslant \frac{3\pi}{16G}\,\frac{1}{t_{\rm ob}^2}, \qquad ...(14)$$

as the Friedmann model is a special case of globally hyperbolic spacetimes.

Now, within the Friedmann model, the contribution to the density due to axions (produced in the early universe) as a function of temperature (T) is (Preskill, Wise & Wilczek 1983; Abbot & Sikivie 1983; Dine & Fischler 1983):

$$\rho_a^F(T) = \frac{3 m_a T^3 f_a^2}{m_{Pl} \Lambda_{QCD}}, \qquad ...(15)$$

where $m_{Pl} = (\hbar c/G)^{1/2}$ is the Planck mass and Λ_{QCD} ($\simeq$ 200 MeV) is the scale parameter in quantum chromodynamics. The axion mass m_a is related to the vacuum expectation value f_a of the scalar field that spontaneously breaks the Peccei-Quinn symmetry invoked to explain the CP-invariance of strong interactions, and is given by (Weinberg 1978; Wilczek 1978)

$$m_a = 1.24 \times 10^{-5} \text{ eV}\left(\frac{10^{12} \text{ GeV}}{f_a}\right). \qquad ...(16)$$

Equation (16) is independent of cosmology. Particle physics, however, does not specify the exact value of f_a; it can lie anywhere between the weak interaction scale and the mass scale of grand unification.

If dark matter is made up entirely of axions, then since $\rho_a^F(T) \leqslant \rho_0^F$ the following general result can be written

$$\rho_a^F(T) \leqslant \frac{3\pi}{16G} \frac{1}{t_{ob}^2}, \qquad ...(17)$$

This gives, using equations (15) and (16),

$$f_a \leqslant \frac{\pi}{16G} \frac{m_{Pl} \Lambda_{QCD}}{T^3} \frac{1}{t_{ob}^2} \frac{1}{(1.24 \times 10^{-2} \text{ GeV}^2)}$$
$$\equiv (f_a)_{max}. \qquad (18)$$

We take $T = 2.73$ K, the present temperature of the universe and $\Lambda_{QCD} = 200$ MeV. Equation (18) thus gives a bound based on cosmological as well as particle physics considerations. This can be substituted in equation (16), which is independent of cosmology, to obtain the following lower bound on the axion mass:

$$m_a \geqslant 1.24 \times 10^{-5} \text{ eV} \left\{\frac{10^{12} \text{ GeV}}{(f_a)_{max}}\right\} = (m_a)_{min}. \qquad ...(19)$$

Observationally, the best lower limits for the age of the universe come from studies of globular clusters of stars in our galaxy. Recent estimates for this are (van den Bergh 1983; Sandage & Tammann 1984) : (13–19) Gyr and (14–20) Gyr (1 Gyr = 10^9 yr). Symbalisty, Yang & Schramm (1980) have suggested a consistent age estimate in the range (13.8–24) Gyr. Nuclear cosmochronometric studies by Thielmann, Metzinger & Klapdor (1983) suggest a range (18.8–24.8) Gyr. Therefore, for our purpose here, we choose t_{ob} to be in the range (13–25) Gyr.

4. Results and discussion

The results of our calculations are shown in table 1. The first column lists possible values of t_{ob}. The second and third columns give the corresponding values of

Table 1. Upper limits on f_a and lower limits on m_a for various values of the observed age of the universe

t_{ob} (in Gyr)	$(f_a)_{max}$ (in 10^{12} GeV)	$(m_a)_{min}$ (in 10^{-5} eV)
13	1.15	1.07
14	0.99	1.24
15	0 87	1.43
16	0.76	1.62
17	0.67	1.83
18	0.60	2.05
19	0.54	2.29
20	0.49	2.53
21	0.44	2.80
22	0.40	3.07
23	0.36	3.41
24	0.33	3.72
25	0.30	4.09

$(f_a)_{max}$ and $(m_a)_{min}$. We find that the range for the (i) upper limit on f_a is $(1.15–0.30) \times 10^{12}$ GeV and (ii) lower limit on m_a is $(1.07–4.09) \times 10^{-5}$ eV. In contrast, arguments based on Friedmann model considerations give $f_a \lesssim 10^{12}$ GeV and a corresponding lower limit for m_a, with the assumption that the axion energy density does not exceed the critical density for the closure of the universe, which involves the knowledge of H_0. Our results are only marginally different from those derived from Friedmann model considerations, excepting an upward revision in the observed age of the universe in which case, values of $(m_a)_{min}$ derived by us would be larger by several factors than that derived on the basis of Friedmann considerations. The interesting feature of our theory is a relationship between $(m_a)_{min}$ and t_{ob}, without explicitly invoking the Hubble parameter. Therefore, if current experimental attempts to determine m_a (Sikivie 1983; Krauss 1985; Moody 1985) are able to shed light on $(m_a)_{min}$, our theory can be turned around to set general upper limits on the age of the universe.

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