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Imaging with insolated mirrors

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Abstract. Modern solar telescope designs are different from the conventional concept of vacuum telescopes. These new designs are "open" telescopes which try to minimize the temperature difference between various parts of the telescope and the ambient air. In this paper, we address a few issues related to the thermal response and image quality of such insolated mirrors. We estimate the distortion produced by thermal and material inhomogeneities and present limiting values of allowable temperature differences and percentage change of expansion coefficients for different aperture diameters, for typical materials under best possible seeing conditions. We predict the evolution of surface temperature of an insolated mirror using a simplified theoretical approach and show that it is compatible with the experimental values to a large extent. The results indicate the possibility of avoiding active cooling of the mirror surfaces, at least for primary mirrors with aperture diameter less than or equal to 50 cm.

Keywords : Sun, Solar telescopes, mirror seeing, thermal distortion of mirrors

1. Introduction

The design of solar telescopes has evolved through various stages. The basic problem is the enhanced flux in the solar image caused by amplifying the f/100 beam of natural sunlight to the much faster beams used in short focus telescopes. The longer the focus, the greater is the cost of making mechanical systems that can track the sun accurately. The coelostat solves this problem by feeding sunlight into a stationary telescope which can

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Table 1. Top panel of this table indicates the maximum allowable temperature (in °K) for different length scales for different materials. The bottom panel indicates percentage tolerances of variation in α over the corresponding length scales. The values were estimated for $\lambda = 500$ nm, $T_o - T_f = 30$ °K.

material	α $10^{-6\circ}\mathrm{K}^{-1}$	0.5 m	1.0 m	1.5 m	2.0 m	4.0 m
8						
CeSiC	1.60	0.261	0.130	0.087	0.065	0.033
Zerodur	0.05	8.340	4.170	2.780	2.085	1.043
Borosilicate	3.25	0.128	0.064	0.043	0.032	0.016
Be	11.40	0.037	0.018	0.012	0.009	0.005
CeSiC	1.60	0.87	0.43	0.29	0.22	0.11
Zerodur	0.05	27.7	13.9	9.3	6.9	3.5
Borosilicate	3.25	0.43	0.21	0.14	0.11	0.05
Be	11.40	0.12	0.06	0.04	0.03	0.02

have arbitrary length. However, contemporary solar research requires accurate polarimetry which becomes difficult in the case of coelostats on account of polarization produced by oblique reflections.

Modern solar telescopes therefore need to be axisymmetric about their optical axis, up to the place where the light is analyzed for polarimetry. The folded designs like Gregorian and Cassegrain need to be employed. Such reflecting telescopes have a common problem when used for solar imaging. The solar flux falling on such mirrors is partially absorbed by the aluminum coating thereby increasing the temperature of the mirror. Any increase in the temperature above the temperature at which the mirror was manufactured distorts their surface due to thermal expansion. Likewise, any increase in temperature above the ambient air temperature would result in convective motions of the air above the mirror surface. This leads to the phenomenon of mirror seeing. Mirror seeing can be avoided by using a vacuum or helium filled telescope. In another approach, as in the case of the Dutch Open Telescope (Rutten et al, 2004), the natural winds are used to flush the heated air. In this case, we need careful mechanical design of the telescope to withstand wind buffeting. Also, the site must have windy conditions.

In this paper, we investigate the response of mirrors to insolation. In the next section, we address the problem of thermal distortion of the mirror. In section 3, we address the mirror seeing, by first presenting a theoretical model of the thermal response of insolated mirrors followed by description of a simple experiment. In section 4, we summarise the results and present our conclusions.

