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On the stability of the hypothetical quintet system of TW Draconis

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Abstract. Aarseth's NBODY 1 code is used to study the stability of the quintet system of DW Draconis postulated by Abhyankar & Panchatsaram (1984). It is found that the observed $(O - C)$ curve based on the ephemeris by Tremko & Kreiner (1981) can be reproduced by taking appropriate initial conditions; but the system becomes unstable outside the range of observations. The nature of instability is studied.

Key words : eclipsing binaries—multiple system—n-body integration

1. Introduction

The run of the photoelectric minimum time residuals of the eclipsing binary TW Draconis according to the ephemeris

$$\text{Hel. Primary Min.} = \text{JD } 2432710.865 + 2.806847E \qquad \ldots(1)$$

given by Tremko & Kreiner (1981) is shown in figure 1. It was interpreted by Abhyankar & Panchatsaram (1984) as the light-time effect in a system of five components. The dashed line in figure 1 indicates their Fourier representation

$$(O - C)_2 = -0.0313 + 0.0033 \sin \frac{360\,(E - 90.4)}{1000} + 0.0093 \sin \frac{360\,(E + 250.5)}{2500} + 0.0190 \frac{\sin(\nu + 310)}{(1 + 0.5 \cos \nu)}. \qquad \ldots(2)$$

They applied the empirical stability criteria for hierarchical systems given by Roy (1982) and for planetary systems given by Graziani & Black (1981) to the quintet system and showed that it is unstable. As it will be interesting to know the nature of the instability we have performed numerical integrations using Aarseth's (1971) NBODY1 program, and the results are presented in this paper.

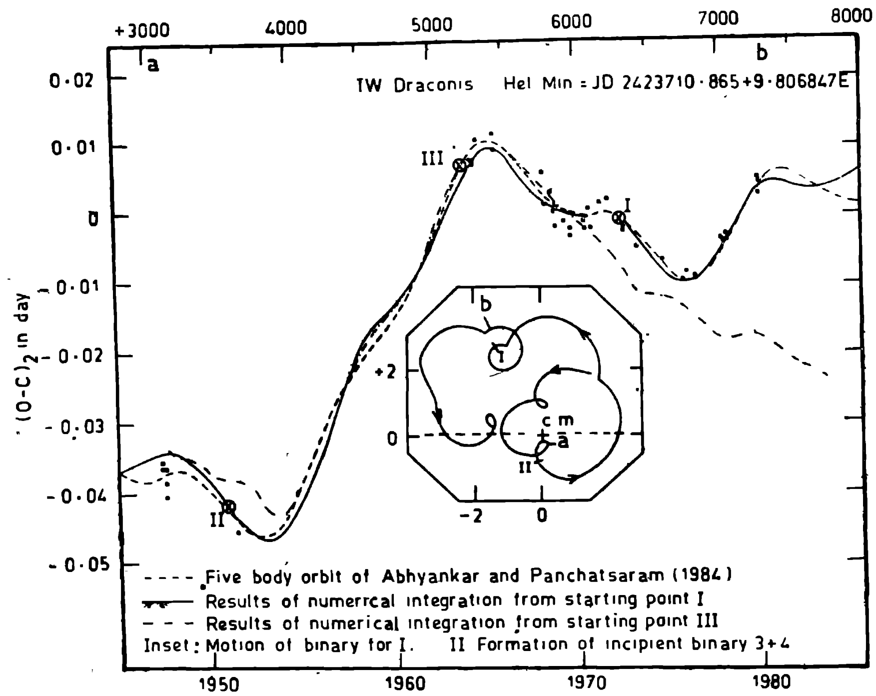

Figure 1. $(O - C)$ diagram of TW Dra according to the ephemeries by Tremko & Kreiner (1981). Results of numerical integration of n-body motion by Aarseth's code are shown as indicated. Epochs a and b indicate the beginning and end of observational data.

