

Coupling of Alfvén and magnetosonic waves in rotating Hall magnetoplasmas

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ABSTRACT

We study the linear theory of magnetohydrodynamic (MHD) waves, namely, the Alfvén and the fast and slow magnetosonic modes in a rotating Hall-MHD plasma with the effects of the obliqueness of the external magnetic field and the Coriolis force and show that these waves can be coupled either by the influence of the Coriolis force or the Hall effects. To this end, we derive a general form of the linear dispersion relation for these coupled modes by the combined influence of the Coriolis force and the Hall effects and analyze numerically their characteristics in three different plasma- β regimes: $\beta \sim 1$, $\beta > 1$, and $\beta < 1$, including some particular cases. We show that while the coupling between the Alfvén and the fast magnetosonic modes is strong in the low- β ($\beta \lesssim 1$) regime and the wave dispersion appears in the form of a thumb curve, in the high- β ($\beta > 1$) regime, the strong coupling can occur between the Alfvén and the slow magnetosonic modes and the dispersion appears in the form of a teardrop curve. Switching of the coupling in the regime of $\beta \sim 1$ can occur, i.e., instead of a thumb curve, a teardrop curve appears when the obliqueness of propagation and rotational angle are close to 70° or more (but less than 90°). Implications of our results to solar and fusion plasmas are briefly discussed.

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I. INTRODUCTION

In classical magnetohydrodynamic (MHD) plasmas, the characteristic timescale of plasma oscillations is much longer than the inverse of the ion-cyclotron frequency, and electrons and ions become tied to the magnetic field, i.e., the electron and ion motions get coupled. In this case, a single fluid MHD model is applicable for describing magnetic field dynamics. However, electron and ion motions can become decoupled when the characteristic timescale is comparable to the inverse of the ion cyclotron frequency, the length-scale of magnetic field variation is akin to or smaller than the ion skin depth, or when plasma is partially ionized. In this situation, electrons drift through ions instead of being carried along with the bulk velocity field, leading to a modification of the generalized Ohm's law by the Hall current, proportional to $\mathbf{J} \times \mathbf{B}$ force, where \mathbf{J} and \mathbf{B} are the current density and magnetic field, respectively. Thus, the classical MHD gets modified to the Hall-MHD (HMHD). The ideal HMHD model has many applications in various astrophysical, space, and laboratory environments, e.g., flux expulsion in neutron star crusts,¹ formation of intensive solar

flux tubes and waves in the solar wind,^{2,3} propagation of whistlers in Earth's ionosphere,⁴ fusion plasmas,⁵ dynamo mechanisms,⁶ magnetic reconnection,⁷ and accretion processes.⁸

The existence of MHD waves, especially the Alfvén wave, was first predicted theoretically by Alfvén in 1942.⁹ Among the earlier researchers, Lighthill developed the standard MHD by including the Hall effect in 1960.¹⁰ In the last few decades, many researchers have studied linear and nonlinear properties of MHD waves in Hall magnetoplasmas.^{11–13} In highly ionized plasmas, the Hall effect appears due to the inertial difference between electrons and ions. However, in partially ionized plasmas, the Hall effect may instead appear as the ions are easily decoupled (compared to electrons) from the magnetic field due to collisions with neutral atoms.⁷ In HMHD plasmas, Kawazura showed that the relativistic factor can influence the characteristics of the phase and group velocities of MHD waves.¹⁴ Ruderman¹¹ studied the nonlinear theory of MHD waves in anisotropic Hall plasmas by deriving a Kadomtsev-Petviashvili (KP) equation. He showed that the fast and slow magnetosonic KP solitons may become unstable due to

transverse perturbations. Recently, Mahajan *et al.*¹⁵ showed how linear waves in HMHD bring a fundamental departure from the standard MHD waves, and the Hall current induces a new mode of circularly polarized waves.

Chandrasekhar and Mon¹⁶ first studied the rotational effects in plasma dynamics and explored the importance of the Coriolis force in the cosmic phenomena, which was later supported in subsequent studies in space and astrophysical plasmas by Lehnert.¹⁷ The influence of Hall electromotive forces on the linear stability of protostellar disks was examined by Steven *et al.* in rotating Hall plasmas.¹⁸ On the other hand, Rax *et al.*¹⁹ investigated the dynamics of torsional Alfvén waves in rotating plasmas and found a new coupling between the orbital angular momentum of the Alfvén waves and the angular momentum of the rotating plasma. Several authors have paid attention to studying different kinds of instabilities, including magnetogravitational instabilities and Jeans instabilities in rotating plasmas in the contexts of space and astrophysical environments.^{20,21} In the nonlinear regime, Hager *et al.*²² studied the characteristics of magnetosonic wave propagation by deriving a Korteweg–de Vries (KdV) equation in rotating quantum plasmas. However, they neglected the Hall effect in the linear and nonlinear analyses. Turi and Misra²³ have studied the modulation instability of fast magnetosonic waves by deriving a nonlinear Schrödinger (NLS) equation in rotating low-beta inhomogeneous magnetoplasmas without the Hall current. The developments in the nonlinear interactions of MHD waves are also seen, e.g., in the context of solar plasmas²⁴ and MHD wave turbulence.²⁵ However, none of the above works have focused on the fundamental MHD wave couplings by the influences of Hall effect and the Coriolis force.

The purpose of this work is to identify the fundamental MHD modes, namely, the Alfvén and fast and slow magnetosonic modes, and how the Hall effect and the Coriolis force can contribute to the coupling of Alfvén and magnetosonic waves.

II. THEORETICAL MODEL AND BASIC EQUATIONS

We consider the propagation of MHD waves in a magnetized rotating Hall-MHD plasma. We assume that the plasma conductivity is high, i.e., the magnetic Reynolds number is larger than unity such that the magnetic field lines tend to remain frozen into the plasma (typically electrons) and move along the fluid flow. In this situation, the term proportional to the plasma resistivity (magnetic diffusion) remains smaller than the fluid flow term (magnetic induction or advection) in the magnetic induction equation (12) and so the plasma resistivity effect can be safely neglected. Also, we consider that the electron and ion collision time scales are much longer than the hydrodynamic timescale of plasma oscillations for which plasma can be treated as collisionless. Furthermore, the length-scale of temperature variation is much larger than the fluid density variation, for which the thermal conduction effect can be ignored. The plasma is supposed to be rotating with uniform angular velocity $\Omega_0 = (\Omega_0 \sin \lambda, \Omega_0 \cos \lambda, 0)$, where λ is the angle made by the axis of rotation with the y -axis. We also consider the wave propagation vector \mathbf{k} along the x -axis and the uniform external magnetic field is in the xy -plane, i.e., $\mathbf{B}_0 = (B_0 \cos \alpha, B_0 \sin \alpha, 0)$, where α is the angle between \mathbf{B}_0 and \mathbf{k} . A schematic diagram for the system configuration is shown in Fig. 1.

For the description of MHD waves, a two-fluid model is necessary to include the Hall current. However, we will see that such a two-fluid description can be reduced to a one-fluid model if we neglect the

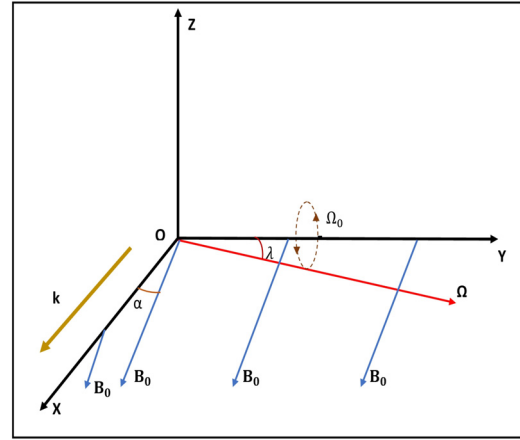


FIG. 1. Schematic diagram showing the geometry of the model configuration.

electron inertia. Each fluid satisfies its own mass, momentum, and energy conservation equations.

The mass conservation equations for electrons and ions are

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot (\rho_e \mathbf{v}_e) = 0, \quad (1)$$

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{v}_i) = 0, \quad (2)$$

where $\rho_j \equiv m_j n_j$ is the mass density (in which m_j is the mass and n_j is the number density) and \mathbf{v}_j is the velocity of j th species fluid with $j = e$ for electrons and $j = i$ for ions.

The momentum conservation equation for electrons (in the absence of the thermal pressure and collisions) in a frame rotating with uniform angular velocity Ω_0 reads

$$\rho_e \left[\frac{\partial \mathbf{v}_e}{\partial t} + (\mathbf{v}_e \cdot \nabla) \mathbf{v}_e \right] = -en_e [\mathbf{E} + \mathbf{v}_e \times \mathbf{B}] - \rho_e (2\Omega_0 \times \mathbf{v}_e) - \rho_e \Omega_0^2 \mathbf{r}, \quad (3)$$

where e is the elementary charge, $\mathbf{r} = (x, y, z)$, and \mathbf{E} and \mathbf{B} are the electromagnetic fields. The three terms on the right-hand side of Eq. (3), respectively, correspond to the electromagnetic, Coriolis, and centrifugal forces. Using the dimensional analysis, we compare the Coriolis and centrifugal forces with the Lorentz force as follows:

$$\frac{|\rho_e 2\Omega_0 \times \mathbf{v}_e|}{|en_e \mathbf{v}_e \times \mathbf{B}|} \sim \frac{m_e 2\Omega_0}{m_i \omega_{ci}}, \quad \frac{|\rho_e \Omega_0^2 \mathbf{r}|}{|en_e \mathbf{v}_e \times \mathbf{B}|} \sim \frac{m_e \Omega_0^2}{m_i \omega_{ci}^2}, \quad (4)$$

where $\omega_{ci} = eB_0/m_i$ is the ion-cyclotron frequency. Since $m_e \ll m_i$, the contributions from the Coriolis and centrifugal forces can become much smaller than the Lorentz force if $\Omega_0 \lesssim \omega_{ci}$. Also, the ratio between $\rho_e d\mathbf{v}/dt$ and the Lorentz force $en_e \mathbf{v}_e \times \mathbf{B}$ scales as $\sim m_e/m_i$. Thus, in the limit of $(m_e/m_i) \rightarrow 0$, Eq. (3) reduces to

$$\mathbf{E} = -\mathbf{v}_e \times \mathbf{B}. \quad (5)$$

Next, the ion momentum conservation equation in a frame rotating with uniform angular velocity Ω_0 reads

$$\rho_i \left[\frac{\partial \mathbf{v}_i}{\partial t} + (\mathbf{v}_i \cdot \nabla) \mathbf{v}_i \right] = en_i [\mathbf{E} + \mathbf{v}_i \times \mathbf{B}] - \nabla P_i - \rho_i (2\mathbf{\Omega}_0 \times \mathbf{v}_i) - \rho_i \mathbf{\Omega}_0^2 \mathbf{r}, \quad (6)$$

where P_i is the ion pressure. Using the Ohm's law (5) and the quasi-neutrality condition: $n_e \approx n_i = n$, Eq. (6) reduces to

$$\rho_i \left[\frac{\partial \mathbf{v}_i}{\partial t} + (\mathbf{v}_i \cdot \nabla) \mathbf{v}_i \right] = \mathbf{J} \times \mathbf{B} - \nabla P_i - \rho_i (2\mathbf{\Omega}_0 \times \mathbf{v}_i) - \rho_i \mathbf{\Omega}_0^2 \mathbf{r}, \quad (7)$$

where $\mathbf{J} = en(\mathbf{v}_i - \mathbf{v}_e)$ is the current density.

