Kodaikanal Obs. Bull. Ser, A (1977) **2**, 19-27

FUNDAMENTAL FLUX TUBES IN THE SOLAR MAGNETIC FIELDS

2. Generation of the 'FFT's and the periodic field-reversal

M. H. GOKHALE

ABSTRACT

It is conceivable that the periodic reversal of the Sun's large-scale magnetic field and the concentration of the field in strong, thin flux tubes on small scales could *both* result from the following *likely* processes near the base of the convection zone

It is likely that in a "base layer" of thickness ~10^5 km near the base of the convection zone a small part (~ 0.4%) of the energy flux from below goes into an azimuthal magnetoacoustic oscillation and once in ~ 11 y or so such an oscillation reaches a critical amplitude ~ 10^4 cm s^{-1} when it undergoes a 'shock transition' to a cellular (azimuthal) convective mode. The shock transition will involve creation of electric current sheaths and magnetic flux sheaths of field intensity ~10^1 G and thickness ~10^2 km each near the 'nodal' meridian planes. The electric current sheaths would provide the 'reversed' magnetic field on one side and the magnetic flux sheaths would eventually degenerate into 'clusters' of thousands of flux tubes of fluxes ~ 10^{18} Mx each.

This model does need further detailed investigation, but if it is valid, it would be free from the difficulties of the earlier models of the solar magnetic cycle.

Key Words : solar activity—solar magnetic fields —solar convection—magnetohydrodynamics

1. Introduction

In paper I (Gokhale, 1977) we have suggested that a large variety of solar magnetic phenomena on different scales might arise from the movements of magnetic flux tubes of fluxes ~10^{17}-10^{18} Mx, which we have named as "fundamental flux tubes" or briefly as "FFT"s.

It will be important to know how such flux tubes could be produced in the Sun.

In principle, the flux tubes of fluxes ~10^{17}-10^{18} Mx observed at the photosphere may either result from a superficial concentration of magnetic flux by some processes near the photosphere, (which would imply that the subphotospheric magnetic fields are weak and diffuse except in small depths below the photosphere); *or*, as suggested in Paper I, they may be arriving at the photosphere after being created in some large depths.

Several processes have been suggested in literature which could locally concentrate the photospheric field in flux tubes. Parker (1976) has shown that most of these processes are inadequate to explain the observed field intensities and has suggested that internal cooling of flux tubes could lead to the field intensities as high as suggested by observations. But none of these processes including the one suggested by Parker can account for the observed order of magnitude of the magnetic flux in the flux tubes. Galloway *et al.* (1977) have been able to account for the observed flux magnitudes. Inevitably, these models involve assumptions about processes in large depths which can neither be proved nor disproved either theoretically or observationally.

However the main weak point of the models for the 'superficial' concentration is that they are based on the assumption that the sub-photospheric fields in the large depths are weak and diffuse. For this assumption to be consistent with the reversal of the Sun's large-scale magnetic field every 11 y or so one has to invoke the turbulent magnetic diffusivity as in the turbulent dynamo models (eg Krause and Rädler, 1971; Stix, 1971). This application involves some serious difficulties. These difficulties pointed out by Piddington (1975b, 1976c) have not found any satisfactory answer from the proponents of turbulent dynamo models, (cf. the discussion following Piddington, 1976c). On the other hand, Piddington *has not given any definite reasons either for the formation of thin flux tubes or for the amounts of flux in them.*

To resolve this stalemate some totally fresh thinking is required for understanding the small-scale flux concentration and the large-scale field reversal without mutual contradiction. In this paper we probe the question whether the basic process responsible for the large-scale field reversal could itself lead to the formation of magnetic flux tubes. If it is so, the flux tubes could later rise across the photosphere and the outer atmosphere (cf. Paper I: Gokhale, 1977), thereby yielding, (presumably), the various phenomena of activity at and above the photosphere (eg. cf. Gokhale, 1975; Piddington, 1975a, 1976a, 1976b).

2. Convection near the Base of the Convection Zone and its Interaction with the Background Magnetic Field

2.1 Definition of Base Layer

The polarity of the Sun's large-scale field is determined by the quiet region fields in high latitudes. Hence the dynamo processes leading to the field reversal must occur in the depths reached by the fields in the high latitudes. Stenflo's (1974) observations on the rotation of the photosperic fields suggest that the high latitude fields extend to larger depths than those reached by active region fields. Models of sunspots (eg. Yun, 1970) indicate that active region fields extend to depths $\gtrsim 10^4$ km. Hence the processes leading to the polarity reversal must occur in some "dynamo layer" which must be in depths $> 10^4$ km. In order to play such a crucial role, the "dynamo layer" must have some crucial distinction compared to the rest of the convection zone. In depths $> 10^4$ km, the only (and perhaps the only one) layer which is crucially distinct from others is the layer of one density scale height near the base of the convection zone. Across such a layer the mode of energy transport changes from "totally radiative" below the layer to "partly convective" above the layer. We shall call this layer as the "base layer". The thickness of this layer would be $\sim 10^{10}$ cm and the mean density in it would be $\sim 10^{-1}$ gm cm^{-3} (eg. Spruit, 1974).