2. Description of the system

(a) *Parameters* : Since Abhyankar & Panchatsaram (1984) had found that the binary itself is stable under perturbations by other bodies we have represented it here by a single particle B of mass $m_B = 2.720\ M_\odot$.* Assuming all orbits to be coplanar the masses of other componets come out to be : $m_3 = 0.306\ M_\odot$, $m_4 = 0.522\ M_\odot$ and $m_5 = 0.742\ M_\odot$. According to equation (2) and as shown in figures 2 and 3, binary B moves around the centre of mass of $B + m_3$ in a circular orbit of radius $a_1 = 0.57$ AU and period $P_1 = 7.685$ yr, having the ascending node with respect to the sky plane at epoch $E_1 = 90.4$. The centre of mass of $B + m_3$ moves around the centre of mass $B + m_3 + m_4$ in a circular orbit of radius $a_2 = 1.61$ AU and period $P_2 = 19.212$ yr with the ascending node at epoch $E_2 = -250.5$. Finally the centre of mass of $B + m_3 + m_4$ moves around the centre of mass of all the five bodies in an elliptical orbit of semi-major axis $a_3 = 4.38$ AU, eccentricity $e = 0.5$ and period $P_3 = 61.478$ yr with periastron at epoch $E_3 = 3625$ which corresponds to $\omega = 310°$. In the circular orbits the mean anomalies are calculated from the ascending nodes and for the elliptical orbit from the periastron.

(b) *Coordinates* : Let x axis be in the sky plane and y axis along the line of sight. Then the orbital coordinates of the three motions are

*Aarseth (1971) has stressed the importance of the influence of such a binary on the rest of the system. However in the present case the two components of the binary are very tightly bound and it would be difficult to treat them as separate particles without proper regularization. Anyway we do not expect significantly different results because the total number of particles is small.

$$\left.\begin{array}{lll} x_1 = a_1 \cos M_1, & x_2 = a_2 \cos M_2, & x_3 = \bar{x} \cos \omega - \bar{y} \sin \omega \\ y_1 = a_1 \sin M_1, & y_2 = a_2 \sin M_2, & y_3 = \bar{x} \sin \omega + \bar{y} \cos \omega \end{array}\right\} \quad \ldots(3)$$

where

$$\bar{x} = r \cos \nu, \quad \bar{y} = r \sin \nu$$

and

$$r = \frac{a_3 (1 - e^2)}{1 + e \cos \nu}. \quad \ldots(4)$$

In Aarseth's program we need the coordinates of the bodies with respect to the centre of mass of the whole system; they are given by :

$$\left.\begin{array}{ll} X_1 = x_1 + x_2 + x_3, & Y_1 = y_1 + y_2 + y_3 \\ X_2 = x_1 - \dfrac{\mathcal{M}_2}{m_2} x_1, & Y_2 = y_1 - \dfrac{\mathcal{M}_2}{m_2} y_1 \\ X_3 = x_3 - \dfrac{\mathcal{M}_2}{m_3} x_2, & Y_3 = y_3 - \dfrac{\mathcal{M}_2}{m_3} y_2 \\ X_4 = - \dfrac{\mathcal{M}_3}{m_4} x_3, & Y_4 = - \dfrac{\mathcal{M}_3}{m_4} y_3 \end{array}\right\} \quad \ldots(5)$$

where

$$\mathcal{M}_i = \sum_{j=1}^{i} m_j. \quad \ldots(6)$$

(c) *Velocities* : Since the unit of length is astronomical unit, and unit of mass is solar mass, the unit of time has to be Gaussian unit, viz., yr/2π in order to make $G = 1$ as required in Aarseth's code. Then the orbital velocity components are given by

$$\left.\begin{array}{l} \dot{x}_1 = \dfrac{-a_1}{P_1} \sin M_1, \dot{y}_1 = \dfrac{a_1}{P_1} \cos M_1 \\ \dot{x}_2 = \dfrac{-a_2}{P_2} \sin M_2, \dot{y} = \dfrac{a_2}{P_2} \cos M_2 \\ \dot{x}_3 = \dot{\bar{x}} \cos \omega - \dot{\bar{y}} \sin \omega \\ \dot{y}_3 = \dot{\bar{x}} \sin \omega + \dot{\bar{y}} \cos \omega \end{array}\right\} \quad \ldots(7)$$

where

$$\left.\begin{array}{l} \dot{\bar{x}} = - \dfrac{a_3}{P_3} \dfrac{\sin \nu}{\sqrt{1 - e^2}} \\ \dot{\bar{y}} = \dfrac{a_3}{P_3} \dfrac{e + \cos \nu}{\sqrt{1 - e^2}}. \end{array}\right\} \quad \ldots(8)$$

The velocities with respect to the centre of mass can then be obtained by transformations similar to equation (5) where the coordinates are replaced by their time derivatives.

3. Initial conditions

(a) *Case A* : The epoch $E = 6282$ (point I in figure 1) represents the centre of gravity of the dominant observations. The configuration of the system at this epoch is shown in figure 2. The initial coordinates and velocities of the four bodies as obtained from the equations of section 2 are given in table 1. On changing the signs of the velocity components we can perform the integration with time running backwards.