The magnetic induction equation can be obtained by using Faraday's law, i.e., $\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$ and Ohm's law (5) as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{v}_i \times \mathbf{B} - \frac{\mathbf{J} \times \mathbf{B}}{en} \right), \quad (8)$$

where the first and second terms on the right side of Eq. (8) correspond to the ion fluid flow and the Hall effect, respectively.

To close the system, we also need the energy equation, i.e., the equation of state, which in the absence of loss mechanism, reads

$$\nabla P_i = c_s^2 \nabla \rho_i, \quad (9)$$

where c_s is the ion-sound speed.

It is convenient to eliminate \mathbf{J} by using Ampère's law, i.e., $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ with μ_0 denoting the magnetic permeability and replace ρ_i by ρ , \mathbf{v}_i by \mathbf{v} , n_i by n , and m_i by m . Thus, in the limit of $m_e/m_i \rightarrow 0$, the Hall-MHD can be described by the following set of fluid equations:^{26,27}

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (10)$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla P}{\rho} + \frac{1}{\mu_0 \rho} (\nabla \times \mathbf{B}) \times \mathbf{B} - 2\mathbf{\Omega}_0 \times \mathbf{v} - \mathbf{\Omega}_0^2 \mathbf{r}, \quad (11)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \frac{m}{e\mu_0 \rho} \nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}], \quad (12)$$

$$\nabla P = c_s^2 \nabla \rho. \quad (13)$$

It is pertinent to express the basic equations (10)–(13) in dimensionless forms. So, we redefine the variables as follows:

$$\begin{aligned} \rho &\rightarrow \rho/\rho_0, & \mathbf{v} &\rightarrow \mathbf{v}/V_A, & \mathbf{B} &\rightarrow \mathbf{B}/B_0, \\ (x, y, z) &\rightarrow (x, y, z)/\lambda_i, & t &\rightarrow \omega_{ci} t, \\ \mathbf{\Omega}_0 &\rightarrow \mathbf{\Omega}_0/\omega_{ci}, & P &\rightarrow P/V_A^2 \rho_0, \end{aligned} \quad (14)$$

where $\rho_0 = n_0 m$ is the unperturbed value of ρ with n_0 denoting the unperturbed number density of electrons and ions, $V_A = B_0 / \sqrt{\mu_0 \rho_0}$ is the Alfvén speed, and $\lambda_i = V_A / \omega_{ci}$ is the ion skin depth (or inertial length). Thus, Eqs. (10)–(13) reduce to

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (15)$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla P}{\rho} + \frac{1}{\rho} (\nabla \times \mathbf{B}) \times \mathbf{B} - 2\mathbf{\Omega}_0 \times \mathbf{v}, \quad (16)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \frac{1}{\rho} \nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}], \quad (17)$$

$$\nabla P = \tilde{c}_s^2 \nabla \rho, \quad (18)$$

where $\tilde{c}_s^2 = c_s^2 / V_A^2$, and we define it as the plasma beta, i.e., $\beta = \tilde{c}_s^2$.

In Eq. (16), we have neglected the contribution from the centrifugal force by assuming that the fluid flow equilibrium is valid to the first-order in $\mathbf{\Omega}_0$. To justify this assumption, we note that the magnitudes of the Coriolis and centrifugal forces compared to the evolution term $d\mathbf{v}/dt$ scale as

$$\left| (2\mathbf{\Omega}_0 \times \mathbf{v}) / \left(\frac{d\mathbf{v}}{dt} \right) \right| \sim 2\mathbf{\Omega}_0, \quad \left| (\mathbf{\Omega}_0^2 \mathbf{r}) / \left(\frac{d\mathbf{v}}{dt} \right) \right| \sim \mathbf{\Omega}_0^2, \quad (19)$$

and similar to Eq. (4), the magnitudes of the Coriolis and centrifugal forces compared to the Lorentz forces scale as

$$\begin{aligned} \left| (2\mathbf{\Omega}_0 \times \mathbf{v}) / \left(\frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{\rho} \right) \right| &\sim 2\mathbf{\Omega}_0, \\ \left| (\mathbf{\Omega}_0^2 \mathbf{r}) / \left(\frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{\rho} \right) \right| &\sim \mathbf{\Omega}_0^2, \end{aligned} \quad (20)$$

where $\mathbf{\Omega}_0$ is normalized by ω_{ci} . Thus, if the magnetic field is strong and the fluid motion is not significantly curved such that the rotational frequency $\mathbf{\Omega}_0$ becomes smaller than the characteristic frequency ω_{ci} , the centrifugal force appearing in Eq. (11) is of the order of $\mathbf{\Omega}_0^2$ and can be neglected in comparison with the other terms. We also mention that in the magnetic induction equation (17), we have neglected the contribution from the plasma resistivity or magnetic diffusivity ($\propto \eta_m$) by considering its magnitude compared to the fluid flow (inductive), time evolution, and Hall terms as follows (after considering the dimension of each variable):

$$\begin{aligned} \frac{|\nabla \times (\mathbf{v} \times \mathbf{B})|}{|\eta_m \nabla^2 \mathbf{B}|} &\sim \frac{v \lambda_i}{\eta_m} \sim \frac{V_A \lambda_i}{\eta_m} \equiv R_m, \\ \frac{|\partial \mathbf{B} / \partial t|}{|\eta_m \nabla^2 \mathbf{B}|} &\sim \frac{V_A^2}{\omega_{ci} \eta_m} \sim \frac{\lambda_i^2 \omega_{ci}}{\eta_m} \sim \frac{\tau_D}{\tau_{ch}}, \\ \frac{\left| \frac{m}{e\mu_0 \rho} \nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}] \right|}{|\eta_m \nabla^2 \mathbf{B}|} &\sim \frac{V_A^2}{\omega_{ci} \eta_m} \sim \frac{\tau_D}{\tau_{ch}}, \end{aligned} \quad (21)$$

where R_m is the magnetic Reynolds number, $\tau_{ch} \sim \omega_{ci}^{-1}$ is the characteristic time of oscillations, and $\tau_D \sim \lambda_i^2 / \eta_m$ is the magnetic diffusion timescale. Thus, in collisionless Hall plasmas, if $R_m \gg 1$ and $\tau_{ch} \ll \tau_D$, τ_{coll} (i.e., if the magnetic Reynolds number is significantly high and the magnetic diffusion and electron-ion collision times are longer than the characteristic time of plasma oscillation), where τ_{coll} is the electron-ion collision time, we can neglect the resistivity effect in the magnetic induction equation (17).

III. DISPERSION RELATION

We study the excitation of fundamental MHD wave eigenmodes and their possible coupling by the influences of the Coriolis force in the fluid motion and the Hall current in the magnetic induction equation. Specifically, we consider three different plasma- β regimes: $\beta \sim 1$, $\beta > 1$, and $\beta < 1$, which, respectively, correspond to the cases of $c_s \sim V_A$, $c_s > V_A$, and $c_s < V_A$, to study the acoustic or magnetic

characters (or both) of MHD wave perturbations in the small-amplitude limit for which the linear theory is valid. To this end, we linearize Eqs. (15)–(18) about the equilibrium state of dependent physical variables and split up the physical quantities into their equilibrium (0 value, or a quantity with suffix 0) and perturbation (with suffix 1) parts according to $\rho = 1 + \rho_1$, $\mathbf{v} = 0 + \mathbf{v}_1$, $\mathbf{B} = \hat{\mathbf{B}}_0 + \mathbf{B}_1$, and $P = P_0 + P_1$, and assume all the perturbations to vary as plane waves with the wave frequency ω (normalized by ω_{ci}) and the wave vector \mathbf{k} (normalized by λ_i) in the form $\exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$. Thus, we obtain from Eqs. (15)–(18) the following linearized equations for the perturbations:

$$\omega \rho_1 - \mathbf{k} \cdot \mathbf{v}_1 = 0, \quad (22)$$

$$\omega \mathbf{v}_1 = k P_1 + [(\hat{\mathbf{B}}_0 \cdot \mathbf{B}_1) \mathbf{k} - (\hat{\mathbf{B}}_0 \cdot \mathbf{k}) \mathbf{B}_1] - 2i\Omega_0 \times \mathbf{v}_1, \quad (23)$$

$$\omega \mathbf{B}_1 + [(\mathbf{k} \cdot \hat{\mathbf{B}}_0) \mathbf{v}_1 - (\mathbf{k} \cdot \mathbf{v}_1) \hat{\mathbf{B}}_0] = i(\hat{\mathbf{B}}_0 \cdot \mathbf{k})(\mathbf{k} \times \mathbf{B}_1), \quad (24)$$

$$P_1 = \tilde{c}_s^2 \rho_1, \quad (25)$$

where $\hat{\mathbf{B}}_0 = (\cos \alpha, \sin \alpha, 0)$. We note that the ratio of the last two terms on the right side of Eq. (23) scales as (in dimensions)

$$\frac{B_0 B_1 k}{\rho_0 \mu_0} \frac{1}{2\Omega_0 v_1} \sim \frac{k B_0}{\sqrt{\rho_0 \mu_0}} \frac{B_1}{v_1 \sqrt{\rho_0 \mu_0}} \frac{1}{2\Omega_0} \sim \frac{k V_A B_1 V_A}{2\Omega_0 B_0 v_1} \sim \frac{\omega_A}{2\Omega_0}, \quad (26)$$