Near the base of the convection zone, a small part of the outward radiative flux must be first converted to the kinetic energy of large-scale flows which feed energy to the turbulence in the upper layers that carries also the rest of the flux convectively near the photospheric layers. Such a 'partial conversion' of the radiative flux into the 'kinetic energy of large-scale flows' cannot occur across a sharply defined boundary like the conventionally defined lower boundary (or briefly "CDLB") which is defined by the equality $\nabla_{rad} = \nabla_{ad}$ ($= \nabla$) in the usual notation. For though the superadiabaticity ($\nabla > \nabla_{ad}$) is a sufficient *local criterion* for the convective instability, the excitation of *large-scale flows* requires that the mean superadiabatic gradient ($<\delta\nabla> = <\nabla - \nabla_{ad}>$) in a *layer of some finite thickness* (eg the "base layer") exceeds a critical value $<\delta\nabla>_c$ so that the Rayleigh number R exceeds its critical value R_c.

This finite thickness of the base layer together with the finite rates of energy input and the finite rates of dissipation by the radiative viscosity (cf. eg. Section 3) would make the convection near the CDLB *non-stationary*.

2.2 The Model Assumed for the Large-Scale Flows in the Base Layer

The exact nature of the convection near the base of the convection zone is however a controversial and complicated problem (eg. Gilman, 1974; Howard, 1975). Durney (1970) has shown that for a certain choice of the basic parameters (eg. the Rayleigh number and the Prandtl number) the convection in a rotating spherical shell may be in the form of about 20 pole-to-pole convective cells each spanning $\sim \pi/_{10}$ radians in longitudes and that the cell pattern may keep on shifting its position like a wave going around the Sun in $\sim$50 y. As indicated by his Figure 6, the circulating flows will be predominently azimuthal near the base. The convection in the real situation may have similar properties *although the number and sizes of the cells and the life of the pattern may have different values* depending upon the real values of the Rayleigh and Prandtl numbers.

On this background we expect here that the convective pattern near the base of the solar convection zone consists of about $\sim$10 "cells" of predominently azimuthal flows, each cell spanning about $\sim \pi/_5$ radians in longitudes (Fig.1). This corresponds to a cell size $\sim 3\times10^{10}$ cm which is the size of the 'giant cells' as commonly expected from the interpretation of various kinds of observations (cf. eg. Simon and

Weiss, 1968; Yoshimura, 1973; Howard and Yoshimura, 1976). The inner flows of the cells must be azimuthal and must be in the "base layer" defined above. These flows will be called "base flows". The meridian planes where they converge will be called "CMP"s and the meridian planes where they diverge will be called "DMP"s (Fig 1).

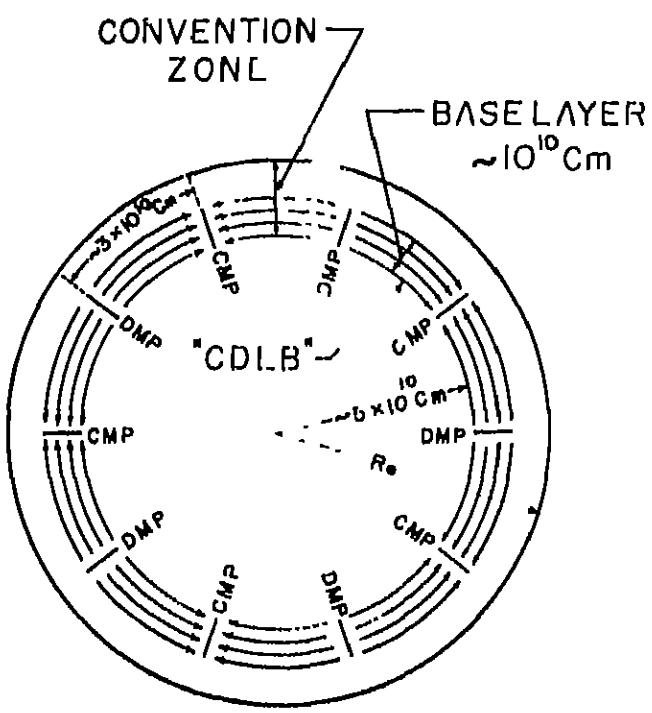

Fig. 1 A schematic diagram showing (i) the equatorial cross-section of the "base layer" near "CDLB" (i.e. the conventionally defined lower boundary of the convection zone). (ii) the "base flows" in the base layer which form the part of the large-scale cellular convective pattern assumed in section 2.2, and (iii) "DMP"s (/"CMP"s) which are the meridian planes at which base flows diverge (/converge).

We shall show in Section 3 that the life of such a pattern can be ~11 y. Here we simply note that such a long lived convective pattern has the following indirect observational verification :

(i) Howard (1971) and Howard and Yoshimura (1976) have observed photospheric east-west flows extending over ranges $\gtrsim 25°$ in longitudes and having lifetimes of at least several months. These suggest long lived azimuthal flows in large depths.

(ii) Motions of various magnetic tracers suggest *Meridional* flows in moderate depths with latitude ranges ~ 30° and lifetimes ~ 11 y (Piddington, 1977). The differential rotation suggests that these meridional flows may be coupled to, and might as well result from, equally long lived azimuthal flows in large depths.