(b) *Case B* : The point III in figure 1 corresponds to $E = 5180$ which represents the midpoint of all observations. The configuration of the system at that epoch is

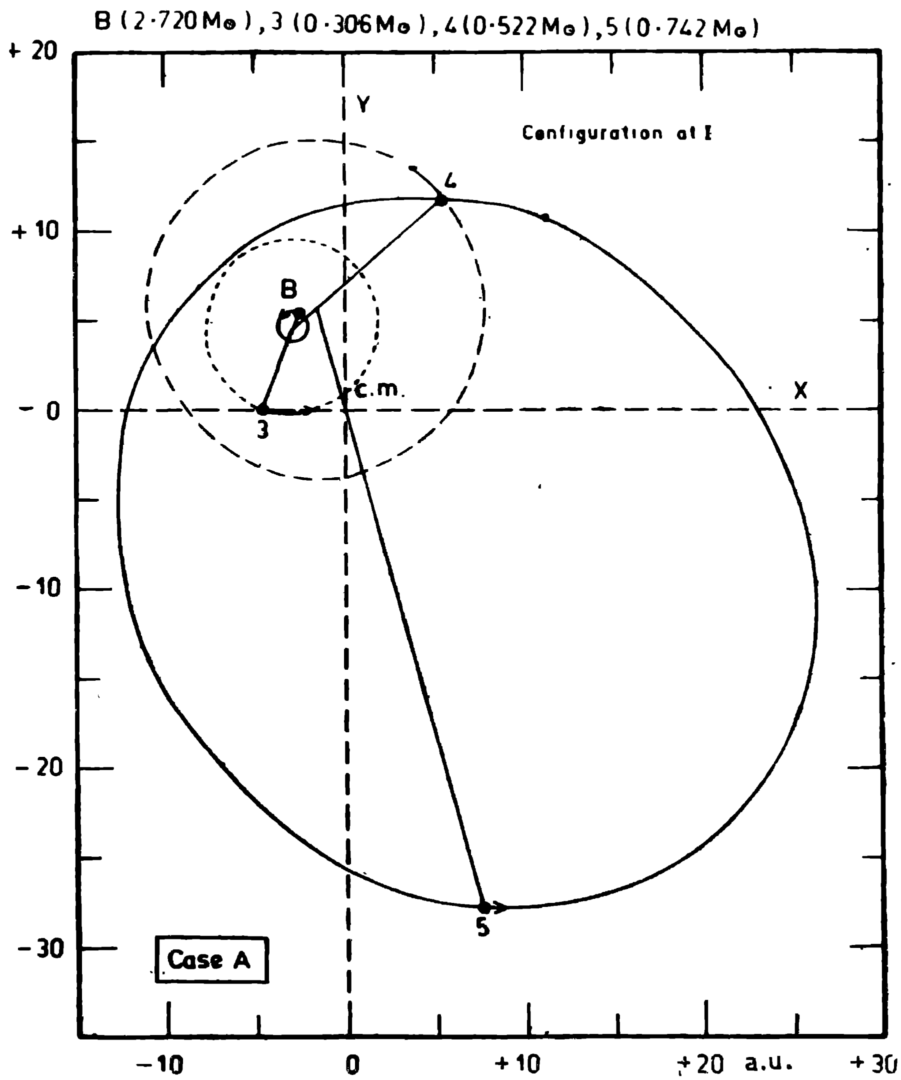

Figure 2. Configuration of the systems of TW Dra for case A.

shown in figure 3. The initial conditions for the same are also given in table 1. Again backward integration can be perfomed by changing the signs of the velocity components.

Table 1. Initial conditions for cases A and B

Star	X	Y	$\dot{X}$	$\dot{Y}$
Case A				
Binary	−2.611	+ 5.289	−0.0624	−0.0323
Star 3	−4.633	+ 0.028	+0.6222	−0.2954
Star 4	+5.485	+11.888	−0.3644	+0.3731
Star 5	+7.622	−27.765	+0.2287	−0.0233
Case B				
Binary	+2.310	+ 6.291	−0.1622	+0.1473
Star 3	−2.457	− 3.282	+0.2293	−0.4730
Star 4	−3.311	− 3.675	+0.3803	−0.1829
Star 5	−5.124	−21.828	+0.2323	−0.2163