where $\omega_A = k V_A$ is the Alfvén frequency and $B_1/B_0 \sim v_1/V_A$. Thus, for Alfvén waves, the Coriolis force dominates over the $\mathbf{J} \times \mathbf{B}$ -force for $\omega_A < 2\Omega_0$. Assuming, for simplicity, the wave propagation vector along the x -axis, i.e., $\mathbf{k} = (k, 0, 0)$ so that $\mathbf{k} \cdot \hat{\mathbf{B}}_0 = k \cos \alpha$ and looking for nonzero solutions of the perturbations, we obtain from Eqs. (22)–(25) the following linear dispersion relation:

$$\begin{aligned} &(\omega^2 - k^2 \cos^2 \alpha)[\omega^4 - (1 + \tilde{c}_s^2)k^2 \omega^2 + \tilde{c}_s^2 k^4 \cos^2 \alpha] \\ &- 4\Omega_0^2 \omega^2 [\omega^2 - \tilde{c}_s^2 k^2 \sin^2 \lambda - k^2 \cos^2(\alpha + \lambda)] \\ &- \omega^2 k^4 \cos^2 \alpha (\omega^2 - \tilde{c}_s^2 k^2) + 4\Omega_0 k^4 \cos^2 \alpha [\omega^2 \sin(\alpha + \lambda) \\ &- \tilde{c}_s^2 k^2 \cos \alpha \sin \lambda] + 4\Omega_0^2 k^4 \cos^2 \alpha (\omega^2 - \tilde{c}_s^2 k^2 \sin^2 \lambda) = 0. \end{aligned} \quad (27)$$

Equation (27) is the general form of dispersion relation for MHD waves in an electron-ion magnetoplasma with the influences of the Coriolis force and Hall current. It generalizes some previous works (e.g., Refs. 11 and 20) where the combined influences of the Coriolis force and the Hall current on the wave modes have not been discussed. By considering the wave propagation parallel to the magnetic field (i.e., $\alpha = 0$) and ignoring the Hall effect, one can recover the same dispersion relation as in Ref. 20 after setting $\theta = 0$, $\nu_c = 0$, and $\omega_j = 0$ therein. In the absence of the Coriolis force and with a substitution of ω^2 by $\omega k \cos \alpha$ in the Hall contributed term, the dispersion equation (27) reduces to Eq. (8) of Ref. 11.

Several terms appearing in Eq. (27) correspond to different physical sources. On the left side, the second term ($\propto 4\Omega_0^2 \omega^2$) appears due to the effects of the Coriolis force, the third term ($\propto k^4 \cos^2 \alpha$) appears due to the Hall effect, and the fourth ($\propto 4\Omega_0 k^4 \cos^2 \alpha$) and fifth ($\propto 4\Omega_0^2 k^4 \cos^2 \alpha$) terms appear due to the combined influence of the Coriolis force and Hall current. Furthermore, in the first term, while the first factor corresponds to the shear Alfvén wave, the second factor gives fast and slow magnetosonic modes. This can be verified by disregarding the Coriolis force and Hall effects in Eq. (27). Thus, the coupling between the Alfvén and magnetosonic modes can occur by the

influence of either the Coriolis force, or the Hall effect, or both of them. When these effects are absent, Eq. (27) gives three decoupled MHD modes: the shear Alfvén wave and the fast and slow magnetosonic modes, to be discussed shortly in Sec. III A. Before we study the coupling of the MHD waves and their characteristics in a general situation, it may be helpful to discuss about some particular cases that correspond to some known results in the literature and to demonstrate the relative influences of the Coriolis force and the Hall effect on the MHD modes.

A. Wave motion without the Coriolis force and Hall effects

We consider the MHD wave propagation in a non-rotating plasma with a frequency much lower than the ion-cyclotron frequency. In this case, the Hall effect can be neglected and there will be no Coriolis force. As a result, the dispersion relation (27) reduces to that for decoupled Alfvén and magnetosonic modes, i.e.,

$$(\omega^2 - k^2 \cos^2 \alpha)[\omega^4 - (1 + \tilde{c}_s^2)k^2 \omega^2 + \tilde{c}_s^2 k^4 \cos^2 \alpha] = 0. \quad (28)$$

The first factor of Eq. (28), when equated to zero, gives the non-dispersive shear Alfvén mode,²⁸ given by

$$\omega = k \cos \alpha. \quad (29)$$

Such transverse waves, driven by the magnetic tension, cannot propagate perpendicular to the static magnetic field but with an angle satisfying $0 \leq \alpha < \pi/2$. Since there is no density or pressure fluctuations associated with the wave, the wave energy flows along the magnetic field lines at the Alfvén speed V_A , which in the dimensionless form gives $\omega/k = 1$ at $\alpha = 0$. On the other hand, equating the second factor of Eq. (28) to zero gives the following dispersion relations for the fast (with suffix “F”) and slow (with suffix “S”) magnetosonic modes propagating obliquely ($0 < \alpha < \pi/2$) to the magnetic field²⁸

$$\omega_{F,S}^2 = \frac{k^2}{2} [(1 + \tilde{c}_s^2) \pm \sqrt{(1 + \tilde{c}_s^2)^2 - 4\tilde{c}_s^2 \cos^2 \alpha}], \quad (30)$$

where the plus and minus signs, respectively, correspond to the fast and slow magnetosonic modes with frequencies ω_F and ω_S . In contrast to the Alfvén wave, the magnetosonic modes are driven by the magnetic pressure and the kinetic pressure gradient forces. In particular, for $\alpha = \pi/2$, i.e., when $\hat{\mathbf{B}}_0 \perp \mathbf{k}$, the fast mode emerges as the classical magnetosonic wave: $\omega_F/k = \sqrt{1 + \tilde{c}_s^2}$ while the slow mode disappears. Also, for $\alpha = 0$, i.e., for $\hat{\mathbf{B}}_0 \parallel \mathbf{k}$, the fast mode turns out to be an acoustic mode: $\omega_F/k = \tilde{c}_s$ and the slow mode becomes the Alfvénic mode: $\omega_S/k = 1$ (i.e., $\omega_S = k V_A$ in dimensional form), or vice versa, depending on the plasma beta, $\beta (\sim \tilde{c}_s^2) > 1$, or $\beta < 1$. On the other hand, for the propagation of MHD waves obliquely to the magnetic field, i.e., $0 < \alpha < \pi/2$, Eq. (30) yields for the fast magnetosonic wave the frequency: (i) $\omega_F \approx \tilde{c}_s k$ in the limit of $\beta \gg 1$, i.e., the fast mode becomes acoustic in character (longitudinal fluid motion), and (ii) $\omega_F \approx k$ in the limit of $\beta \ll 1$, i.e., the fast mode becomes magnetic in character (fluid motion transverse to the magnetic field). Thus, the fast mode (when the fluid and magnetic pressure variations are in phase) is basically an acoustic wave in the high- β regimes such as those in the solar convection zone, photosphere, and lower chromosphere. However, in the low- β regimes (e.g., solar corona and upper

chromosphere), it becomes more like an Alfvén wave.²⁸ On the other hand, Eq. (30) gives for slow magnetosonic modes (when the fluid and magnetic pressure variations are out of phase) propagating obliquely to the magnetic field ($0 < \alpha < \pi/2$) the frequency, $\omega \approx k\tilde{c}_s \cos \alpha$ in the limit of $\beta \ll 1$ and the frequency $\omega \approx k \cos \alpha$ in the limit of $\beta \gg 1$. It also follows that, in contrast to the fast mode and in the low- β regime (e.g., solar corona), the slow magnetosonic mode is more acoustic than magnetic. Thus, it may be predicted that if there is a possibility of coupling between the Alfvén and magnetosonic modes by the influence of either the Coriolis force or the Hall effect, or both, it may be likely that in the low- β regime, this coupling can occur between the fast magnetosonic mode (more magnetic) and the Alfvén mode (magnetic), and in the high- β regime, the same can be between the slow magnetosonic (more magnetic) and Alfvén (magnetic) modes. We will justify these assertions and give a clearer picture about the couplings in Secs. III B–III D.

B. Wave motion with the Coriolis force but without the Hall effect

We consider the propagation of MHD waves with a wave frequency much lower than the ion-cyclotron frequency as in Sec. III A but in rotating magnetoplasmas. In this case, the Hall effect can be neglected, and the dispersion equation (27) reduces to

$$(\omega^2 - k^2 \cos^2 \alpha)[\omega^4 - (1 + \tilde{c}_s^2)k^2 \omega^2 + \tilde{c}_s^2 k^4 \cos^2 \alpha] - 4\Omega_0^2 \omega^2 [\omega^2 - \tilde{c}_s^2 k^2 \sin^2 \lambda - k^2 \cos^2(\alpha + \lambda)] = 0. \quad (31)$$

It is evident that not only are Alfvén and magnetosonic waves coupled but also modified by the influence of the Coriolis force (the term proportional to Ω_0^2). To reveal the effects of this force on the fundamental modes, we first consider the magnetosonic mode at an angle $\alpha = \pi/2$ (at which the Alfvén mode disappears). The dispersion equation (31) then reduces to

$$\omega^4 - [(1 + \tilde{c}_s^2)k^2 + 4\Omega_0^2]\omega^2 + 4\Omega_0^2 k^2 \sin^2 \lambda (1 + \tilde{c}_s^2) = 0, \quad (32)$$

from which the frequencies for the fast and slow modes are found to be

$$\omega_{F,S}^2 = \frac{1}{2}[\Lambda^2 \pm \sqrt{\Lambda^4 - 16\Omega_0^2(1 + \tilde{c}_s^2)k^2 \sin^2 \lambda}]. \quad (33)$$

Here, $\Lambda^2 = (1 + \tilde{c}_s^2)k^2 + 4\Omega_0^2$, and the plus (minus) sign before the radical sign stands for the fast (slow) magnetosonic mode. The cutoff frequencies (at $k = 0$) for the fast and slow modes, respectively, are $2\Omega_0$ and 0. In particular, if $\lambda = \pi/2$, i.e., the axis of rotation coincides with the propagation vector, the fast and slow modes become decoupled with frequencies, $\omega_F = k\sqrt{1 + \tilde{c}_s^2}$ and $\omega_S = 2\Omega_0$, i.e., while the former appears as a non-dispersive magnetosonic mode, the latter emerges as a constant oscillation with a frequency twice the plasma rotational frequency. On the other hand, when $\lambda = 0$, i.e., the axis of rotation is perpendicular to the propagation direction, the fast mode emerges as a dispersive magnetosonic mode with a frequency, $\omega = \sqrt{(1 + \tilde{c}_s^2)k^2 + 4\Omega_0^2}$, which gets modified by the rotational frequency and agrees with Eq. (15) of Ref. 20 if one disregards the effects of the cosmic pressure and gravitational force therein. However, the slow mode disappears. The cutoff frequency for the fast

magnetosonic mode is $2\Omega_0$, which is the same as obtained from Eq. (33), i.e., independent of the angle of rotation.