2. Thermal distortion of the mirror

We shall examine the thermal distortion of the mirror due to the solar radiative flux. Here, we are interested in the quantitative extent of the temperature rise due to thermal absorption and the resulting temperature gradient. A thermal gradient inside the mirror due to non-uniform heat absorption will result in a differential expansion, distorting the shape of the mirror and consequently the image quality. If we consider a mirror, figured at a temperature of T_f and operating at a temperature T_o, with a coefficient of thermal expansion α, the result of thermal expansion can be written as

$$x = x_o[1 + \alpha(T_o - T_f)] \tag{1}$$

and

$$z = z_o[1 + \alpha(T_o - T_f)] \tag{2}$$

where x and z can be considered as any length scale, respectively parallel and perpendicular to surface of the mirror, while x_o and z_o are the corresponding dimensions at figured temperature T_f. The total distortion in the surface geometry can be written as

$$d = \alpha(T_o - T_f)({x_o}^2 + {z_o}^2)^{1/2} \tag{3}$$

For a homogeneous mirror at uniform temperature T_o and expansion coefficient α, the result of increase in temperature from T_f to T_o is a change of radius (for a parabolic surface) which can be directly compensated by a focusing mechanism. The problem becomes a little more complicated for non-parabolic surfaces, which we will not discuss here beyond noting that special care must be taken for complicated surfaces. The additional distortion Δd produced by material and thermal inhomogeneities can be estimated as

$$\Delta d = ({x_o}^2 + {z_o}^2)^{1/2}[\Delta\alpha(T_o - T_f) + \alpha\Delta T_o] \tag{4}$$

For a meniscus mirror with parabolic profile, the the mean thickness z_o is $D/6$ while the change in z_o across the profile is given by $D/(16N)$, where N is the f- number and D is the diameter. For simplicity, we will ignore z_o relative to x_o. Then the equation for Δd, after some algebra, can be written as

$$\Delta d = x_o\alpha(T_o - T_f)\left[\frac{\Delta\alpha}{\alpha} + \frac{\Delta T_0}{T_o - T_f}\right] \tag{5}$$

Of course, a manufacturing defect in the figure of the mirror surface will also lead to further distortion due to thermal expansion. In fact, this effect constrains the tolerance on the optical quality specified for the manufacturer. The ZEMAX package includes such a module for calculating thermal effects. However, this thermal distortion of the aberrated surface is a second order effect and is not considered in this paper. We can equate the distortion shown in Equation 5 to the root mean square (*rms*) wave-front error (Angel et al., 1988) and set limits on the maximum allowable difference in temperatures and co-efficient of thermal expansion across different portions of the mirror. There are different ways by which one can set the limit on the *rms* wave-front error. One way is to say that the performance of the mirror must be better than the atmosphere under "ideal" seeing conditions which will correspond to Fried's parameter r_0 equal to D. Thus, the *rms* wave-front error ΔW would be given by $\Delta W = (0.417\ \lambda)\ (D\ /r_o)^{(5/6)}$ which reduces to $\Delta W = 0.417\lambda$ for the so-called ideal seeing conditions. The condition that the thermal distortion be less than the wave-front error induced by a "benign" atmosphere with Fried's parameter equal to aperture diameter translates into $\Delta d < \Delta W$, which leads to the condition

$$x_o\alpha(T_o - T_f)\left[\frac{\Delta\alpha}{\alpha} + \frac{\Delta T_0}{T_o - T_f}\right] < 0.417\lambda \tag{6}$$

The two terms on the L.H.S of the above equation are comparable. Hence we can use the R.H.S to individually constrain the tolerance in ΔT_o and $\Delta\alpha$ respectively. Here the maximum value of $x_o = D$ has been used to get the strictest tolerance. The result of this equation has been applied to four materials, viz. SiC, Zerodur borosilicate glass and Beryllium and presented in Table 1. We find that Zerodur allows for a larger variation in temperature and α than other materials.

3. Mirror seeing

There are a few experimental investigations on how a mirror, which is warmer than its surroundings, will affect the image quality and cause 'mirror seeing' (Lowne, 1979, Zago, 1995, Wilson, 2001). Ambient temperature would be cooler than the mirror surface for both solar and night-time applications to begin with. The solar heating increases till noon and then decreases and so does the ambient temperature. In comparison, at night, the ambient air cools faster than an uncooled mirror. As the sunlight falls on a mirror, usually coated with aluminium, its surface temperature would increase. However, the amount and rate of increase would depend on the net flux absorbed and the thermal response time of the mirror. When the surface temperature of the mirror is higher than the ambient temperature, it gives rise to mirror seeing, which in turn degrades the image quality. This effect is severe when the mirror is pointing vertically up, as convection is initiated by buoyancy due to a vertical thermal gradient. Thus, it is conventionally preferable to maintain the surface temperature of the mirror at ambient temperature.