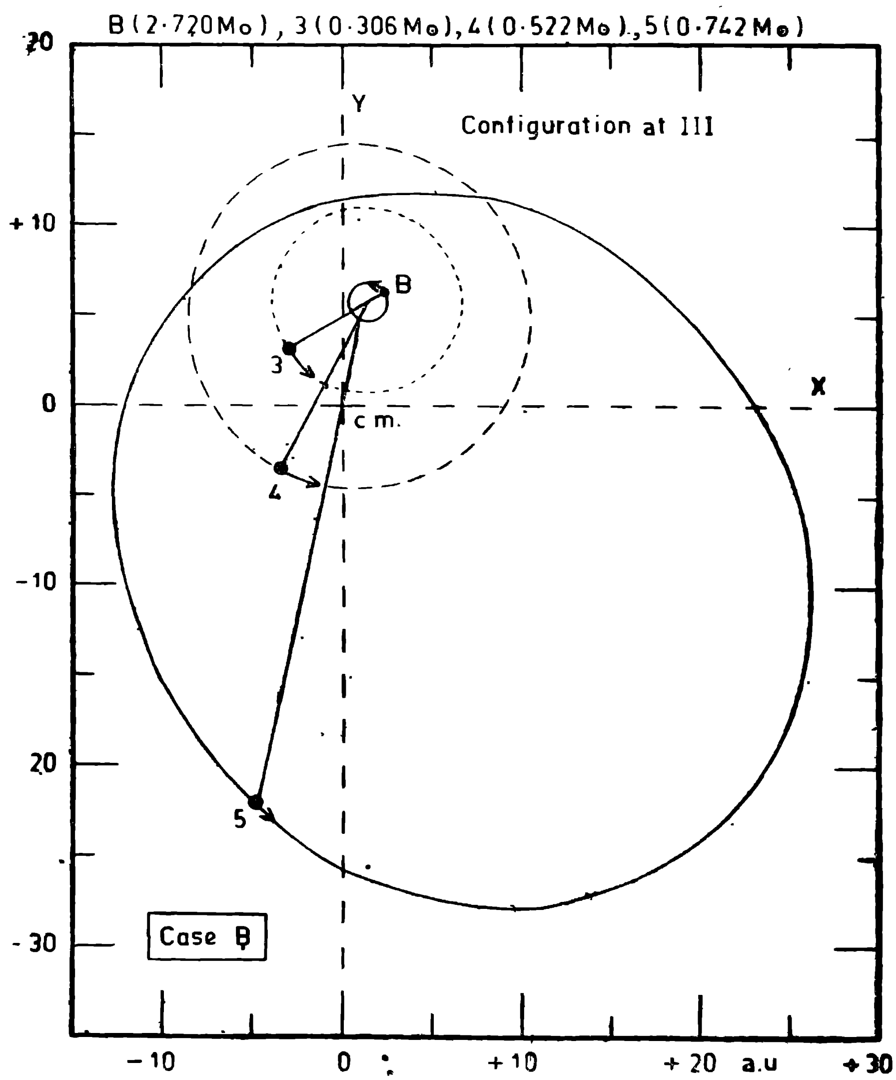

Figure 3. Configuration of the system of TW Dra for case B.

(c) *Cases C and D* : We have also considered the following two extreme configurations of the four bodies. In case C we put star 5 at periastron and star 4 in conjunction with it. This configuration is shown at the top of figure 4 where X′ axis points towards the periastron. In case D the star 5 is put at apastron and star 4 farthest from it at opposition point. This configuration is shown at the bottom of figure 4 where X″ axis points towards the apastron. The initial conditions for these two cases in X′ Y′ and X″ Y″ coordinate systems, respectively are given in table 2.

4. Results for cases A and B

(a) *The (O — C) curve* : For case A the computed trajectories of the stars 3, 4 and 5 with respect to the centre of mass are shown in figure 5 and with respect to the