Next, we consider the propagation parallel to the magnetic field ($\alpha = 0$) and study the effects of the Coriolis force on the wave dispersion. In this case, Eq. (31) reduces to

$$(\omega^2 - k^2)\{\omega^4 - (1 + \tilde{c}_s^2)k^2 \omega^2 + \tilde{c}_s^2 k^4\} - 4\Omega_0^2 \omega^2 \times (\omega^2 - \tilde{c}_s^2 k^2 \sin^2 \lambda - k^2 \cos^2 \lambda) = 0. \quad (34)$$

A novel feature of Eq. (34) is that even in the case of MHD wave propagation parallel to the static magnetic field, not only are both the magnetic and acoustic characters of the magnetosonic modes retained but the modes also get coupled to the Alfvén mode by the influence of the Coriolis force. The reason why the magnetic character of the magnetosonic mode persists even at $\alpha = 0$ may be both the Lorentz and the Coriolis forces are similar in that both are proportional to the particle velocity and act perpendicular to it.²⁹ In the absence of the Coriolis force, both the Alfvén and magnetosonic modes get decoupled, giving the fast mode to appear as the Alfvén mode $\omega = k$ and the slow mode as the acoustic mode with $\omega = \tilde{c}_s k$. In particular, for $\lambda = \pi/2$ (i.e., when the axis of rotation is along the propagation vector), Eq. (34) gives the magnetosonic wave as an acoustic mode with $\omega = \tilde{c}_s k$ and the fast and slow Alfvén waves,²⁰ given by

$$\omega = \pm\Omega_0 + \sqrt{\Omega_0^2 + k^2}. \quad (35)$$

It shows that the Alfvén waves propagating with a frequency lower than the ion-cyclotron frequency in a rotating magnetoplasma become dispersive due to the Coriolis force. The cutoff frequency for the slow Alfvén mode is zero, but it is shifted by $2\Omega_0$ for the fast mode. However, at $\lambda = 0$ (i.e., when the axis of rotation is perpendicular to the wave vector), we obtain from Eq. (34) a non-dispersive Alfvén mode, $\omega = k$, and dispersive fast and slow magnetosonic modes, given by

$$\omega_{F,S}^2 = \frac{1}{2}[\Lambda^2 \pm \sqrt{\Lambda^4 - 4\tilde{c}_s^2 k^4}], \quad (36)$$

with the cutoff frequencies, $2\Omega_0$ and 0, respectively.

In what follows, we consider a general situation in which $0 < \alpha, \lambda < \pi/2$, and obtain from Eq. (31) approximate dispersion relations for both the Alfvén and magnetosonic waves. For the Alfvén wave, we have

$$\omega^2 = d_1 + k^2 \cos^2 \alpha, \quad (37)$$

where d_1 is the correction term, obtained by replacing ω by $k \cos \alpha$ in the term proportional to Ω_0^2 and divided by the factor associated with the magnetosonic wave, given by

$$d_1 \sin^2 \alpha = 4\Omega_0^2 [\tilde{c}_s^2 \sin^2 \lambda - \cos^2 \alpha + \cos^2(\alpha + \lambda)]. \quad (38)$$

Typically, the approximate dispersion relation (37) describes an Alfvén mode that is hybridized by the coupling effect of the magnetosonic mode in the presence of the Coriolis force. Furthermore, the dispersion relation is in the form of a Langmuir wave in classical plasmas. So, it can correspond to a fundamental MHD mode, whose nonlinear evolution as Alfvénic wave envelopes can be described by a nonlinear Schrödinger (NLS)-like equation.^{30,31} From Eq. (37), it is also evident that the hybridized Alfvén mode can propagate only for the wave number exceeding a critical value, i.e., $k > k_c$, where the critical value k_c is given by

$$k_c = 4\Omega_0 \sqrt{\cos^2 \alpha - \cos^2(\alpha + \lambda) - \tilde{c}_s^2 \sin^2 \lambda / \sin 2\alpha}, \quad (39)$$

provided $\tilde{c}_s^2 \leq \sin(2\alpha + \lambda) / \sin \lambda$, or $\lambda \leq \lambda_c$ with $\lambda_c \equiv \tan^{-1}[(1 - \sin 2\alpha) / \cos 2\alpha]$ denoting the critical value of λ . In particular, in the long-wavelength limit, $k \ll 1$ with $d_1 > 0$, Eq. (37) reduces to

$$\omega \approx \sqrt{d_1} + \frac{1}{2} \left(\frac{\cos^2 \alpha}{\sqrt{d_1}} \right) k^2. \quad (40)$$

Thus, low-frequency, long-wavelength Alfvén waves have second order dispersion, implying that its nonlinear evolution as Alfvénic solitary waves cannot be precisely described by the Korteweg de-Vries (KdV)-like equations but by NLS-like equations.

Similarly, looking for an approximate dispersion relation for the magnetosonic wave (hybridized by the coupling effect of the Alfvén wave), we obtain from Eq. (31) the following relation:

$$\omega_{F,S}^2 = \frac{k}{2} \left[(1 + \tilde{c}_s^2)k + \sqrt{b_{1\pm} + k^2 \{2a_{\pm}^2 - (1 + \tilde{c}_s^2)\}^2} \right], \quad (41)$$

where the plus and minus signs correspond to the fast and slow modes with the phase velocities, $a_{\pm} \equiv \omega_{F,S}/k$, given by Eq. (30), and $b_{1\pm}$ is given by

$$b_{1\pm} = \frac{16\Omega_0^2 a_{\pm}^2 [a_{\pm}^2 - \tilde{c}_s^2 \sin^2 \lambda - \cos^2(\alpha + \lambda)]}{a_{\pm}^2 - \cos^2 \alpha}. \quad (42)$$

In the long-wavelength limit $k \ll 1$, Eq. (41) reduces to

$$\omega_{F,S} \approx b_{1\pm}^{1/4} K + \frac{(1 + \tilde{c}_s^2)}{b_{1\pm}^{1/4}} K^3 + \frac{\{2a_{\pm}^2 - (1 + \tilde{c}_s^2)\}^2}{b_{1\pm}^{3/4}} K^5, \quad (43)$$

where $K = \sqrt{k/2}$. Equation (43) discerns that low-frequency long-wavelength magnetosonic waves have a cubic or higher order dispersion. So, the corresponding weakly nonlinear evolution of magnetosonic solitary waves can be governed by a KdV-like equation with or without higher-order (than cubic) dispersion.

From the results obtained in this section, we may conclude that the Alfvén and magnetosonic waves propagating in a rotating magnetoplasma get coupled by the influence of the Coriolis force. The Coriolis force plays an important role in retaining the magnetic properties of the magnetosonic mode even when the wave propagation is parallel to the magnetic field. This is in contrast to non-rotating plasmas, where the magnetosonic modes emerge as acoustic-like modes.²⁵ The reason may be that the Coriolis and Lorentz forces have mathematical similarities, i.e., they are proportional and perpendicular to the particle velocity. The Alfvén and magnetosonic waves also become dispersive due to the Coriolis force effect with even (≥ 2) and odd (≥ 3) orders of dispersion, implying that their nonlinear evolution as solitary waves can be described by NLS- and KdV-like equations, respectively, and the nonlinear coupling of these waves (to be governed by a coupled KdV- and NLS-like equations) could be more pronounced in the presence of the Coriolis force.

C. Wave motion with the Hall effect but without the Coriolis force

In this section, we consider the MHD wave propagation in a non-rotating plasma and assume that the characteristic scale length (k^{-1}) is

comparable to or smaller than the collisionless ion skin depth and the characteristic timescale of hydrodynamic oscillations is comparable to the ion gyroperiod such that the Hall effect may no longer be negligible. So, disregarding the terms involving Ω_0 in Eq. (27), we obtain^{11,12}

$$(\omega^2 - k^2 \cos^2 \alpha)[\omega^4 - (1 + \tilde{c}_s^2)k^2 \omega^2 + \tilde{c}_s^2 k^4 \cos^2 \alpha] - \omega^2 k^4 \cos^2 \alpha (\omega^2 - \tilde{c}_s^2 k^2) = 0. \quad (44)$$

We note that the third term on the left side of Eq. (44) appears due to the Hall effect, giving the Alfvén and magnetosonic modes coupled, and the coupling persists until the wave propagation direction remains oblique to the magnetic field, i.e., $0 \leq \alpha < \pi/2$. The dispersion equation (44) agrees with Eq. (8) of Ref. 11 when one considers the appropriate normalizations for the physical quantities and approximates the factor ω^2 by $\omega k \cos \alpha$ in the leading factor of the term associated with the Hall effect (the last term on the left side). Such an approximation may be valid when the Hall contribution remains small compared to the other effects, and the characteristic length scale of wave excitation is much larger than the characteristic length of wave dispersion.¹¹ In this situation, the Hall contributed term may be considered a correction to the dispersion relation. We, however, do not make any approximation at this stage; rather, we consider some particular cases. For example, we note that when $\alpha = \pi/2$, i.e., the wave propagation is perpendicular to the magnetic field, the Hall contribution disappears, and only the magnetosonic mode emerges with a frequency, given by $\omega^2 = (1 + \tilde{c}_s^2)k^2$, which means that the Hall contribution can be effective for Alfvén and magnetosonic modes when the propagation angle remains within the interval: $0 \leq \alpha < \pi/2$. On the other hand, for $\alpha = 0$, i.e., when the wave propagation is along the magnetic field, Eq. (44) gives the magnetosonic modes to emerge as the non-dispersive acoustic mode with frequency $\omega = k\tilde{c}_s$ and the Alfvén mode with the following dispersion relation:

$$\omega = \frac{k}{2} (k + \sqrt{4 + k^2}), \quad (45)$$

in which the higher-order dispersion than the first-order (see the first term and the second term under the radical sign on the right side) appears due to the Hall effect.

