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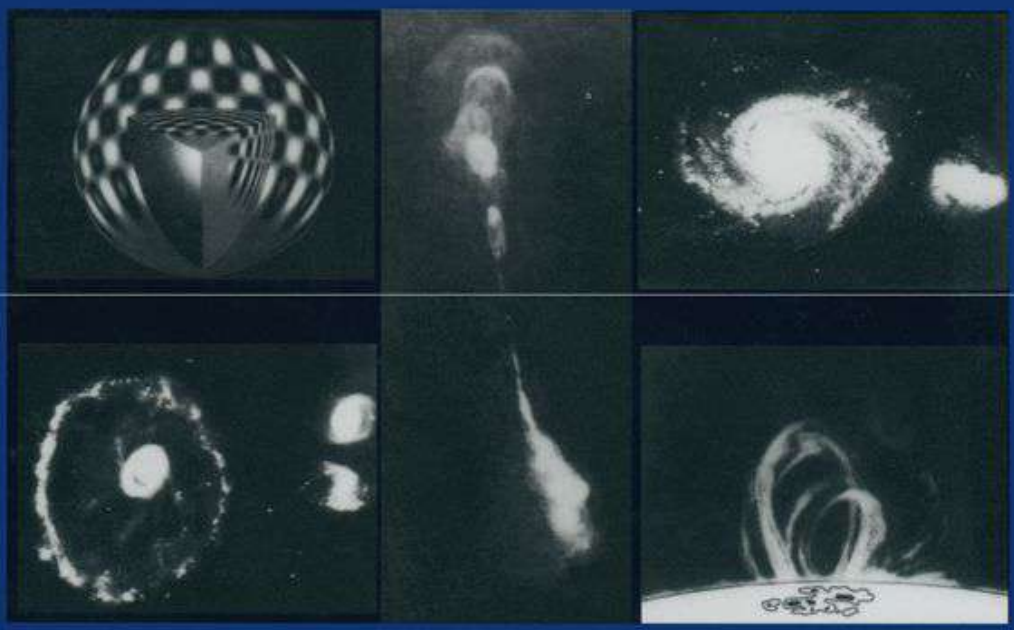
VINOD KRISHAN



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by

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Dedication

To everyone, I have known.

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Preface

Life was simple when the dynamic, the spectral and the resolving powers of our instruments were small. One observed whole objects – planets, stars, sunspots, galaxies, often in rainbow colours. Then the revolution occurred: we acquired the centimetric eyes, the millimetric eyes, the infrared eyes, the ultraviolet eyes, the X-ray eyes and the γ-ray eyes. With these we see mottles on the surface of stars, streams in sunspots, and spirals in nuclei of galaxies. We see regions of multiple mass densities and temperatures in a precarious balance, losing it occasionally, exhaling flares. The universe is timed, cosmic phenomena are clocked; eternity is lost and variability is bought. Microarcsecond resolutions revealed stirring and sizzling interiors underneath serene surfaces. Short durations and small scales demanded employing a discipline with similar attributes – the discipline of Plasmas and Fluids – known more for its complexity than for its felicity. Some would like to wish it away.

We shall learn about plasmas for it is too little familiarity that breeds fear. Complexity can be systemized, to a large extent, by looking for a common denominator among apparently disparate phenomena.

It is not immediately obvious what the contents and the style of a graduate level course on plasmas and fluids aimed at understanding astrophysical phenomena should be. Plasmas and fluids are huge subjects by themselves. The cosmic phenomena where plasmas and fluids play a definite role are equally diverse and numerous. It is not possible to achieve proficiency of comparable levels both in the physics of plasmas and fluids as well as in astrophysical phenomena in a one-semester course. Since a graduate student of astronomy has opportunities to learn about astrophysical phenomena in other astronomy courses, we have chosen to emphasize the physics of plasmas and fluids in this text, illustrating it with examples from the observed astrophysical phenomena. There is no dearth of good text books on plasma physics and fluid dynamics; however, to the best of my knowledge and assessment, no treatment designed to teach a graduate student the essentials of plasmas and fluids in the context of astrophysical phenomena in a span of one semester, exists under

one cover. The choice of topics and the pedagogical treatment, I hope, will prove to be one of the attractive features of the present book.

I have strongly felt that the physics of plasmas and fluids could be presented in a more logical sequence than has usually been done. In looking at some books, it appears to me that some authors, not being sure of the competence of the readers, tend to go back and forth in dealing with presumably complex topics. For example, the fluid and the magnetohydrodynamic descriptions of plasmas are presented in earlier parts of a book whereas their equations are derived in later parts, after describing the kinetic treatment. One wonders, if it is not more desirable to systematically establish the mathematical frame work in a fairly complete manner and then study the various topics as special cases.

With this view, I begin with the Liouville equation from which originates all our knowledge of a large system. In this approach, there is absolutely no confusion whether the single particle picture is a special case of a fluid picture or vice-versa. All descriptions emerge from the Liouville equation, depending upon the assumptions and simplifications thrown in from the physics of a system. This style also helps us to remain aware of the inter-relationships of the single-fluid, the two-fluid, and kinetic treatments. The rest of the contents of the book, essentially, highlight the two major astrophysical issues — the configurational concerns and the radiative requirements — which often influence each other. Another way in which this book differs from a standard text book on plasma physics is by containing a lengthy chapter on nonconducting fluids. Since a zeroth order comprehension of many astrophysical phenomena is attempted by studying fluids in gravitational fields, a familiarity with the basics of fluid dynamics, including the all pervading turbulence, is a must. I have also aspired to make chapter one as informative as possible. Some of the topics, such as strongly coupled plasmas and dusty plasmas have been dealt with in chapter one, since these topics are still in an exploratory stage, and perhaps it is premature to include them with the core course. The reader may find that some concepts appear more than once at different places in the book; this, I believe, cannot hurt. Having said what this book is, I must say

what it isn't. Throughout the writing of this book, I had to often remind myself that it is neither a book exclusively on astrophysics, nor on plasma physics and fluids, so the temptation to specialize was curbed. After taking a course based on this text, I expect that those who are interested will be able to easily venture into the world of rigour, while those not so inclined to do so will get much more out of seminars on plasma instabilities and magnetohydrodynamics.

It is now time to express my heartfelt gratitude to my friends and critics, who over the last twenty years that I have been in the field, have always demanded more from me than I delivered. This constant phase lag has kept me on my toes without toppling. I am most grateful to Professor Paul J. Wiita, an astrophysicist at the Georgia State University, USA, for reading every word in this book and suggesting changes in its substance and style. I also thank Professor Som Krishan, a plasma physicist at the Indian Institute of Science, Bangalore, who has also read every word in this book, for exacting standards of conceptual clarity in my presentation. I have tried. All lapses, however, are mine.

I am extremely grateful to Dr. Baba Anthony Varghese of the Indian Institute of Astrophysics, Bangalore, without whose help, my thoughts could not have become printed words. I also thank Ms Pramila N.K. for her help in typing and correcting the manuscript. And lastly, as I have enjoyed writing this book, I hope you will enjoy reading it.

Vinod Krishan