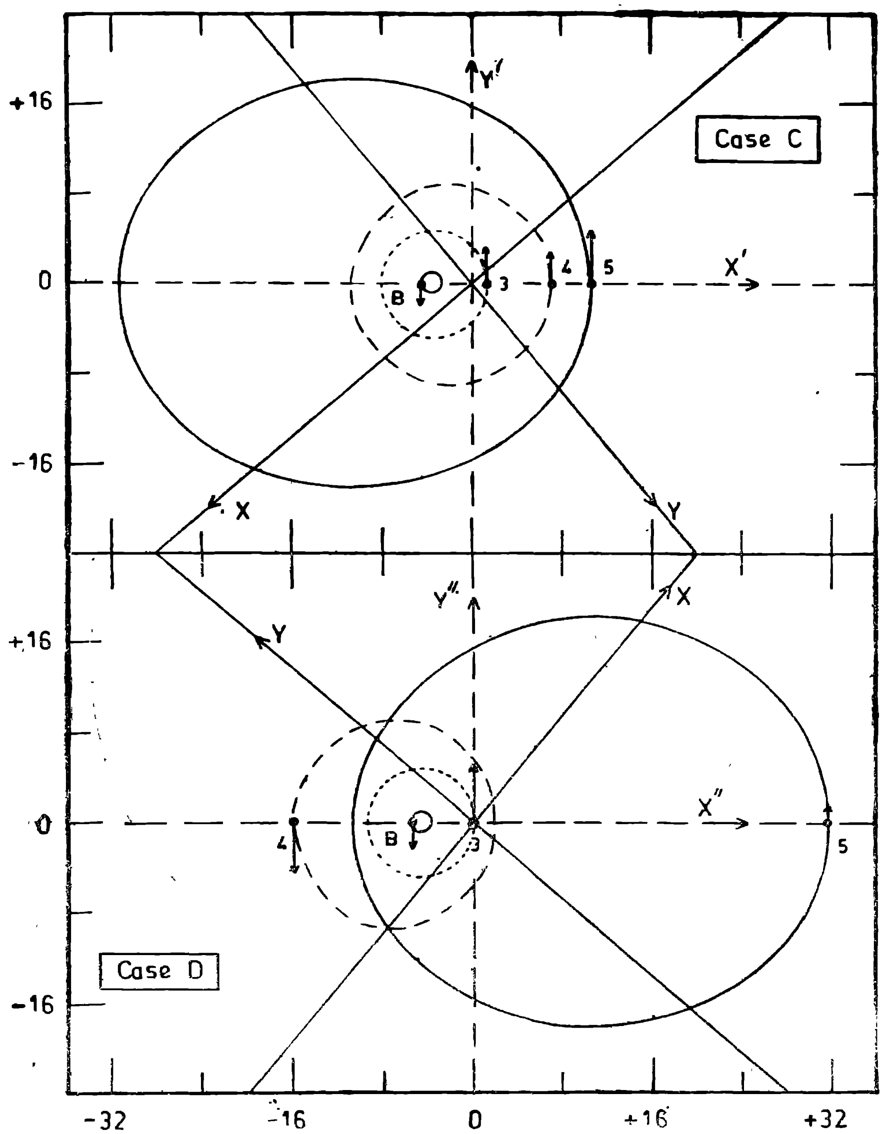

Figure 4. Configurations of the systems of TW Dra for cases C and D.

Table 2. Initial conditions for cases C and D

Star	X′ or X″	Y′ or Y″	$\dot{X}'$ or $\dot{X}''$	$\dot{Y}'$ or $\dot{Y}''$
Case C				
Binary	− 4.370	0	0	−0.2814
Star 3	+ 1.267	0	0	+0.4521
Star 4	+ 7.143	0	0	+0.3624
Star 5	+10.473	0	0	+0.5900
Case D				
Binary	− 5.530	0	0	−0.0315
Star 3	+ 0.107	0	0	+0.7020
Star 4	−15.903	0	0	−0.5269
Star 5	+31.417	0	0	+0.1967

binary B in figure 6. The trajectory of the binary itself with respect to the centre of mass is shown in the inset of figure 1. The latter gives the $(O - C)$ curve from the relation (*cf.* equation 2)

$$O - C = 0.0313 + (Y_1/c) \qquad ...(9)$$

where c is the velocity of light. It is shown by the continuous curve in figure 1. We find that the agreement with observations (dots) is good, which means that the postulated quintet system is viable for the duration of the observations for the assumed initial conditions. However, calculations made for case B, which are