Next, we consider the case of oblique propagation of MHD waves with $0 < \alpha < \pi/2$ and obtain approximate dispersion relations for both Alfvén and magnetosonic modes in the same way as in Sec. III B. For the Alfvén waves, modified by the coupling effect of the magnetosonic wave, we obtain the following dispersion relation:

$$\omega^2 = k^2 \cos^2 \alpha + d_2 k^4, \quad (46)$$

where $d_2 = (\tilde{c}_s^2 - \cos^2 \alpha) \cot^2 \alpha$, which is positive in a high-beta regime ($\beta \sim \tilde{c}_s^2 > 1$) and can be negative in a low-beta regime ($\beta < 1$) depending on the angle α . In the long-wavelength limit, $k \ll 1$, Eq. (46) reduces to¹¹

$$\omega = k \cos \alpha + \frac{1}{2} d_2 k^3 \sec \alpha. \quad (47)$$

This hybridized low-frequency Alfvén mode has a cubic (or higher) order dispersion, implying that it would be meaningful to describe the nonlinear evolution of small-amplitude Alfvén waves by a KdV-like equation. In the same way, an approximate dispersion relation for the magnetosonic wave can be obtained from Eq. (44) as

$$\omega_{F,S}^2 = \frac{k^2}{2} \left[(1 + \tilde{c}_s^2) + \sqrt{\{2a_{\pm}^2 - (1 + \tilde{c}_s^2)\}^2 + b_{2\pm}k^2} \right], \quad (48)$$

where a_{\pm} is defined by Eq. (30) and $b_{2\pm}$ (where the plus sign is for the fast mode and the minus sign for the slow mode) is given by

$$b_{2\pm} = \frac{4a_{\pm}^2(a_{\pm}^2 - \tilde{c}_s^2)\cos^2\alpha}{(a_{\pm}^2 - \cos^2\alpha)}. \quad (49)$$

In the long-wavelength limit, $k \ll 1$, Eq. (48) reduces to¹¹

$$\omega = a_{\pm}k + b_{\pm}k^3, \quad (50)$$

where $a_- < a_+$ and b_{\pm} are given by

$$b_{\pm} = \frac{a_{\pm}(a_{\pm}^2 - \tilde{c}_s^2)\cos^2\alpha}{2(a_{\pm}^2 - \cos^2\alpha)\{2a_{\pm}^2 - (1 + \tilde{c}_s^2)\}}, \quad (51)$$

with $b_+ > 0$ and $b_- < 0$. Like the Alfvén wave, the low-frequency long-wavelength magnetosonic mode also has a cubic or higher-order dispersion for which the KdV theory may be applicable for the evolution of weakly nonlinear magnetosonic solitary waves.

Thus, we may conclude that the Alfvén and magnetosonic waves get coupled and have higher-order dispersion by the Hall effect. Also, they have the same form of dispersion relation with odd (cubic or higher) order of dispersion in the long-wavelength limit, implying that both can propagate as low-frequency long-wavelength fundamental modes, whose evolution as weakly nonlinear solitary waves can be governed by KdV-like equations; however, their nonlinear couplings may not be effective by the Hall effect.

D. Wave motion with the Coriolis force and Hall effects

We consider a general situation in which both the Hall and Coriolis effects are present in the model. Before we study the general dispersion equation (27) in Sec. IV, we consider some particular cases and obtain approximate dispersion relations for the Alfvén and magnetosonic waves. For example, when $\alpha = \pi/2$, Eq. (27) reduces to the same as Eq. (32), i.e., the Hall contribution disappears and so does the Alfvén mode, and we have only the fast and slow magnetosonic modes. However, for propagation parallel to the static magnetic field ($\alpha = 0$), Eq. (27) reduces to

$$\begin{aligned} &(\omega^2 - k^2)\{\omega^4 - (1 + \tilde{c}_s^2)k^2\omega^2 + \tilde{c}_s^2k^4\} - 4\Omega_0^2\omega^2 \\ &\times (\omega^2 - \tilde{c}_s^2k^2\sin^2\lambda - k^2\cos^2\lambda) - \omega^2k^4(\omega^2 - \tilde{c}_s^2k^2) \\ &+ 4\Omega_0^4k^4\sin\lambda(\omega^2 - \tilde{c}_s^2k^2) + 4\Omega_0^2k^4(\omega^2 - \tilde{c}_s^2k^2\sin^2\lambda) = 0. \end{aligned} \quad (52)$$

In the low-beta regime, $\beta \sim \tilde{c}_s^2 \ll 1$ and if $\omega \gg k\tilde{c}_s$, Eq. (52) further reduces to

$$\begin{aligned} &(\omega^2 - k^2)^2 - 4\Omega_0^2(\omega^2 - k^2\cos^2\lambda) - k^4\omega^2 \\ &+ 4\Omega_0^4k^4\sin\lambda + 4\Omega_0^2k^4 = 0, \end{aligned} \quad (53)$$

which gives two stable kinetic Alfvén wave modes modified by the Hall and Coriolis force effects. An approximate dispersion relation for the shear Alfvén wave with $0 < \alpha < \pi/2$ can be obtained in the same way as in Sec. III B as

$$\omega^2 = d_1 + d_3k^2 + d_2k^4, \quad (54)$$

where

$$\begin{aligned} d_3 = \frac{1}{\sin^2\alpha} &[\sin^2\alpha\cos^2\alpha + 4\Omega_0\cos\alpha \\ &\times \{\cos\alpha\sin(\alpha + \lambda) - \tilde{c}_s^2\sin\lambda\} + 4\Omega_0^2 \\ &\times (\cos^2\alpha - \tilde{c}_s^2\sin^2\lambda)]. \end{aligned} \quad (55)$$

The term proportional to d_3 appears due to the combined influence of the Hall and Coriolis force effects, and it vanishes for $\alpha = \pi/2$ and $\lambda = 0$. In the long-wavelength limit, $k \ll 1$, Eq. (54) reduces to

$$\omega \approx \sqrt{d_1} + \frac{d_3k^2}{2\sqrt{d_1}} + \frac{d_2k^4}{2\sqrt{d_1}}, \quad (56)$$

where $d_1 > 0$. This dispersion equation has the same form as Eq. (37) except for the higher-order dispersion ($\propto k^4$), which appears due to the Hall effect. So, the evolution of weakly nonlinear Alfvén wave envelope can be described by a NLS-like equation. In the same way, an approximate dispersion relation for the magnetosonic wave with $0 < \alpha < \pi/2$ can be obtained as

$$\omega^2 = \frac{k}{2}[(1 + \tilde{c}_s^2)k + \sqrt{b_{1\pm} + \{2a_{\pm}^2 - (1 + \tilde{c}_s^2)\}^2k^2 - b_{3\pm}k^2 + b_{2\pm}k^4}], \quad (57)$$

where a_{\pm} is defined in Eq. (30) and $b_{3\pm}$ is given by

$$\begin{aligned} b_{3\pm} = \frac{16\Omega_0\cos^2\alpha}{(a_{\pm}^2 - \cos^2\alpha)} &[a_{\pm}^2\sin(\lambda + \alpha) - \tilde{c}_s^2\sin\lambda\cos\alpha] \\ &+ \Omega_0(a_{\pm}^2 - \tilde{c}_s^2\sin^2\lambda). \end{aligned} \quad (58)$$

The term proportional to $b_{3\pm}$ appears due to the combined influence of the Hall and Coriolis force effects, and it vanishes at $\alpha = \pi/2$. In the long-wavelength limit, $k \ll 1$, Eq. (57) reduces to

$$\begin{aligned} \omega \approx \frac{b_{1\pm}^{1/4}}{\sqrt{2}}k^{1/2} &+ \frac{(1 + \tilde{c}_s^2)}{2\sqrt{2}b_{1\pm}^{1/4}}k^{3/2} + \frac{[\{2a_{\pm}^2 - (1 + \tilde{c}_s^2)\}^2 - b_{3\pm}]}{4\sqrt{2}b_{1\pm}^{3/4}}k^{5/2} \\ &+ \frac{b_{2\pm}}{4\sqrt{2}b_{1\pm}^{3/4}}k^{9/2}. \end{aligned} \quad (59)$$

This dispersion equation has the same form as Eq. (43) except for the higher-order dispersion ($\propto k^{9/2}$), which appears due to the Hall effect. So, in the nonlinear regime, the magnetosonic mode can evolve as solitary waves, whose evolution can be described by the KdV-like equations.

IV. RESULTS AND DISCUSSION

In this section, we numerically study the dispersion characteristics of Alfvén and magnetosonic waves and their coupling by the influences of the Coriolis force and Hall current, as well as the obliqueness of the propagation vector to the magnetic field and the fluid rotational angle λ about the y -axis. From the general dispersion relation (27), we observe that there are five terms, and in each term, there appear two main factors, one of which corresponds to an Alfvén mode and the other to magnetosonic waves. Specifically, for the first term, the first factor corresponds to the Alfvén wave, whereas the second factor corresponds to the magnetosonic modes. We can verify it by disregarding the Coriolis force and Hall effects and equating each factor to zero.