2.3 The Generation of the Convective Mode

2.3.1 Role of the simple (magneto-) accoustic modes

The generation of any new convective mode in the base layer requires first the creation of simple waves (Courant and Friedrichs, 1948, Sec. 29, p.59). For generating azimuthal flows the wave vectors also must be azimuthal. In the presence of the poloidal magnetic field the waves would be magnetoaccoustic in nature, but for a weak field like the Sun's general field (eg. $B_r \sim 3$ G) these waves may be treated as practically "accoustic".

The possibility that such waves may indeed be excited *before* the excitation of convective modes follows from the following argument. Chandrasekhar (1961) has shown that for an incompressible fluid layer rotating about a perpendicular axis the critical Rayleigh number $R_c^{(o)}$ for "overstability" is smaller than the critical Rayleigh number $R_c^{(c)}$ for "convective instability" when Prandtl number is below some critical value. In Sun the Prandtl number is extremely small and the rotation vector has nonzero normal component everywhere except near the equator. Although the "base layer" (as defined above) is not incompressible, we may expect Chandrasekhar's conclusion to be qualitatively applicable. Thus we may expect overstability, but the overstability may generate "accoustic" modes which we have assumed to be in the azimuthal direction. Thus, once the mean temperature gradient $\langle \frac{dT}{dz} \rangle$ in the base layer reaches a value $\langle \frac{dT}{dz} \rangle_c$ corresponding to $R_c^{(o)}$ the excess flux corresponding to the difference $\left[\langle \frac{dT}{dz} \rangle_{rad} - \langle \frac{dT}{dz} \rangle_c \right]$ will be fed to the fastest growing accoustic mode.

2.3.2 The shock transition from the accoustic mode to a cellular convective mode

The transition from the simple (magneto-) accoustic modes to the cellular convective mode would be an irreversible transition and would involve creation of discontinuities in the form of extremely large gradients of physical quantities like fluid velocity, pressure, etc., across very thin layers (viz. 'shocks'). For example, the motions in the non-azimuthal direction will be generated when the velocity gradient $\left| \frac{du}{dx} \right|$ in such a thin layer exceeds the frequency of the "accoustic" mode (Here x is the x-coordinate of a *local Cartesian system* with x-axis along the azimuthal direction). This condition can be written as :

$$\left|\frac{du}{dx}\right| > \frac{1}{T_{aco}} \quad , \qquad (1)$$

where T_{aco} is the period of the accoustic wave.

Since the base layer has finite boundaries, the conservation of momentum requires that the 'shocks' yielding the transition are created in pairs of equally strong shocks propagating in opposite directions. For setting up a cellular convective pattern like the one assumed in Section 2.2, about 5 *pairs of shocks* must be created *simultaneously* near the meridian planes which become (after the shock transition) the DMP's of the cellular pattern.

2.4 Generation of Electric Current Sheaths and Magnetic Flux Sheaths during the Transition

During the generation of the pairs of shock layers leading to the above mentioned 'transition' the pressure gradient across each shock layer will go in :

(i) accelerating the plasma at a rate $\sim \varrho u^2 \left|\frac{du}{dx}\right|$, (ii) overcoming the ohmic dissipation $\sim j^2/\sigma$ through an electric current

$$j = \sigma(E + u \times B) \qquad (2)$$

and (iii) amplifying the current $|j|$ along with the fluid velocity so as to maintain the equation (2) which must be maintained at each instant (since the current relaxation time for the relevant values of ϱ and T is $\sim 10^{-16}$ s).

Here we note that due to the initial proximity of the two shocks in the shock pair near any DMP, the electric currents, which will be directed mutually opposite in the two shocks, will close the circuit for each other and hence can avoid charge accumulation. This way the possibility of $|E + u \times B|$ becoming *too small* compared to $|u \times B|$ *may be avoided.* Thus, in order of magnitude, Eq (2) can be written as :

$$|j| \approx \sigma |u \times B| \quad . \qquad (3)$$

If we assume that the energy going in process (ii) is comparable to that input in process (i), then *in order of magnitude* one must have :

$$\frac{j^2}{\sigma} \approx \varrho u^2 \left|\frac{du}{dx}\right| \qquad (4)$$

Using the approximation (3), we see that one must have ;

$$\varrho u^2 \left|\frac{du}{dx}\right| = \sigma u^2 B^2$$

i.e.

$$\left|\frac{du}{dx}\right| \approx \left|\frac{du}{dx}\right|_c \qquad (5)$$

where

$$\left|\frac{du}{dx}\right|_c = \frac{\sigma B^2}{\varrho}$$

For the values $\sigma \approx 10^{-6}$ e m u, $B \approx B_i \approx 3$ G and $\varrho \approx 10^{-1}$ gm cm^{-3}, which are appropriate to the base layer, we have

$$\left|\frac{du}{dx}\right|_c \approx 10^{-3}\ s^{-1} \quad . \qquad (6)$$

Thus the gradients $|du/dx|$ must reach values $\sim 10^{-3}$ s^{-1} across the shock layers.