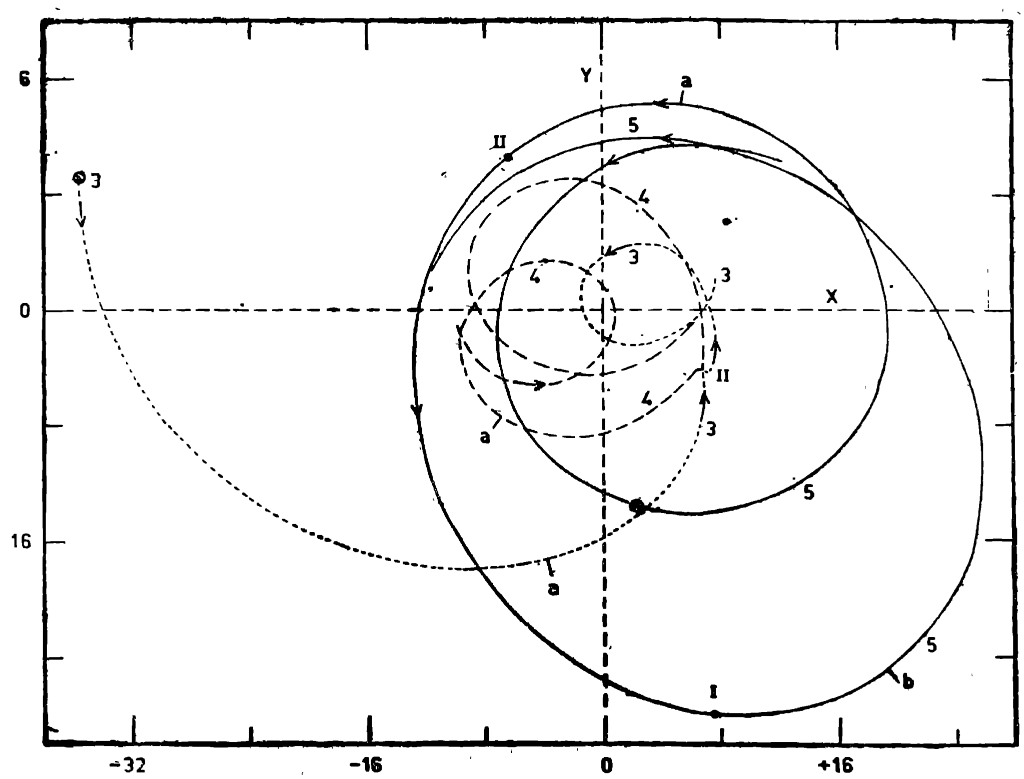

Figure 5. Trajectories of the components of TW Dra for case A with respect to the centre of mass of the system.