As the surface of the mirror is heated, it simultaneously starts cooling by means of convection, conduction, and radiation into the ambient air. Let F be the heat flux absorbed by the mirror surface and F_k, F_c, F_σ be the amount of flux lost by the surface to the surroundings by conduction, convection and radiation respectively. The difference between the flux falling on the mirror and that flowing out of the surface, H, will be used to heat the material, as given by Equation 7.

$$F - [F_k + F_c + F_\sigma] = H. \tag{7}$$

or

$$F - [h_k + h_c + h_\sigma](T_s - T_a) = H. \tag{8}$$

where h_k, h_c, h_σ are respectively the exchange coefficients for conduction, convection and radiation, while T_s, T_a are the temperatures of mirror surface and the ambient air respectively. Essentially, we treat the problem as a pure surface phenomenon, assuming that thin aluminium layer is heated due to the input flux. The substrate adjoining the aluminum layer too is heated up. The thickness of the heated substrate will be approximately the thickness through which heat can diffuse into the substrate in a given time. Thus, as time increases, the thickness of heated substrate will also increase. When this thickness reaches the total thickness of the substrate, the entire substrate will participate in the thermal process. In a heuristic manner, we can write the equation for heat balance as

$$F - h(T_s - T_a) = \epsilon \rho C_p \frac{\partial T_s}{\partial t} \qquad (9)$$

where, $h = h_k + h_c + h_\sigma$, ρ is the density of the medium and C_p is its specific heat and $\epsilon = \sqrt{kt/\rho C_p}$, is the diffusion thickness (k-is thermal conductivity). Assuming that the ambient temperature does not vary with time, and treating ϵ to be slowly varying in time and therefore quasi-static, the solution of Equation 8 is given by

$$T_s = T_a + \frac{F}{h}\left[\ 1 - exp(-t/\tau)\ \right]; \tau = \frac{\epsilon \rho C_p}{h} \qquad (10)$$

The above solution is strictly true for time intervals longer than the time taken for heat to diffuse through the entire thickness L of the substrate. This diffusion time is given by $\rho C_p L^2/k$. For conventional meniscus mirrors, the thickness is usually 1/6 of the mirror diameter. For example, in the case of a mirror with 50 cm diameter, this turns out to be about 8 cm. Thus, the diffusion time in SiC and Zerodur substrates with thickness 8 cm is respectively 50 s and 10000 s. The value of τ in Equation 10 for completely thermalised substrate of 8 cm thickness for both SiC and Zerodur is approximately 10000 s and is independent of the thermal conductivity. The important conclusion is that the asymptotic thermal behaviour of an insolated mirror is independent of the thermal conductivity of the substrate, but depends only on the ratio of heat flux absorbed by the mirror surface to heat flux removed from the mirror surface. Thus, after sufficiently long time, the surface temperature of the mirror would be higher than the ambient temperature by an amount F/h.

3.1 Experiment

The main aim of this experiment was to measure the rise in surface temperature of an insolated mirror as a function of time. A glass flat (density= 2500 ± 125 kgm^{-3}) of dimensions 50X20x8 mm coated with aluminium on one of the largest sides was placed inside a slot made in a piece of *thermocol*, with the coated side facing up. The *thermocol* was fixed on a wooden piece, which in turn was mounted in a big plastic bowl. The bowl was mounted on a telescope drive, which tracks the Sun. Thus the sunlight falls normally on the coated surface of the mirror and the walls of the bowl reduce the flow of wind on the surface. An AD590 temperature sensor (Figure 1) was used to measure the surface