Similarly, for the second (due to the Coriolis force or rotational effect) and third (due to the Hall effect) terms, the factor ω^2 is for the Alfvén mode, and the factors in the square brackets and parentheses are for the magnetosonic modes. Inspecting the fourth and fifth terms (which appear due to combined influences of the Hall current and Coriolis force), we find that the factor $k^2 \cos^2 \alpha$ can correspond to the Alfvén mode and the factors in the square brackets and parentheses to the magnetosonic modes. Thus, upon viewing the appearance of the Alfvén mode factor in the fourth and fifth terms with the Hall effects, it may be reasonable to replace the factor ω^2 in the third term (associated with the Hall effect) by $k^2 \cos^2 \alpha$. From our numerical results, we will see that such an approximation captures more precisely the coupling between the Alfvén and fast magnetosonic waves and the Alfvén and slow magnetosonic waves in the forms of thumb and teardrop-like curves, respectively (see, e.g., Fig. 2). Here, we note that Eq. (27) also exhibits similar couplings between the modes but outside the domains: $\omega, k < 1$ and with a different set of values of α and λ . The advantage of considering Eq. (60) is that it exhibits coupling within the domains of definitions of ω and k , i.e., $\omega, k \leq 1$. For a similar approximation of replacing ω^2 by $k^2 \cos^2 \alpha$, we may refer to Ref. 11. Thus, the modeling of the dispersion equation by Eq. (60) instead of Eq. (27) may be reasonable, especially while predicting or relating to astrophysical MHD wave phenomena. So, from Eq. (27), we obtain the following reduced dispersion relation:

$$\begin{aligned}
 &(\omega^2 - k^2 \cos^2 \alpha)[\omega^4 - (1 + \tilde{c}_s^2)k^2 \omega^2 + \tilde{c}_s^2 k^4 \cos^2 \alpha] \\
 &- 4\Omega_0^2 \omega^2 [\omega^2 - \tilde{c}_s^2 k^2 \sin^2 \lambda - k^2 \cos^2 (\alpha + \lambda)] \\
 &- k^6 \cos^4 \alpha (\omega^2 - \tilde{c}_s^2 k^2) + 4\Omega_0 k^4 \cos^2 \alpha [\omega^2 \sin(\alpha + \lambda) \\
 &- \tilde{c}_s^2 k^2 \cos \alpha \sin \lambda] + 4\Omega_0^2 k^4 \cos^2 \alpha (\omega^2 - \tilde{c}_s^2 k^2 \sin^2 \lambda) = 0. \quad (60)
 \end{aligned}$$

We contour plot the dispersion relation (60) in three different regimes of β : (a) $\beta \sim 1$, (b) $\beta > 1$, and (c) $\beta < 1$, to exhibit the dispersion curves for three fundamental modes, namely, the Alfvén wave (ω_A) and the fast (ω_F) and slow (ω_S) magnetosonic modes, as well as their coupling as shown by the solid lines in Fig. 2. The dashed lines represent the curves corresponding to Eq. (27). We observe that while the coupling between the Alfvén and fast magnetosonic waves become viable in the regimes of $\beta \leq 1$, such as those in the solar corona and upper chromospheric regions, between the Alfvén and slow magnetosonic waves it occurs in the high- β ($\beta > 1$) regimes, e.g., in the solar photosphere and lower chromospheric regions. Comparing the dispersion curves corresponding to Eqs. (27) and (60), we see that the dispersion equation (60) exhibits stronger couplings in the forms of thumb and teardrop-like curves between the modes than Eq. (27) and that the dispersion curves of Eqs. (27) and (60) mostly agree in the propagation domains $0 \leq (\omega, k) \leq 1$. However, in contrast to Eq. (27), Eq. (60) predicts the cutoff frequencies for the slow mode at finite values of k . An important point is that all the MHD waves corresponding to Eq. (60) propagate with a frequency below the ion-cyclotron frequency, and the coupling between the modes occurs within the domain $0 \leq k \leq 1$.

Next, we study the influence of the Coriolis force on the wave coupling and dispersion properties of Alfvén and magnetosonic modes using Eq. (60). The results are shown in Fig. 3. In Sec. III B, we have seen that the Coriolis force not only plays a crucial role in the wave coupling but also retains the magnetic properties of the magnetosonic modes even when wave propagation is parallel to the magnetic field. So, an analysis of how the Coriolis force influences the MHD modes is pertinent. From subplot (a), we see that as the angular velocity Ω_0 is reduced, or the contribution from the Coriolis force weakens, the thumb curve splits, resulting in separation of the fast magnetosonic

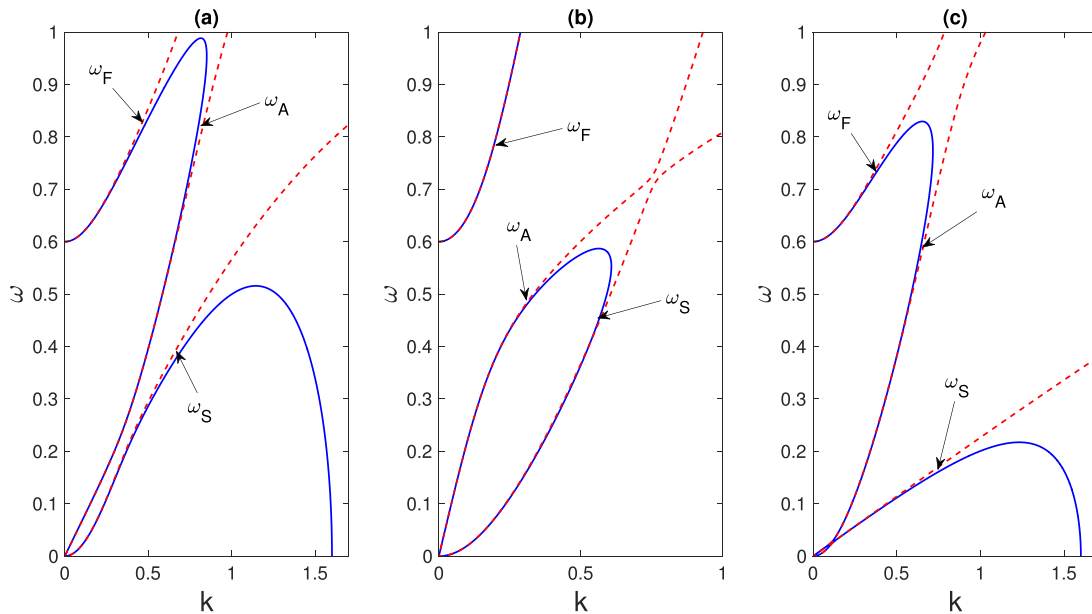


FIG. 2. Dispersion relation is contour plotted in the ωk plane to show three fundamental MHD modes: the Alfvén wave (ω_A) and the fast (ω_F) and slow (ω_S) magnetosonic waves, as indicated in the figure when both the Coriolis force and the Hall effects are included in the model. The dashed and solid lines, respectively, correspond to the dispersion relations (27) and (60). Unless otherwise stated, the fixed parameter values are $\Omega_0 = 0.3$, $\alpha = 40^\circ$, and $\lambda = 50^\circ$. Subplots (a), (b), and (c) show the modes in three different regimes: (a) $\beta \sim 1$ ($\tilde{c}_s \sim 0.85$), (b) $\beta > 1$ ($\tilde{c}_s \sim 3$), and (c) $\beta < 1$ ($\tilde{c}_s \sim 0.3$), respectively.

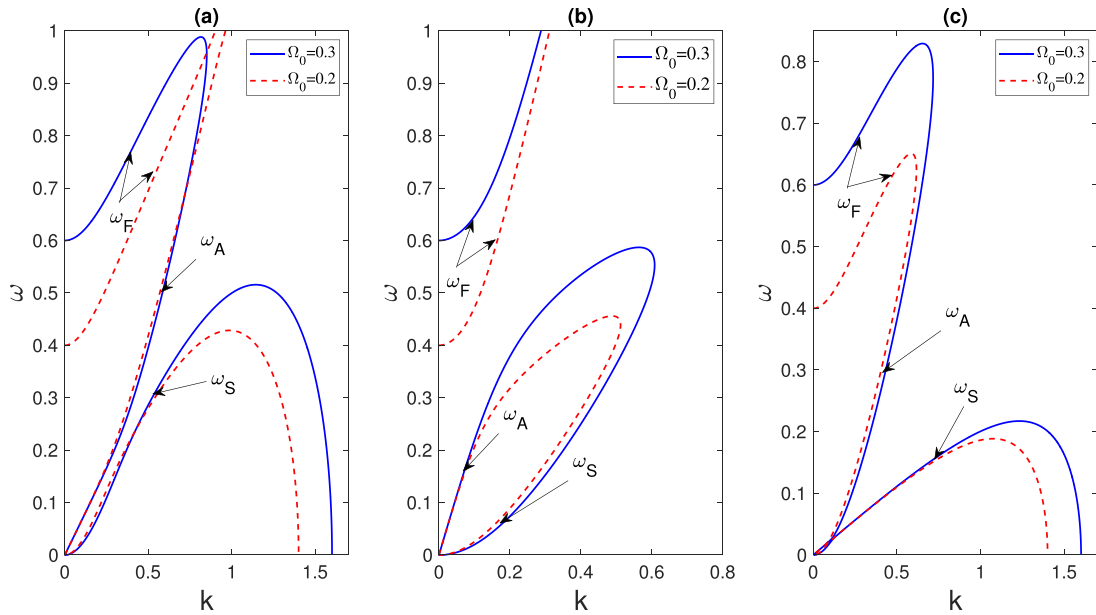


FIG. 3. Dispersion relation [Eq. (60)] is contour plotted in the ωk plane to show three fundamental MHD modes: Alfvén wave (ω_A) and fast (ω_F) and slow (ω_S) magnetosonic waves as indicated in the figure when both the Coriolis force and the Hall effects are included in the model. The solid and dashed lines, respectively, correspond to two different values of Ω_0 : $\Omega_0 = 0.3$ and $\Omega_0 = 0.2$, or as in the legends. Unless otherwise stated, the fixed parameter values are $\alpha = 40^\circ$ and $\lambda = 50^\circ$. Subplots (a), (b), and (c) show the modes in three different regimes: (a) $\beta \sim 1$ ($\bar{c}_s = 0.85$), (b) $\beta > 1$ ($\bar{c}_s = 3$), and (c) $\beta < 1$ ($\bar{c}_s = 0.3$), respectively.

and Alfvén modes within the frequency domain ($0 < \omega < 1$) and exhibiting weak coupling between the modes, i.e., larger the Coriolis force, the stronger is the coupling between the fast magnetosonic and the Alfvén modes, especially in the regime $\beta \sim 1$. However, the values of Ω_0 should be limited such that the values of ω do not greatly exceed unity. In the regime of $\beta \lesssim 1$, subplots (a) and (c) show that while the fast magnetosonic mode frequency is significantly reduced, the frequency of the Alfvén wave remains almost unchanged with a reduction of Ω_0 . In the latter at $\Omega_0 = 0.2$, the frequency of the slow magnetosonic mode also gets reduced but at higher values of k having a cutoff at a lower value of k than when $\Omega_0 = 0.3$ and its coupling with the Alfvén mode remains stronger in the form of a thumb curve. On the other hand, in the high- β regime with $\beta > 1$, the fast magnetosonic mode has a similar property with a wave-frequency reduction as in the cases $\beta \lesssim 1$ [subplots (a) and (c)]. However, while the Alfvén wave frequency gets reduced, the frequency of the slow mode gets enhanced. Both the modes have stronger coupling, exhibiting a teardrop-like curve even with a reduction of Ω_0 from $\Omega_0 = 0.3$ to $\Omega_0 = 0.2$. From the analysis, we may conclude that to have strong coupling between the Alfvén and fast magnetosonic modes in the regime of $\beta \sim 1$, the values of Ω_0 must exceed a critical value. For the coupling of the other modes in other regimes of β , there is no such critical value.