It follows that the time-scales τ_c of the transition from the accoustic to the convectic mode must be :

$$\tau \approx 10^3 \quad s \quad .$$

2.4.1 Justification for the assumed cell-size

Conditions (1) (5) and (6) imply that only those accoustic modes which have frequencies $< 10^{-3}$ s^{-1} (i.e. periods $> 10^3$s) will be able to generate cellular convective patterns through the processes discussed so far. Since the accoustic speed in the base layer is $\sim 10^{7.5}$cm s^{-1}, (eg. as follows from the values of P and ϱ in Spruit's (1974) model) the accoustic mode *must have wavelengths* $\gtrsim 10^{10.5}$ cm. But since the relevant wavevectors are azimuthal, the wavelenghths must be $\sim 2\pi R/m = 3 \times 10^{11}/m$ cm where m is an integer not too small, and $R \approx 6 \quad 10^{10}$ cm. Thus the wavelength of the accoustic mode *cannot be much larger* than $\sim 10^{10.5}$ cm.

This justifies the cell size assumed in Section 2.2.

2.4.2 The Dimensions of the generated current sheaths

It follows from Section 2.4 that the creation of the shock pair at each DMP (say DMP1 in Figure 2a will also create a pair of current sheaths of antiparallel currents with one sheath on each side of the DMP (eg. 'D1E', 'D1W' at DMP1, etc. in Fig. 2a) and that in each of these current sheaths, the velocity gradient $\left|\frac{du}{dx}\right|$ will be $\sim 10^{-3}$ s^{-1}.

For justification which will be given in the *appendix A* we make a crucial assumption that the shock transition occurs when the velocity amplitude of the accoustic mode reaches a value $u_0 \approx 10^4$ cm s^{-1}. Then the flow velocity just after the shock will also be

$u_o \approx 10^4$ cm s^{-1} and hence the thickness of each generated current sheath will be

$$d \approx u_o \Big/ \left| \frac{du}{dx} \right|_* \approx 10^7 \text{ cm} \quad , \qquad (7)$$

and the average magnetic field across it will be

$$B_* \approx \sqrt{4\pi\varrho}\, u_o \approx 10^4 \text{ G} \quad . \qquad (8)$$

2.4.3 *The magnetic structures corresponding to the generated current sheaths*

The magnetic structure of each current sheath will be a magnetic "flux roll" consisting of two flux sheaths : (i) the "front" flux sheath with its field in the same direction as the background field (eg. 'DIEe', "D1Ww", "D2Ee", "D2Ww" etc. and (ii) the "back" flux sheath with its field directed opposite to the background field (eg. "D1Ew", "D1We", "D2Ew", "D2We", etc.) (cf. Figure 2a).

Fig. 2 Schematic diagrams showing steps (a), (b), (c), (d) in the reversal of Sun's general magnetic field (of Section 2.6). The diagrams show the part of the approximately cylindrical central section of the "base layer" between two DMP's and between ~ 40 N and ~ 40 S mapped on the plane of the paper. The photospheric areas poleward of ~ 40 N and ~ 40 S are mapped on the horizontal lines marked "photosphere". The areas beyond those lines represent the space external to the Sun (cf. diagram 2a) Arrowheads in the shading represent the direction of the field in the flux rolls.

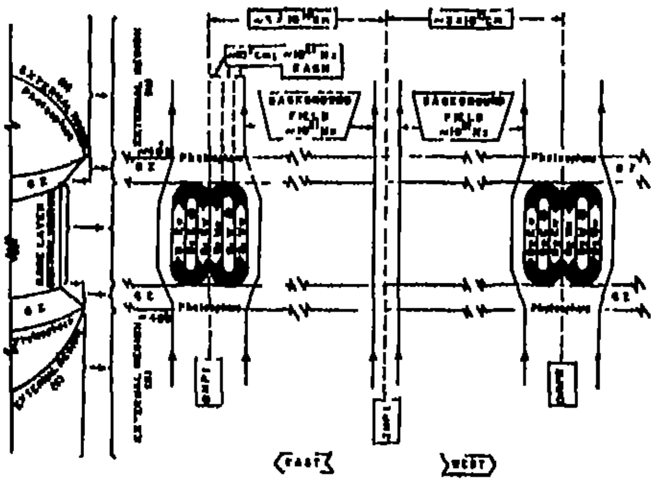

Fig. 2a The cross sections of the electric current sheaths (D1E, D1W, D2E, D2W) and the associated magnetic flux sheaths (D1Ee, D1Ew etc) generated (cf. Sections 2.4.2 and 2.4.3) during the birth of a cellular convective pattern (like the one assumed in Section 2.2) from the azimuthal flows near the CDLB as suggested in Section 2.3.

The mean intensity B_* and the thickness d of each flux sheath will be $\approx 10^4$ G and $\approx 10^7$ cm respectively. The width of each flux sheath along the $u \cdot B$ (i.e. radial) direction will be roughly equal to the thickness of the base layer i e. ~ 10^{10} cm. Hence the flux content of each flux sheath will be ~ 10^{21} Mx which is comparable to the magnetic flux of a typical active region.

2.4.4 *Formation of 'fundamental flux tubes'*

It is reasonable to expect that each of the flux sheaths will eventually split into a 'cluster' of about ~10^3 flux tubes ('FFT's) each of thickness $\approx d$ (i.e. $\approx 10^7$cm) and magnetic flux $\approx B_* d^2 \approx 10^{18}$ Mx.