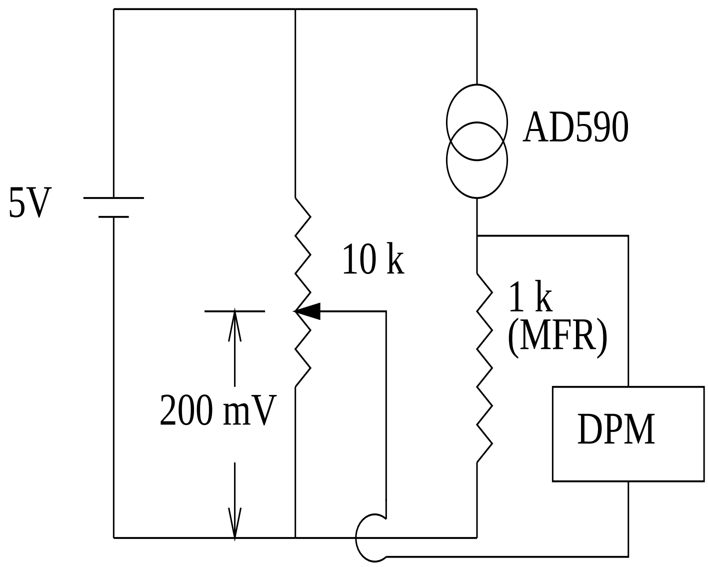

Figure 1. Circuit used for measuring the surface temperature of mirror using AD590 sensor.

temperature of the mirror by fixing it in contact with the insolated mirror surface. The sensor was covered so that direct sunlight does not fall on it. Aluminium being a good conductor, conducts the heat along the surface very fast and hence the entire surface is at the same temperature. The diffusion time across the mirror surface is $\rho C_p R^2/k$, where R is the radius of the mirror. The diffusion time across 50 mm (longer dimension of the flat) works out to 0.25 s, thus temperature equilibrium is rapidly established at the surface. The sensor measures the temperature at a single position on the surface. The potential difference between the ground and the potentiometer (Figure 1) was set to 200 mV before starting the experiment. The voltage measured in the multimeter plus 200 mV gives the temperature of the surface directly in °K. The sensor was calibrated at ice point. The output voltage of the sensor was converted by an A/D card available in a digital multi-meter, the digital voltage was passed onto a computer through RS232 cable. The data was recorded with a cadence of one second.

Figure 2 shows the measured temperature as a function of time for one of the data sets (8 mm thick mirror). It shows that the surface temperature rises in the beginning and reaches a constant value after nearly 25 minutes, or 1500 s. The e-folding time τ (time taken for the temperature to reach 63.2% of its final value), as measured from the experiment, can be put into the the expression for τ in Equation 10, to obtain $h = \rho C_p \epsilon/\tau$. We also see a gradual increase in the surface temperature and some fluctuations. The slow increase in temperature could be due to increase in the temperature of the supporting structures and the fluctuations could be because of ambient wind flow (which was much

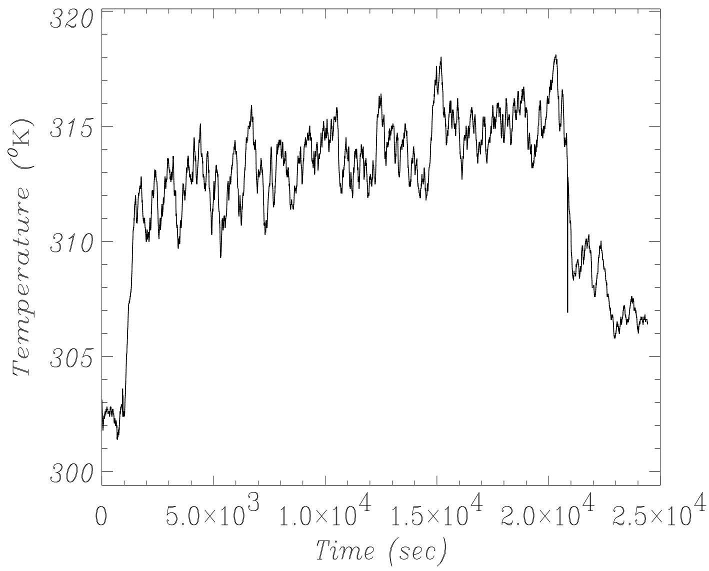

Figure 2. Temporal evolution of the surface temperature of the flat of 8 mm thickness, irradiated by direct sunlight.