We also study the effects of the angle of propagation α and the rotational angle λ on the dispersion properties of the MHD waves and their coupling. The results are displayed in Fig. 4. From subplots (a) and (c) (in the regimes of $\beta \sim 1$ and $\beta < 1$), it is interesting to note that depending on the values of λ and α , the thumb curve for the coupling of the fast magnetosonic mode and the Alfvén wave may form or split into two separate curves. For example, for subplot (a), the thumb curve forms (exhibiting strong coupling) in the regimes: $0 < \alpha \lesssim 40^\circ$,

$0 < \lambda < 90^\circ$; $\alpha \approx 45^\circ$, $0 < \lambda < 65^\circ$, and $\alpha \approx 50^\circ$, $0 < \lambda < 25^\circ$. For the subplot (c), the regimes are $0 < \alpha \lesssim 59^\circ$, $0 < \lambda < 25^\circ$; $\alpha \approx 60^\circ$, $75^\circ < \lambda < 90^\circ$; and $\alpha \approx 61^\circ$, $80^\circ < \lambda < 90^\circ$, otherwise the thumb curve splits, resulting in a weak coupling between the fast magnetosonic and the Alfvén modes. On the other hand, subplot (b) shows that in the regimes of $\beta > 1$, the coupling between the Alfvén and slow magnetosonic modes occurs in the form of a teardrop curve, which remains for wide ranges of values of α and λ in $0 < (\alpha, \lambda) < 90^\circ$. A novel feature in the regime of $\beta \sim 1$ [subplot (a)] is that a switching of the coupling occurs, i.e., the thumb curve coupling between the fast magnetosonic and Alfvén modes [which is also the case in the low- β regime with $\beta < 1$; see subplot (c)] shifts to the teardrop coupling between the Alfvén and slow magnetosonic modes [which is typically the case in the high- β regime with $\beta > 1$; see subplot (b)] when both α and λ assume values close to 70° or more, e.g., in the regimes $68^\circ \lesssim \alpha < 90^\circ$, $72^\circ \lesssim \lambda < 90^\circ$ and $69^\circ \lesssim \alpha < 90^\circ$, $68^\circ \lesssim \lambda < 90^\circ$. From subplots (a) and (c), we also observe that unless α and λ assume values close to 70° or more, the slow magnetosonic mode exhibits cutoffs at different finite values of the wave number k as the parameters α and λ vary.

Finally, we study the relative or combined influence of the Coriolis force and Hall effects on the dispersion characteristics of MHD waves. The results are shown in Fig. 5 in three different regimes of β : (a) $\beta \sim 1$, (b) $\beta > 1$, and (c) $\beta < 1$. We consider the cases: (i) when both the effects are present and (ii) when any of them is present. Comparing the solid lines with the dashed or dotted lines, we observe that the strong coupling between the fast magnetosonic and Alfvén modes (thumb curve) and between the Alfvén and slow magnetosonic modes (teardrop curve) are possible only when both the effects are present in the model. Also, from the dashed and dotted lines, we note that

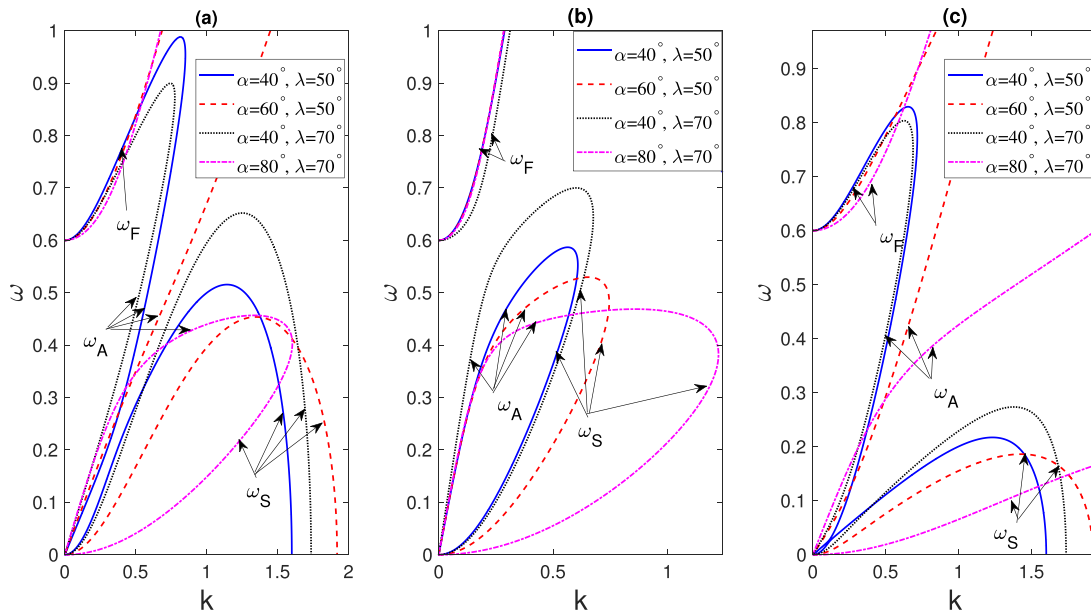


FIG. 4. Dispersion relation [Eq. (60)] is contour plotted in the ωk plane to show three fundamental MHD modes: Alfvén wave (ω_A) and Fast (ω_F) and slow (ω_S) magnetosonic waves as indicated in the figure when both the Coriolis force and the Hall effects are included in the model. The solid, dashed, and dotted lines, respectively, correspond to different values of the angles α (made by the external magnetic field with the x -axis) and λ (made by the axis of rotation with the y -axis) as indicated in the legends but with a fixed $\Omega_0 = 0.3$. Subplots (a), (b), and (c) show the modes in three different regimes: (a) $\beta \sim 1$ ($\bar{c}_s = 0.85$), (b) $\beta > 1$ ($\bar{c}_s = 3$), and (c) $\beta < 1$ ($\bar{c}_s = 0.3$), respectively.

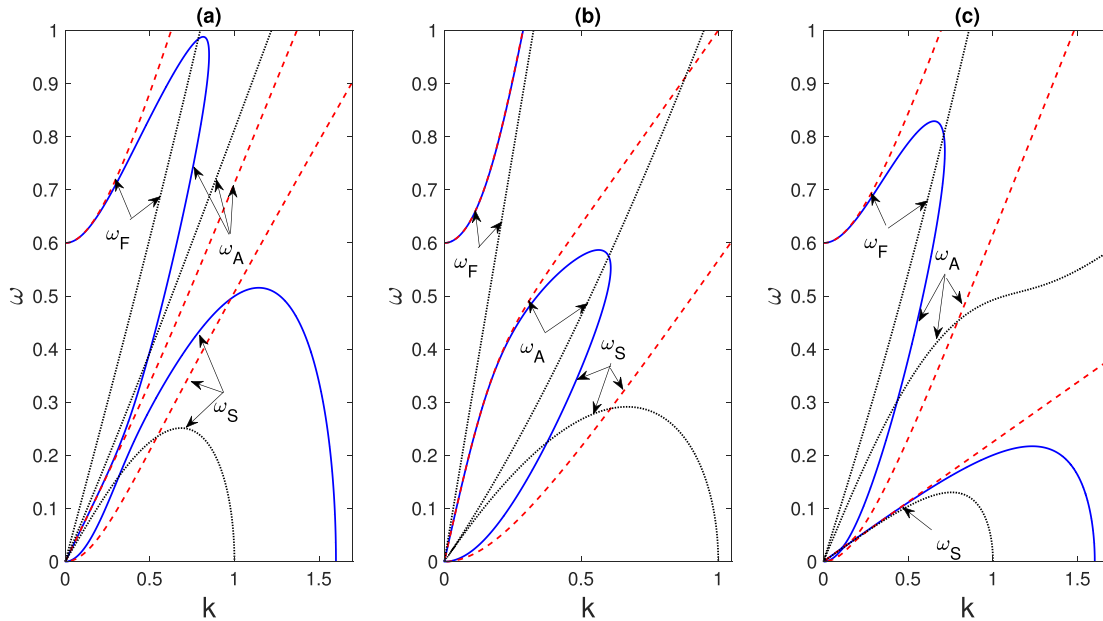


FIG. 5. Dispersion relation [Eq. (60)] is contour plotted in the ωk plane to show three fundamental MHD modes: Alfvén wave (ω_A) and fast (ω_F) and slow (ω_S) magnetosonic waves as indicated in the figure. The solid, dashed, and dotted lines, respectively, correspond to the cases when (i) both the Coriolis force (with $\Omega_0 = 0.3$) and Hall effects are present, (ii) the Coriolis force is present (with $\Omega_0 = 0.3$) but in the absence of the Hall effects, and (iii) the Hall effects are included but in the absence of the Coriolis force. The other fixed parameter values are $\alpha = 40^\circ$ and $\lambda = 50^\circ$. Subplots (a), (b), and (c) show the modes in three different regimes: (a) $\beta \sim 1$ ($\bar{c}_s = 0.85$), (b) $\beta > 1$ ($\bar{c}_s = 3$), and (c) $\beta < 1$ ($\bar{c}_s = 0.3$), respectively.

the slow magnetosonic modes exhibit cutoffs at finite values of k by the influence of the Hall effect. Thus, we conclude that the combined influence of the Coriolis force and the Hall effect is the prerequisite for the strong coupling of Alfvén and magnetosonic modes. The slow mode exhibits cutoff due to the influence of the Hall current. The rotational frequency should be above a critical value but be limited (< 1) such that the centrifugal force may remain much smaller than the Coriolis force.