2.5 *Formation of Flux Sheaths near the "CMP"s*

As the shocks proceed from the DMPs to CMPs they will compress the flux of the background magnetic fields in flux sheaths near CMPs (eg. "C1E", "C1W" etc. in Fig. 2b; Cf. eg. Weiss, 1966). Since the distance D between a DMP and the neighbouring CMP is ~$10^{10.5}$ cm and the thickness H of the base layer is ~10^{10}cm, the flux of each flux sheath so formed near the CMPs will also be : B_l DH $\approx 10^{21}$ Mx. Also, the field intensity in such flux sheaths will be $\approx \sqrt{4\pi\varrho}\, u' \approx 10^4$ G and hence the thickness will be $\approx 10^7$cm. Hence, like the "front" flux sheaths of the newly created flux rolls, these flux sheaths formed near the CMP's also may split in "clusters" of ~ 10^3 flux tubes ('FFT's) of flux amounts $\approx 10^{18}$ Mx each.

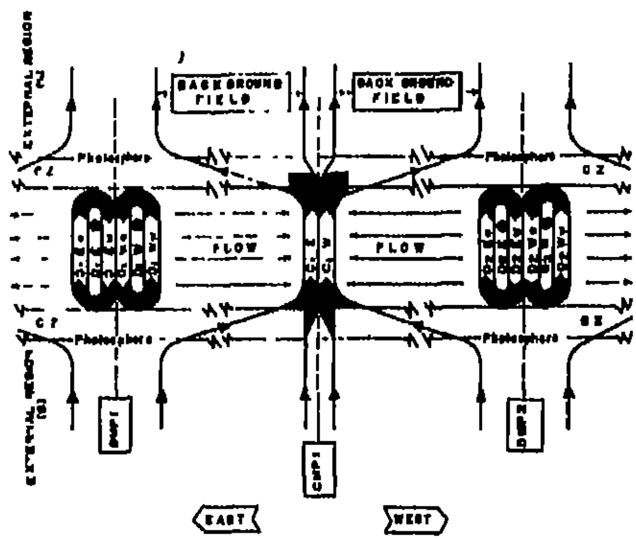

Fig. 2b The magnetic configuration after the background field is 'packed' into flux sheaths near the CMP's (eg C1E, C1W). These flux sheaths are expected to have a thickness ~ 10^7 cm and flux ~ 10^{21} Mx each.

2.6 *Reversal of the Sun's General Field*

As shown in Paper I, the flux of the flux sheaths "C1E", "C1W" etc. formed near the CMP's will rise to the layers *above* the base layer, (possibly in the form of clusters of 'FFT's as indicated before). There, for several years (cf. Paper I) the 'FFT's will be elongated and wound by the differential rotation and will finally rise above the photosphere part by part yielding a series of 'active regions' (eg. as in Babcock's. 1961, model). The trailing parts of the 'FFT's *in between* the active regions will keep on rising above the photosphere thereby removing the photospheric intersections which form the active region fields (eg. Gokhale, 1975). Finally the emergence of the polewardmost subphotospheric segments (either with or without reconnections) will remove the polewardmost intersections of these 'FFT's. In this way all the flux of the background' field in the base layer collected in the flux sheaths C1E, C1W etc. near the CMP's will *leave the Sun, carrying away with it also the associated ('background') photospheric fields in the high latitudes* (Fig. 2c).

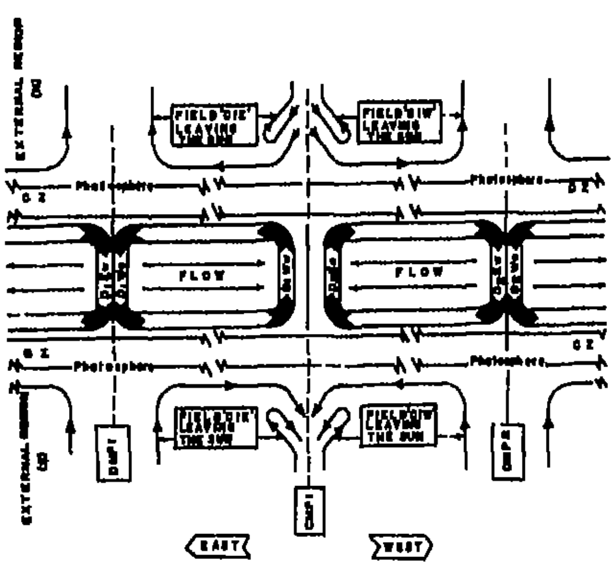

Fig. 2c The magnetic configuration after the magnetic flux in C1E and C1W has gone above the photosphere after yielding a series of "active regions" in the central latitudes.

Similarly the flux in the "front" flux sheaths eg. "D1Ww", "D2Ee" etc. of the newly generated "flux rolls" will also rise, yielding a *second* series of active regions with the same polarity orientations as those in the first series given by the flux sheaths formed near the *CMPs*. Photospheric fields of the active regions of this second series will also be annihilated by the emergence of the intermediate flux tube portions above the photosphere. *However unlike the flux of the flux sheaths formed near the CMPs, the flux in the 'front' flux sheaths D1Ww, D2Ee etc. of the "flux rolls" cannot leave the Sun as it is connected to the 'back' flux sheaths of the 'flux rolls'. Instead, the polewardmost intersections of the FFTs of these 'front' flux sheaths will provide the "reversed" photospheric field in the high latitudes (Fig. 2d).*