reduced after we placed the mirror inside the bowl, in initial experiments done without the bowl, the fluctuations were more than that shown in the figure). When the mirror is pointed away from the Sun (by tilting the bowl), its surface temperature decreases rapidly. We repeated the experiment for another flat of dimensions 10x10x2 mm. Figure 3 shows the measured temperature as a function of time for this mirror. It shows a similar behaviour like the thicker mirror, though the time-scale for attaining the maximum temperature is only about 6 minutes, or 360 s. The reduction in time scales with the thickness of the flat, which is consistent with our theoretical model. In order to eliminate the slow increase in temperature, we performed the experiment under controlled environment, by irradiating the mirror surface by the light from a solar simulator (Oriel SUN SIMULATOR model no. 81172) in an air-conditioned room with the same sensor and recorded the surface temperature manually with a cadence of 15 seconds. Figures 4 and 5 show the temporal evolution of surface temperatures under controlled environment. The kink near the maximum in the rising part of Figure 4 is due to transient power failure. We find that the gradual variation has been removed and fluctuations have reduced drastically. The time-scale for attaining the maximum temperature is once again proportional to the thickness of the mirror. Table 2 shows the experimentally determined values of τ along with derived values of h and F. The results are subject to the uncertainties posed by the slow increase in temperature for the sunlit experiment, which is absent in the simulator experiment.

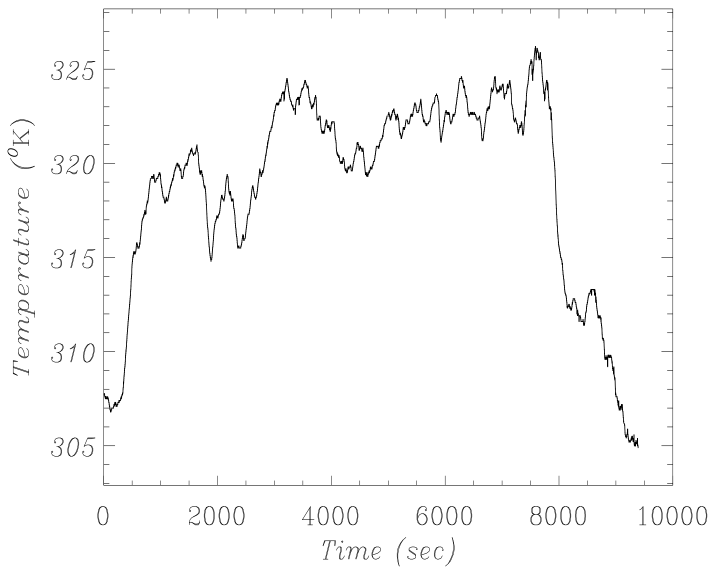

Figure 3. Temporal evolution of the surface temperature of the flat of 2 mm thickness, irradiated by direct sunlight.

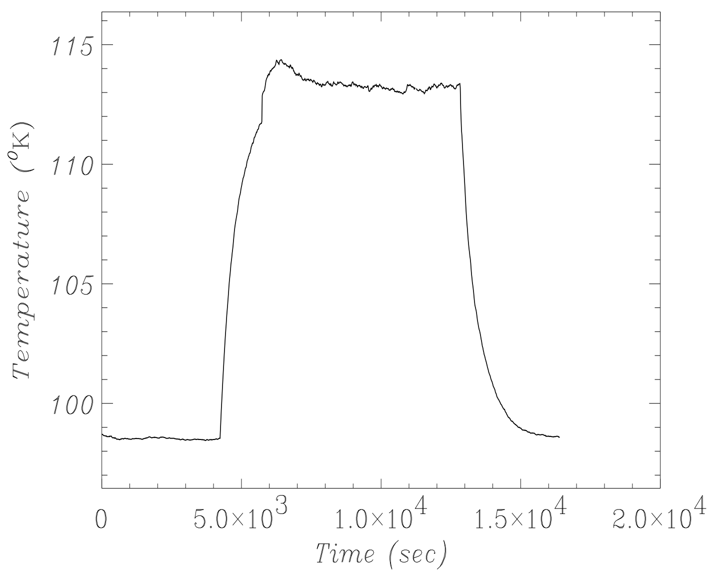

Figure 4. Temporal evolution of the surface temperature of the flat of 8 mm thickness, irradiated by solar simulator. The kink near the maximum in the rising part of the curve is due to transient power failure.