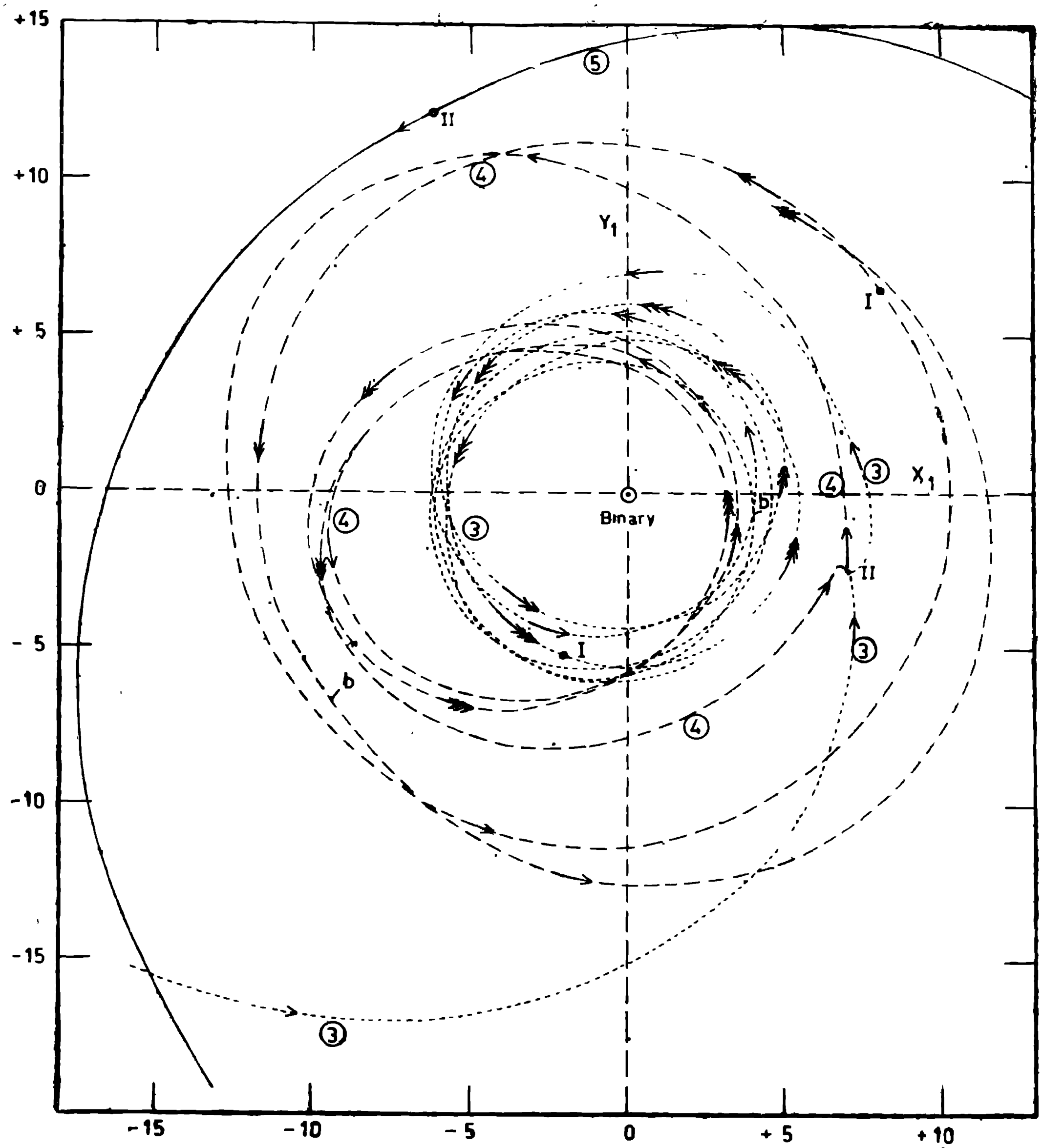

Figure 6. Trajectories for case A with respect to the binary B.

shown by the dash-dot curve in figure 1, show that there are quick departures from the observed $(O - C)$ curve. Hence the initial conditions are very important for even temporary agreement with the observed data, which is a pointer to the instability of the system.

(b) *Treatment of singularity* : The backward integration for case A brings us to point II (shown in figures 1, 5 and 6) where the stars 3 and 4 come very close and Aarseth's NBODY1 program does not work owing to the resultant singularity*. For

*Singularity can be avoided by introducing a softening parameter which we want to avoid in this rigorous calculation. It may be pointed out that Aarseth's NBODY3 or NBODY4 programs, which employ regularization, would have been more suitable for the present problem. However we could not use them because of nonavailability of a larger computer.

example the program stopped working at $t = -133.3$ in Gaussian units. At this instance $r_{34} = 0.2765$ and the derivatives of force, which depend on r_{34}^{-n} become very large and the step size becomes too small. In order to overcome this difficulty, we treated stars 3 and 4 as forming an incipient binary for a short duration. Its motion was resolved into the motion of their centre of mass and their relative motion. During a short interval of time following the breakdown of Aarseth's algorithm the motion of binary B, star 5 and the centre of mass of stars 3 and 4 were simply linearly extrapolated. The relative motion of star 3 with respect to star 4 was represented by a highly elliptic orbit with $a = 1.940357$, $n = 0.336661$, $e = 0.99707$, $\omega = 111.^\circ 313$ and $t_0 = -133.3794$. The singularity was removed on reaching $t = -133.662$ when $r_{34} = 0.77635$. Hence new initial conditions were obtained by recombining the motion of the centre of mass of $m_3 + m_4$ and their relative motion at this epoch. It was verified that their was no change in the total energy of the system between $t = -133.3$ and $t = -133.662$. Integration could then be continued with Aarseth's code without further difficulty. The results of this procedure are quite similar to an elastic collision between stars 3 and 4.

(c) *Probable history of the system* : Figures 5 and 6 show that prior to the encounter at II, star 3 was in a highly elliptical, perhaps even parabolic, orbit. It appears that the star has been captured recently with the following results (see figure 6) : (i) Star 4 which was originally close to the binary has been pushed out into a more open orbit, and (ii) star 3 has ensconced itself in an almost circular orbit close to the central binary.

At first thought we are tempted to attribute the increase in the period of TW Draconis before 1947 inferred by Slovokhotova (1954) to the capture of star 3, because the binary would move out to meet the interloper. However, the mass of the interloper $m_3 = 0.306\ M_\odot$ is too small to account for the observed increase in period.

5. Results for cases C and D

These cases represent the situations when stars 4 and 5 have maximum and minimum mutual perturbations; the final outcomes for both of them are quite similar.

(a) *Case C* : This is the worst case from the stability point of view. The results for this case are shown in figure 7 where the trajectories are drawn with respect to the centre of mass. The salient features are :

(i) Star 3 remains close to the binary B in an eccentric circular orbit with slight perturbation as shown in the inset.

(ii) Star 4 is carried along by star 5 and both shepherd each other in a large elliptical orbit.

Thus star 3 close to the binary and stars 4 and 5 forming a separate close system farther out appears to be a stable configuration.