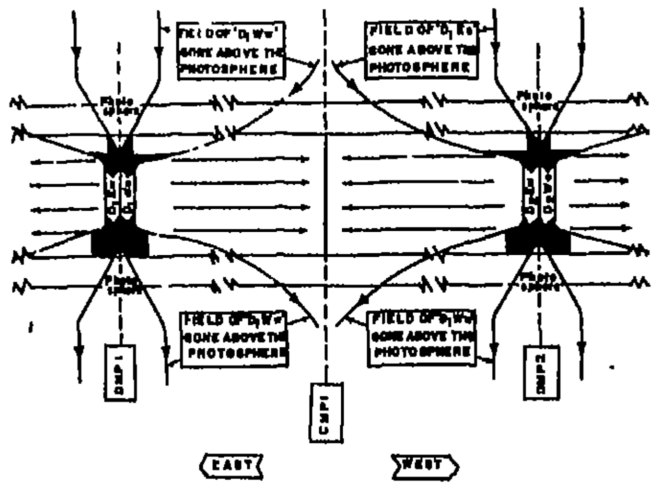

Fig. 2d The configuration after the magnetic flux in the sheaths D1Ww and D2Ee have gone above the photosphere, yielding another series of active regions.

Finally the magnetic flux in the 'back' flux sheaths "*D1We*", "*D2Ew*", .. etc. will be spread over the whole base layer between the *CMPs* and *DMPs* to provide a "reversed" field of ~ 3G for the start of a new magnetic cycle at the time of the generation of the new cellular pattern by the "next" shock transition (cf. Section 3.2). This spreading of flux can occur on time scales < 11 y if the flux tubes formed from the flux sheaths have their currents concentrated near the surface in sufficiently thin current layers and/or if the base flows create enough turbulence at their boundaries (cf. Paper I).

3. The '11-y' Periodicity

3.1 *The Decay of the Base Flows*

At the time of the shock transition the kinetic energy of the accoustic mode per "unit column" (i.e. column of unit cross section and height H same as that of the base layer) will be

$$E^{(0)}_{acc} \approx \tfrac{1}{2}\rho u_0^2 H \approx 5 \times 10^{10} \text{ erg cm}^{-2} \quad . \qquad (9)$$

The *initial* velocity of the established base flows will also be $\approx u_0 \approx 10^4$ cm s^{-1}, and the *initial energy* E_0 of the base flows per "unit column" will be

$$E_0 \approx E^{(o)}_{acc} \quad . \qquad (10)$$

This energy will be gradually transferred by viscous dissipation to the turbulence in the layers above the base layer.

At any subsequent instant, the kinetic energy content of the base flow per "unit column" will be given by

$$E \sim \tfrac{1}{2}\varrho u^2 H \qquad (11)$$

where u is the instantaneous velocity amplitude of the flow. The rate of depletion of E due to the turbulence in the upper layer will be :

$$\dot{E} \approx -\nu_{turb}\,\varrho' u \frac{du}{dz} \approx -\nu_{turb}\,\varrho' \frac{u^2}{H} \qquad (12)$$

where ϱ' is the density in the upper layer, ν_{turb} is the turbulent kinematical viscosity, and

$$\frac{du}{dz} \approx \frac{u}{H} \qquad (13)$$

is the velocity gradient across the top of the base layer. This depletion constitutes an upward *non-radiative* flux

$$\Delta F \sim -\dot{E} \qquad (14)$$

across the top of base layer.

From Equations (11) and (12) it follows that E varies with time as :

$$\frac{\dot{E}}{E} = -\frac{1}{T_c} \qquad (15)$$

where

$$T_c \approx \frac{H^2}{2(\varrho'/\varrho)\,\nu_{turb}} \approx \frac{H^2}{2(\varrho'/\varrho)\,bVL} \qquad (16)$$

where L and V are the mean eddy size and mean eddy velocity in the turbulence near the top of the base layer, and b is a fraction $\approx 1/3$. This gives :

$$E = E_0 \exp(-t/T_c), \qquad (17)$$

where t is the time measured from the *birth* of the cellular pattern.

3.2 The 'Next' Shock Transition

It follows from Eqns (14) and (17) that as E decreases with time, the non-radiative flux ΔF at the top of the base layer will go on reducing from an *initial value*

$$(\Delta F)_0 \approx E_0/T_c \qquad (18)$$

to almost nil at the end.

However, the total energy flux coming up from the Sun's radiative core across the base of the base layer will be roughly constant. Therefore as ΔF dwindles, the energy will go on *accumulating* in the base layer at a rate

$$(\Delta F)_0 - (\Delta F). \qquad (19)$$

For the reasons mentioned earlier in Section 2.3.1 this accumulation will be in the form of some fastest growing simple (magneto-) accoustic mode. The energy per unit column so accumulated upto a time t will be given by

$$E_{acc}(t) = \int_0^t [\,(\Delta F)_0 - (\Delta F)\,]\,dt \qquad (20)$$

which, using (14) (17) and (18) gives :

$$E_{acc}(t) = E_0\left[\frac{t}{T_c} - 1 + \exp(-t/T_c)\right] \quad . \qquad (21)$$

From this and from Eq. (10) it follows that at about $t = 2T_c$ the energy E_{acc} accumulated in the new accoustic mode will reach the same value $E^{(o)}_{acc}$ at which the earlier accoustic mode got converted to the cellular mode by the shock transition at $t = o$. Thus at about $t \sim 2T_c$ the new accoustic mode may be expected to get converted to a *new* cellular flow pattern. At this moment the energy of the old cellular pattern will have been reduced by a factor e^{-2} (cf. Eq 17) i.e. the old cellular convection will have almost come to a halt.