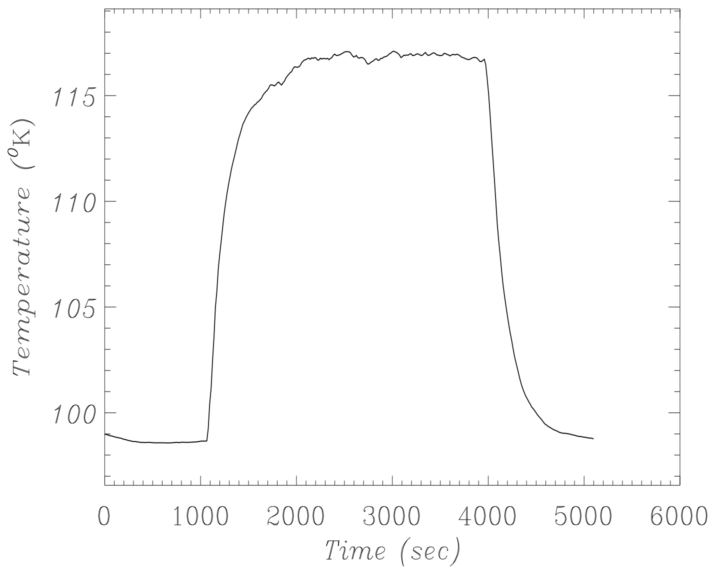

Figure 5. Temporal evolution of the surface temperature of the flat of 2 mm thickness irradiated by solar simulator.

Table 2. Table gives concise results of the experiments. The values have been rounded off to nearest integers.

source of heating	thickness (mm)	ΔT(°K)	τ (s)	h ($Wm^{-2}K^{-1}$)	F (Wm^{-2})
sunlight	8	12	665	20	240
simulator	8	18	885	15	270
sunlight	2	15	200	17	255
simulator	2	18	195	17	306

4. Results and discussions

We find that in order to prevent the thermal distortion of the mirror, the mirror has to be maintained within a few degree temperature range, as well as small tolerance in $\Delta\alpha$ which depends on the material. For CeSiC mirrors, when used for imaging the Sun, the temperature difference between different parts of the mirror should be less than about 0.1 K, while $\Delta\alpha$ is constrained to be less than 0.9%. The predicted value of the evolution of the surface temperature based on a simplified theory is consistent with the observations.

The surface temperature of the mirror increases by several degrees when exposed to the sunlight. Blowing air over the mirror surface can reduce the amount of mirror seeing by shifting the domain of turbulence from natural to forced regime (Dalrymple, 2002). For a given excess of temperature, Dalrymple (2002) has shown that the mirror seeing can be reduced by 2 orders of magnitude, if the convection is shifted from natural to forced convective regime. This is essentially because the thickness of the convecting layer decreases by 1 order of magnitude for forced convection. Using Dalrymple's (2002) equations 3 and 16, we obtain the *rms* phase fluctuation $\Phi = 0.03\Delta T$ for a 1 m/s wind blown across a .5 m mirror. This yields, following equation 8 of Dalrymple (2002), an estimate of the blur angle θ_{blur} as $1.00005\theta_D$, where θ_D is the diffraction limit for the aperture, for $\Delta T = 10^\circ K$. We therefore come to the rather interesting conclusion that a 0.5 m diameter insolated telescope can remain with negligible mirror seeing even without cooling from the back-side, at the cost of merely maintaining an air-flow of 1 m/s over its front surface. Referring back to our table 1, we can also conclude that for a temperature excess of about 10 degrees which could arise without active cooling, Zerodur seems to be the appropriate material of choice for the substrate. Clearly, a few experiments are needed to determine the actual level of mirror seeing and to verify Dalrymple's model for forced convection.

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