对具有最大曲率的广义相对论的 Born–Infeld 型修正

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摘　　要

由于挠率与自旋的基本单位 $\hbar$ 有关，空时拓扑的缺陷应当以 Planck 长度的倍数出现，Planck 长度出自空时结构的内禀缺陷，这意味着 Planck 长度是最小的基本长度，进而意味着有一个最大曲率存在。

由此可得出一些物理结论　当坍缩物体接近其 Schwarzschild 半径时，熵不会变为无穷大，而且我们还得出黑洞的最小质量（不为零且等于 M_{Pl}）。具有这一最小质量的黑洞可能在早期宇宙中形成，或者作为较大的黑洞在蒸发中的残余幸存下来。

关键词　挠率—最大曲率—黑洞

A Born-Infeld Type of Modification of General Relativity with Maximal Curvature

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Abstract

Based on the fact that torsion is related to fundamental unit of spin $\hbar$, defects in space-time topology should occur in multiples of Planck length that follows from the intrinsic defect built into the structure of space-time, implying a minimal fundamental length which, in turn, implies a maximal curvature. This results has some physical consequences. entropy do not become infinite as the collapsing object approaches the Schwarzschild radius and moreover we have also a minimal mass ($\neq 0$ and

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$=M_{pl}$) of black holes that could form in the early universe or survive as a remnant in the evaporation of larger black holes.

Key words black holes—torsion—maximal curvature

In an attempt to overcome the problem of the infinite self energy of a point charge and other singular problems in classical electrodynamics, Born and Infeld[1] suggested a modification of the Maxwell equations incorporating the notion of a maximal field strength E^2_{max} which an electric field can have. In the weak field limit defined by $F_{\mu\nu}F_{\mu\nu} \ll E^2_{max}$ one recovers the usual Maxwell theory. The modified field equations have the form:

$$\frac{\nabla \cdot E}{(1 - E^2/E^2_{max})^{1/2}} = 0 \tag{1}$$

which for $E \ll E_{max}$ becomes $\nabla \cdot E = 0$, as usual The term $(1 - E^2/E^2_{max})^{1/2}$ corresponds to an effective field-dependent dielectric constant $\varepsilon = (1 - E^2/E^2_{max})^{1/2}$.

As a result of the modification, the minimal radius of a point charge is not zero, but $\approx (e/E_{max})^{1/2}$ and one does not have the singular situation of Maxwell's theory. One can raise the question as to whether a similar modification of Einstein's general relativity could resolve similar singular problems associated with that theory.

First of all we can give an estimate about the value of E_{max} referring to one of our previous works[2] where we have given a minimum radius for the electron when considering torsion and strong gravity In fact we have

$$r_e \cong \left(\frac{G_f \hbar^2}{m_e c^4}\right)^{1/3} \tag{2}$$

(where G_f is the strong gravity constant), so that

$$E_{max} \cong \left(\frac{m_e c^4}{G_f \hbar^2}\right)^{2/3} e \tag{3}$$

and as (see ref.[2]):

$$e \cong (G_f \hbar^2 m_e^2 c^2)^{1/6} \tag{4}$$

we have

$$E_{max} \cong \frac{m_e c^3}{(G_f \hbar^2)^{1/2}} \tag{5}$$

We can write for r_{min}

$$r^2_{min} \cong \frac{e(G_f)^{1/2}\hbar}{m_e c^3} \tag{6}$$

For gravitational field also we can find a minimal length observing that we can go from electromagnetism to gravitation with the usual substitution $e \to m\sqrt{G_f}$. With this we have for r_{min}:

$$r^2_{min} \simeq \frac{G_f \hbar}{c^3} \tag{7}$$

But we can reach this result in a better way. In fact in a recent work[3] it was again emphasized that by relating torsion to the intrinsic fundamental unit of spin $\hbar$, defects in space-time topology should occur in multiples of the Planck length $(\hbar G/c^3)^{1/2}$. This intrinsic defect built into the structure of space-time would then define a minimal fundamental length, i.e. the Planck length $(\hbar G/c^3)^{1/2}$(*) entering through the minimal unit of spin or action $\hbar$. The notion of 'length' or distance would lose its physical meaning beyond this length. This in turn would also imply a maximal curvature which turns out to be