Thus the whole process of amplification of the accoustic mode and its eventual conversion to a cellular mode by a shock transition may repeat itself with a period

$$T_0 \approx 2T_c \quad . \qquad (22)$$

3.3 The '11-Y' Periodicity

From Sections 2.4-2.6 and 3.2 follows the periodicity of the Sun's large-scale field reversal. For striking an agreement of T_0 with the observed periods ~11 y, one should have :

$$T_c \approx 1.5 \times 10^8\ \mathrm{s} \quad . \qquad (23)$$

For this, all one needs is that in formula (16),

$$\left(\frac{\varrho'}{\varrho}\right) bVL \simeq 10^{11.5}\ \mathrm{cm^2\,s^{-1}}.$$

If $V \approx 10^{3.5}$ cm s^{-1} and $L \approx 10^{8.5}$ cm (eg. Spruit, 1974), one needs(ϱ'/ϱ) b $\approx \frac{1}{30}$; and this is certainly possible within reasonable limits $\left(\text{eg. } \frac{\varrho'}{\varrho} \approx \frac{1}{10}, b \approx \frac{1}{3}\right)$.

3.4 *The Maintenance of the Level of the Reversal Operation*

Conventional models like Babcock's (1961) model have no explanation why the level of the reversal processes does not go on rising with successive cycles (Piddington, 1972). In our present model, as the "FFT"s of the "front" flux sheaths go on rising higher and higher, the mass in them will flow back longitudinally along the FFT's and will be ultimately dumped in the "back" flux sheaths like "D1We", "D2Ew" etc. Some mass, of course will be indeed carried away in the form of solar wind along the field lines reaching the interplanetary distances. However, it may be verified that during the entire life of the Sun, this mass loss will hardly displace the centre of gravity of the mass associated with the background field in the base layer (viz. the total mass of the "base layer"). This will maintain the level of the field reversal operation of our model in the "base layer".

4. Concluding Remarks

As mentioned in the Introduction we have done here a preliminary exploration of the possibility that the small-scale flux concentration and the large-scale field reversal could result from one and the same process. We have also identified the shock transition from (magneto-) accoustic mode to the convective mode at the base of the convection zone as the 'likely' process responsible for the two phenomena. It is conceded that at this stage the suggested processes are not yet well understood. But the order-of-magnitude agreement with the widely different kinds of observational constraints shows that the suggested processes *certainly deserve further study*. For example, the detailed structure of the shock layer, its generation and propagation *need mathematical treatment*. It is also necessary to derive a theoretical formula for the amplitude u_0 of the accoustic mode at which it gets converted to the "convective" mode.

Appendix A

Comments Justifying the Choice of u_0

(1) In Section 2.4.2 the value $u_0 \approx 10^4$ cm s^{-1} was chosen somewhat arbitrarily. However it is important to note that this single choice yields $B_* \approx 10^4$ G and d $\approx 10^7$ cm simultaneously from *independent* considerations (Eqns. (7) and (8)) and that these values of B_* and d automatically satisfy the following constraints *simultaneously*:

(i) the flux of the newly created 'reversed' field (viz. the 'back' flux sheaths) becomes roughly same as that of the original background field thereby maintaining the overall magnetic flux roughly of the same order from one 'cycle' to the next cycle and

(ii) the flux of the possible 'flux tubes' becomes roughly same as that of the small-scale field concentrations observed at the photosphere.

Thus the single choice of u_0 is consistent with *two different theoretical constraints* (Eqns. (7) and (8)) and *also with two different observational constraints*. This indicates that the suggested processes may be real and the chosen value may be realistic.

(2) The mixing length models fail to account for the mean non-radiative flux $(\Delta F)_*$ across the 'last mixing length' which 'ends' at the *'CDLB'* (conventionally defined lower boundary; cf. Section 2.2). The attempt to avoid this difficulty by choosing the mixing length to vanish at the boundary (eg. Bohm and Stuckl, 1964) implies a transfer of energy from the *smaller* eddies at the *'CDLB'* to *larger* eddies in the upper layers, which seems against the second law of thermodynamics. Our present suggestion that in a 'base layer' (of finite thickness) the energy goes first in an accoustic mode accounts for the $(\Delta F)_*$ if the average non-radiative flux across the base layer in our model--which is $\approx (\Delta F)$ defined in Section 3.1—is also comparable to $(\Delta F)_*$.

The amount of this 'unaccounted' flux $(\Delta F)_*$ is, in fact

$$(\Delta F)_* \approx F_0\,(1 - \nabla_{\text{actual}}/\nabla_{\text{rad}})$$

where $F_0 = 7 \times 10^{10}$ erg cm^{-2} s^{-1} is the constant total flux and ∇_{actual} and ∇_{rad} are the values of the 'actual' and the 'radiative' temperature gradients in the 'last' step of inward integration in the mixing length model. Values from Spruit's model yield $(\Delta F)_* \approx 3 \times 10^8$ erg cm^{-2} s^{-1} requiring $(\Delta F) \approx 3 \times 10^8$ erg cm^{-2} s^{-1}. This according to Eq. 18 requires $E_0 \approx 4 \times 10^{14}$ erg cm^{-2}, which corresponds to $u_0 = 10^4$ cm s^{-1}. This is another reason why the choice $u_0 = 10^4$ cm s^{-1} may indeed be appropriate.