V. APPLICATIONS TO SOLAR AND FUSION PLASMAS

The Sun's atmosphere, which typically consists of the photosphere, chromosphere, and corona, is an active medium with a wide range of temperatures and densities. Due to, e.g., convective gas motions, the three fundamental MHD modes, namely, the Alfvén wave and the fast and slow magnetosonic modes, can be excited in the medium. Typically, in a uniform plasma, these modes propagate independently. However, in an inhomogeneous medium (e.g., coronal plasma), they propagate as coupled modes. We have seen in Secs. III B–III D that even in a homogeneous medium, the coupling between the modes can be possible by the influence of either the Coriolis force or the Hall current effect, and the coupling can be strong by their combined influences. While the solar photosphere and chromosphere are typically partially ionized plasmas, the highly ionized corona is collisionless and its low-frequency dynamics can be described by the ideal MHD or the HMHD model.¹³ Furthermore, ground- and space-based observations have confirmed small-amplitude (3–10 km/s) as well as large-amplitude (more than 10 km/s) oscillatory motions in the Solar atmosphere, which have been interpreted in terms of propagating MHD modes.

In what follows, we study the relevance of the MHD modes, especially the Hall and Coriolis force effects in the Solar corona. To this end, we consider typical plasma parameters and examine the Hall time and length scales, together with the rotational angle and the angular velocity associated with the Coriolis force. Coronal heating by MHD waves has been a feasible mechanism for interpreting high coronal temperatures compared to the lower photosphere. In the latter, the MHD waves are excited by the footprint motion of the magnetic field lines, which later emerge into the corona. In this context, Alfvén waves, while propagating in the corona, may lose energy to plasma particles by resonance damping and thus can accelerate solar plasmas from coronal holes. The power spectra of horizontal motions in the photosphere also indicate the existence of MHD waves with frequencies ranging from 10^{-5} to 0.1 Hz at a few solar radii. So, one can expect an observable frequency of the order of 1 Hz. Also, low-frequency whistler waves can exist close to the footprint of the magnetic flux tube with reduced neutral density. These waves later turn into Alfvén waves, which propagate into the corona and heat it. Due to the decreasing neutral density and increasing ionization, such waves propagate almost undamped in the corona. Thus, the Hall effect may be responsible for exciting whistler waves in the lower photosphere or lower part of the corona. Typically, the Hall scale size is approximately more than a few tens of kilometers. Thus, the MHD modes due to the Hall effects could be important for modeling various coronal heating.

In the present fully ionized plasma model, the Hall timescale is $\tau_H \sim \omega_{ci}^{-1}$, i.e., the inverse of the ion-cyclotron frequency and the Hall length-scale is $L_H \sim V_A/\omega_{ci} \equiv \lambda_i$, i.e., the ion skin depth, implying that in fully ionized plasmas (e.g., Solar corona), the Hall effects become important for $\omega \gtrsim \omega_H \sim \omega_{ci}$, or $\omega \lesssim \omega_{ci}$ and the length-scale comparable to the ion skin depth, i.e., $L_H \sim \lambda_i$. For typical parameters

(relevant for coronal loops) with the magnetic field strength $B_0 \sim 100$ G, the temperature $T \sim 6 \times 10^6$ K, the number density $n_0 \sim 1.4 \times 10^9 \text{ cm}^{-3}$, and assuming the ion mass as the proton mass, we have the Alfvén speed $V_A \sim 6 \times 10^6$ m/s, the ion skin depth or the Hall length-scale $L_H \sim 6$ m, and the Hall frequency or the ion-cyclotron frequency $\omega_H \sim \omega_{ci} \sim 9.7 \times 10^5 \text{ s}^{-1}$. Adopting a coronal region of radius $a \sim 10$ m, or 1 m, one can have the Alfvén frequency as $\omega_A = V_A/a \sim 6 \times 10^5 \text{ s}^{-1}$, or $6 \times 10^6 \text{ s}^{-1}$, i.e., the Hall effect is important either for $\omega \lesssim \omega_{ci}$, or for $\omega \gtrsim \omega_{ci}$. In the former, the Hall effect is comparable to the Coriolis force effect since $\omega_A \sim 2\Omega_0 \sim 6 \times 10^5 \text{ s}^{-1}$, in the latter, the Hall effects can dominate over the Coriolis force. It also follows that the Hall frequency is much higher, but the Hall length-scale is much lower than the corresponding scales for the solar photosphere at an altitude of about 10^3 km or ionospheric plasmas at an altitude of about 150 km (or lower).

On the other hand, the Hall effect can play a crucial role in tokamak discharges or non-Ohmic current drive schemes.⁵ It is also important near the wall region of a tokamak. Since near the wall region, the ionization rate is high ($n_n/n_i \sim 10^{-4}$ – 10^{-3}) with n_n denoting the neutral density, the Hall time and length scales will remain the same as for fully ionized coronal plasmas. For typical fusion plasma parameters¹³ with $B_0 \sim 10$ kG, $n_0 \sim 1.4 \times 10^9 \text{ cm}^{-3}$, and assuming the ion mass as the proton mass, we have $V_A \sim 2.2 \times 10^6$ m/s, $L_H \sim \lambda_i \sim 0.023$ m, and $\omega_H \sim \omega_{ci} \sim 9.7 \times 10^7 \text{ s}^{-1}$. Adopting a major radius of the tokamak, $a \sim 1$ m, we have $\omega_A \sim 2.2 \times 10^6 \text{ s}^{-1}$, i.e., the Hall scales are much smaller than for coronal plasmas. Thus, in fusion plasmas (near the wall region of a tokamak), the Hall effect may be important for $\omega \sim \omega_{ci}$ and the length-scale comparable to the ion skin depth, i.e., $L_H \sim \lambda_i$.

VI. SUMMARY AND CONCLUSION

We have studied the existence and coupling of obliquely propagating Alfvén and magnetosonic waves in a rotating Hall-MHD magnetoplasma, as well as analyzed the characteristics of these modes in three different regimes of β by the influences of the obliqueness of wave propagation, the angular velocity, and rotational angle associated with the Coriolis force, and the Hall current effect. We have seen that the coupling between the Alfvén and magnetosonic waves can occur by the Coriolis force or Hall effects. Also, while in the regime $\beta \leq 1$, the coupling between the fast magnetosonic and Alfvén modes becomes prominent, and they appear in the form of a thumb curve, the teardrop-like coupling between the slow magnetosonic and Alfvén modes occurs in the high- β ($\beta > 1$) regime. We summarize the main results as follows.

- (1) The MHD waves propagate with a frequency below the ion-cyclotron frequency, and the coupling between the modes occurs within the domain $0 \leq k \leq 1$.
- (2) *Without Hall effect:* In the absence of the Hall effect, an interesting feature in a rotating plasma is that even for MHD wave propagation parallel to the static magnetic field, not only are both the magnetic and acoustic characters of the magnetosonic modes retained, but the modes also get coupled to the Alfvén mode by the influence of the Coriolis force. However, in the absence of the Coriolis force, both the Alfvén and magnetosonic modes get decoupled, giving the fast mode to appear as the Alfvén mode $\omega = k$ and the slow mode as the acoustic mode $\omega = \tilde{c}_s k$.

- (3) *Without Hall effect*: The Alfvén and magnetosonic waves become dispersive due to the Coriolis force effect with even (≥ 2) and odd (≥ 3) orders of dispersion, implying that their nonlinear evolution as solitary waves can be described by NLS- and KdV-like equations, respectively, and the nonlinear coupling of these waves (to be governed by a coupled KdV- and NLS-like equations) could be more pronounced in the presence of the Coriolis force.
- (4) *Without Coriolis force*: In the absence of the Coriolis force (non-rotating plasmas), the Alfvén and magnetosonic waves also get coupled and have higher-order dispersion (than by the Coriolis force) by the Hall effect. Also, they have the same form of dispersion relation with odd (cubic or higher) order of dispersion in the long-wavelength limit, implying that both can propagate as low-frequency long-wavelength fundamental modes, whose evolution as weakly nonlinear solitary waves can be governed by KdV-like equations; however, their nonlinear couplings may not be effective by the Hall effect.
- (5) *With Coriolis force and Hall effect*: To have strong coupling between the Alfvén and fast magnetosonic modes in the regime of $\beta \sim 1$, the values of Ω_0 must exceed a critical value. For the coupling of the other modes in other regimes of β , there is no such critical value. In the regime, $\beta \sim 1$, a switching of the coupling occurs, i.e., the thumb curve coupling between the fast magnetosonic and Alfvén modes (which is the case for $\beta < 1$) shifts to the teardrop coupling between the Alfvén and slow magnetosonic modes (which is typically the case for $\beta > 1$) when both α and λ assume values close to 70° or more (but less than 90°).

To conclude, the combined influence of the Coriolis force and the Hall effect is the prerequisite for the strong coupling of Alfvén and magnetosonic modes in which the slow mode exhibits cutoff by the influence of the Hall current and the rotational frequency should be above a critical value but be limited ($< \omega_{ci}$) such that the centrifugal force remains smaller than the Coriolis force. We have noted that the coupling between the slow magnetosonic and Alfvén modes is typically weak (except in a particular domain of Ω_0 stated above; cf. Fig. 5) and that between the fast magnetosonic and Alfvén modes is rather strong in the low- β regime. Since Alfvén wave can drive both the slow and fast modes, the process of driving the fast modes can be enhanced by the phase mixing effect due to magnetic field inhomogeneity. Furthermore, it has been shown that the inhomogeneities in the pressure gradient and magnetic field-line curvature can play crucial roles in the MHD wave couplings.³² So, considering these inhomogeneity effects in the model could enrich the coupling mechanism and modify their dispersion characteristics, which could be a project for future research. Also, in the present HMHD model, we have neglected the electron inertial effect. However, since electrons can drift more freely than ions with the magnetic field, such effects could be important at frequencies higher than the ion-cyclotron frequency. So, extending the HMHD with electron inertia included could be another project of future study.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Amar P. Misra: Conceptualization (lead); Formal analysis (equal); Investigation (lead); Methodology (equal); Supervision (lead); Validation (lead); Writing – original draft (equal); Writing – review & editing (equal). **Rupak Dey**: Conceptualization (supporting); Formal analysis (equal); Investigation (equal); Methodology (equal); Validation (equal); Writing – original draft (equal). **Vinod Krishan**: Conceptualization (supporting); Formal analysis (supporting); Investigation (supporting); Methodology (supporting); Validation (equal); Writing – review & editing (supporting).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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