$$R_{max} \approx c^3/\hbar G \approx 10^{66}\text{cm}^{-2} \tag{8}$$

Again this is consistent with the argument in ref.[4], that the notion of "maximal acceleration" which is implied in many independent approaches[5], would through the equation of geodesic deviation imply a maximal curvature R_{max} the same as above. As in gravitation curvature plays the role of field strength, the notion of maximal curvature would suggest by analogy a Born-Infeld type of modification of general relativity A suitable action incorporating the maximal curvature R_{max}, would be:

$$\begin{aligned} I &= (c^3/16\pi G)\int\frac{\sqrt{-g}R}{(1-L_{Pl}^2 R)}\,d^4x + L_{matter} \\ &= (c^3/16\pi G)\int\frac{\sqrt{-g}R}{(1-R/R_{max})}\,d^4x + L_{matter} \end{aligned} \tag{9}$$

In the limit of small curvatures (i.e. field strengths) $R \ll R_{max}$, this would reduce to the usual Hilbert action for general relativity.

Expanding the above Lagrangian for $R \approx R_{max}$, we have the effective Lagrangian in powers of the curvature:

$$\begin{aligned} L_{eff} = \int d^4x\sqrt{-g}(1/16\pi)[Rc^3/G + \hbar R^2 + \hbar L_{Pl}^2 R^3 \\ + \cdots\cdots + \hbar L_{Pl}^{2(n-2)} R^n] \end{aligned} \tag{10}$$

It is remarkable that L_{eff} has the same form as that suggested earlier[6] by one of the present authors by very different considerations through arguments involving scale and Weyl invariance of gravity and construction of a "high energy" theory of gravity by analogy with the QCD theory of strong interactions.

Also it appears to be consistent with the approach to gravity suggested by Sakharov[7] where the zero point energy associated with the curving of background space can be expressed by expanding the gravity Lagrangian as a series of powers of the curvature of space Again it may be noted that the field theory limit of superstring theories which also involve a minimal length[8], gives rise to a similar Lagrangian.

We note that in the above expression the remarkable fact that the Hilbert term

(*) In the case of strong gravity relevant for hadronic physics, i.e. energy scales of ~ GeV, the appropriate coupling constant would be the strong gravity constant G_f. At Planck scale $G = G_N$ the Newtonian constant (see ref [8] for details) (which is the term dominating at ordinary energy scales $R \ll R_{max}$, $L \gg L_{pl}$) is the only one not having $\hbar$, in the coupling. Thus the higher power terms are to be pictured as quantum gravity corrections appearing when the curvature is near R_{max} or at distance scales of L_{pl}. The terms involving higher powers of the curvature are suppressed at ordinary energies E by high powers of (E/E_{pl}), where $E_{pl} \approx 10^{19}$GeV. Thus for all practical purposes the low energy effective action would just have the usual Hilbert term of genaral relativity. The higher powers of curvature correspond to higher powers of $\hbar$, i. e. quantum corrections of higher order.

The field equations of the above action would have the form:

$$\frac{R}{(1 - L_{pl}^2 R)^2} \approx \chi \rho_{matter} \tag{11}$$

which would indicate that even in the hypothetical case of infinite density of point particles or infinite number of particles (i.e. the right hand side becoming infinite), the curvature R would not become infinite but would tend to the maximal value $R_{max} = 1/L_{pl}^2 = c^3/\hbar G$. This would eliminate curvature singularities in the solutions of the field equations including cosmological models. This would be explored in a later work.