(3) It may finally be noted that if any substantial part of the mass flux in the 'cellular pattern' reaches

observable layers then the velocity $u_0 \approx 10^4$ cm s^{-1} of base flows would imply surface velocities too large compared to those observed (eg. Howard, 1971). However if most of the 'reverse' mass flux in the 'cells' is confined to, say, only one or two scale heights immediately above the base layer (or, for example, if the reverse mass flux in the upper layers goes on decreasing as fast as the density) then there will be no contradiction with observations.

Appendix B

Merits of the Present Model for the Field Reversal

(1) It is free from the difficulties (pointed out by Piddington, 1972) in the conventional models by Babcock (1961) and Leighton (1964, 1969) and in the turbulent dynamo models (eg. Krause and Rädler, 1971).

(2) It provides a plausible theoretical link between : (a) the flux content of the small-scale magnetic flux tubes (which seem to constitute most of the field structures directly or indirectly observed) and (b) the large-scale phenomena like the solar magnetic cycle.

(3) It also incorporates the *non-axisymmetry* which seems essential to explain the occurrence of the "preferred longitudes" and the "sector structures" of the large-scale photospheric field (cf. Stix, 1971). Although Yoshimura (1976) has also given a model based on a non-axisymmetric convection, he has used the turbulent magnetic diffusivity on global scales which may not be applicable to the solar dynamo problem when the fields have the small-scale structures like those observed (cf. Piddington, 1975b).

(4) The model developed in sections 3.1-3.3 for the periodicity of the convection near the CDLB may account for the likely variation of the solar constant associated with the 11-y cycle of activity (eg. Labs and Nickel, 1971).

References

Babcock, H. W., 1961, *Astrophys. J.* 133, 572.
Chandrasekhar, S., 1961, *Hydrodynamic and Hydromagnetic Stability*, Oxford University Press, London.
Courant, R., Friedrichs, K. O., 1948, *Supersonic Flow and Shock Waves*, Inter Science Publishers, Inc New York.
Durney, B., 1970, *Astrophys. J.* 161, 1115.
Galloway, D. J., Proctor, M. R. E., Weiss, N. O., 1977, *Nature*, 266, 686.
Gilman, P. A., 1974, *Ann. Rev. Astron. & Astrophys.* 12, 47.
Gokhale, M. H., 1975, *Solar Phys.* 41, 381.
Gokhale, M. H., 1977, *Kodaikanal Obs. Bull. Ser. A*, 2, 10.
Howard, R., 1971, *Solar Phys.* 16, 21.
Howard, R., 1975, *Scient. Am.* 232, 106.
Howard, R., Yoshimura, H., 1976, IAU Symp 71, *Basic Mechanisms of Solar Activity*, p 19, eds. V. Bumba, J. Kleczek, D. Reidel Publishing Company, Dordrecht-Holland.
Krause, F., Rädler, K. H., 1971, IAU Symp. 43, *Solar Magnetic Fields*, p.770, ed. R. Howard, D. Reidel Publishing Company Dordrecht-Holland.
Labs, D., Neckel, H., 1971, *Solar Phys.* 19, 3.
Leighton, R. B., 1964, *Astrophys. J.* 140, 1547.
Leighton, R. B., 1969, *Astrophys. J.* 156, 1.
Parker, E. N., 1976, *Astrophys. J.* 204, 259.
Piddington, J. H., 1972, *Solar Phys.* 22, 3.
Piddington, J. H., 1975a, *Astrophys. Space Sci.* 34, 347.
Piddington, J. H., 1975b, *Astrophys. Space Sci.* 35, 269.
Piddington, J. H., 1975c, *Astrophys. Space Sci.* 38, 157.
Piddington, J. H., 1976a, *Astrophys. Space Sci.* 40, 73.
Piddington, J. H., 1976b, *Astrophys. Space Sci.* 41, 79.
Piddington, J. H., 1976c, IAU Symp. 71, *Basic Mechanisms of Solar Activity*, p 389, eds. V. Bumba, J. Kleczek, D. Reidel Publishing Company, Dordrecht-Holland.
Piddington, J. H., 1977, *Astrophys. Space Sci.* 47, 231.
Simon, G. W., Weiss, N. O., 1968, *Z. Astrophys.* 69, 435.
Spruit, H. C., 1974, *Solar Phys.* 34, 277.
Stenflo, J. O., 1974, *Solar Phys.* 36, 495.
Stix, M., 1971, *Astron. Astrophys.* 13, 203.
Weiss, N. O., 1966, *Proc. R. Soc. London Ser A*, 293, 310.
Yoshimura, H., 1973, *Solar Phys.* 33, 131.
Yoshimura, H., 1976 *Astrophys. J. Suppl.* 29, 464.
Yun, H. S., 1970, *Astrophys. J.* 162, 957.