We can apply eq.(11) to self-gravitating radiation system (SGRS) Sorkin et al.[9] considered the entropy of SGRS confined to a spherical box. To avoid producing infinite entropy they pointed out that configurations of SGRS should not approach their own Schwarzschild radii arbitrarily close to zero, and that the proper distance of the radiation from its Schwarzschild radius be at least one radiation wavelength

$$D(r) = g_{rr}^{1/2}\varepsilon = r^{1/2}\varepsilon^{1/2} \geqslant \lambda(r) \tag{12}$$

$D(r)$ is the proper distance, $\lambda(r)$ denotes a typical proper wavelength of the radiation at that radius and

$$\varepsilon(r) = r - 2Gm(r)/c^2 > 0 \tag{13}$$

$$g_{rr} = [1 - 2Gm(r)/c^2 r]^{-1} = r/\varepsilon \tag{14}$$

$$m(r)c^2 = \int_{L_{pl}}^{r} \rho 4\pi r^2 \mathrm{d}r \tag{15}$$

Eq.(15) is different from the corresponding eq. in ref. [9] since L_{pl} is taken as the lowest bound on all physical length scales following eq.(8).

It is interesting to note the analogous behaviour of both time and entropy in the collapse to the Schwarzschild radius. It was noted above that entropy becomes infinite as the collapsing configuration approaches its Schwarzschild radius. This is also analogous to the infinite time taken by an object approaching its Schwarzschild radius as measured by a distant observer. However the notion of maximal curvature which also implies a minimal physical length $\simeq L_{pl}$ and consequently also a minimal time $\simeq L_{pl}/c = (\hbar G/c^5)^{1/2}$ (if one considers the temporal component of the torsion induced defect) would drastically alter the situation.

The quantum fluctuations in the metric induced by torsion would be governed by

$$\delta g/\langle g\rangle \sim L_{pl}^2/L^2 \quad (16)$$

implying that the Schwarzschild radius would fluctuate by:

$$\delta R_s \sim L_{pl}^2/R_s \quad (17)$$

so in the integration for the time as measured by an external observer to fall to R_s would be modified as

$$\Delta t \approx \int_{R_s+\delta R_s}^{r} \mathrm{d}r/[1-R_s/r] \quad (18)$$

This can be shown to give a *finite* quantity $\Delta t \approx R_s \ln(R_s/L_{pl})$ (19)

If the minimal length (or time) tends to zero, i.e $L_{pl}\to 0$ (or alternatively the maximal curvature $\sim 1/L_{pl}^2 \to \infty$), we recover the usual $\Delta t \to \infty$, of classical general relativity.

Eq. (8) can also be interpreted as a modification of the energy density ρ of SGRS as:

$$\rho = c^4/8\pi G r^2(1+L_{pl}^2/r^2) \quad (\text{with } r > L_{pl}) \quad (20)$$

With eq (20) one can prove that inequality given by eq (13) would hold everywhere.

The maximal curvature as given by eq.(8) would imply from Einstein's equations that the maximum possible energy density of SGRS is

$$\rho_{max} = \rho|_{r=L_{pl}} = c^4/4\pi G L_{pl}^2 \quad (21)$$

substituting eq.(21) into eq (11) we have:

$$R/(1-L_{pl}^2 R)^2 = 2/L_{pl}^2 \quad (22)$$

giving for this case $R=[(5\pm 3)/4](1/L_{pl}^2)$ and considering that $R_{max}\approx 1/L_{pl}^2$, we must choose the smaller solution, as the other one exceeds the value for maximal curvature, that is $R=(1/2)(1/L_{pl}^2)$ is a good solution

Since the density of a black hole scales as $1/M^2$, i e.

$$\rho_{B.H} \approx 3c^6/32\pi G^3 M^2 \propto 1/M^2 \quad (23)$$

eqs. (22) and (23) would suggest that the existence of a *maximal* curvature would constrain the *minimum* mass of black holes that could form in the early universe or survive as a remnant in the evaporation of larger black holes to be $\approx M_{pl}=(\hbar c/G)^{1/2}$.

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