(b) *Case D* : Results for this case are shown in figure 8 where the trajectories are drawn with reference to the binary B as origin. The motion of the binary with respect to the centre of mass is shown in the inset. Inspection of figure 8 shows the following :

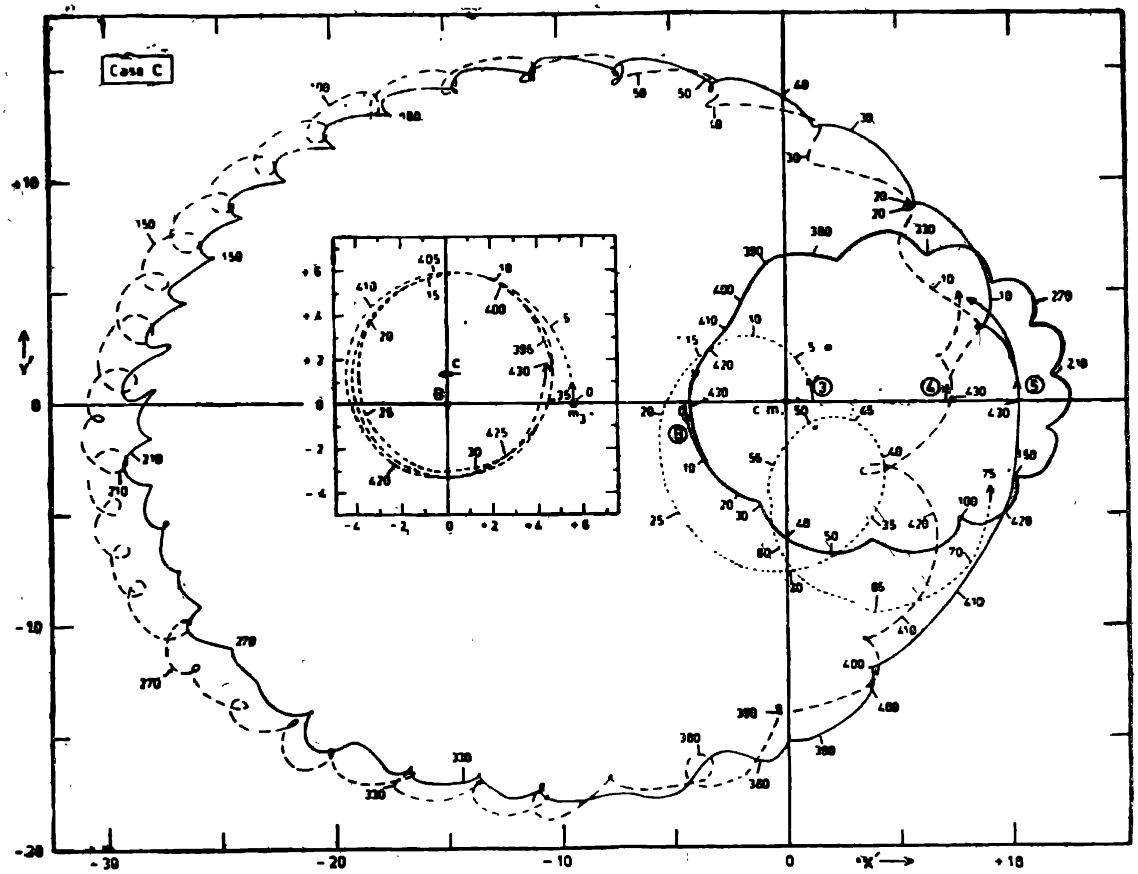

Figure 7. Trajectories for case C with respect to the centre of mass. Inset shows the motion of star 3 with respect to the binary B.

(i) At first, stars 4 and 5 interchange positions by crossing each other's orbit.

(ii) Then star 5 drags star 3 out of its almost circular orbit around the binary, which is shown by the dotted region, and later forms an incipient binary at $t = 362$. The resultant singularity was treated in the manner indicated in section 4b.

(iii) Then, after repeated close encounters between stars 3 an 5, star 3 goes into a tighter elliptical orbit around the binary B and star 5 returns to its original orbit.

(iv) Finally stars 4 and 5 are headed towards each other and may end up into shepherding each other as in case C.

Thus star 3 in a close orbit around B and stars 4 and 5 in an outer shepherding orbit might again be a stable configuration.

6. Conclusions

Calculations for cases A and B show that the observed $(O - C)$ curve can be reproduced by taking appropriate initial conditions. However a small change in the starting conditions spoils the agreement which indicates that the hypothetical quintet system of TW Draconis is not stable.

Both cases C and D indicate that the outer two stars are likely to form a binary system. From observational point of view they could as well be a single object.

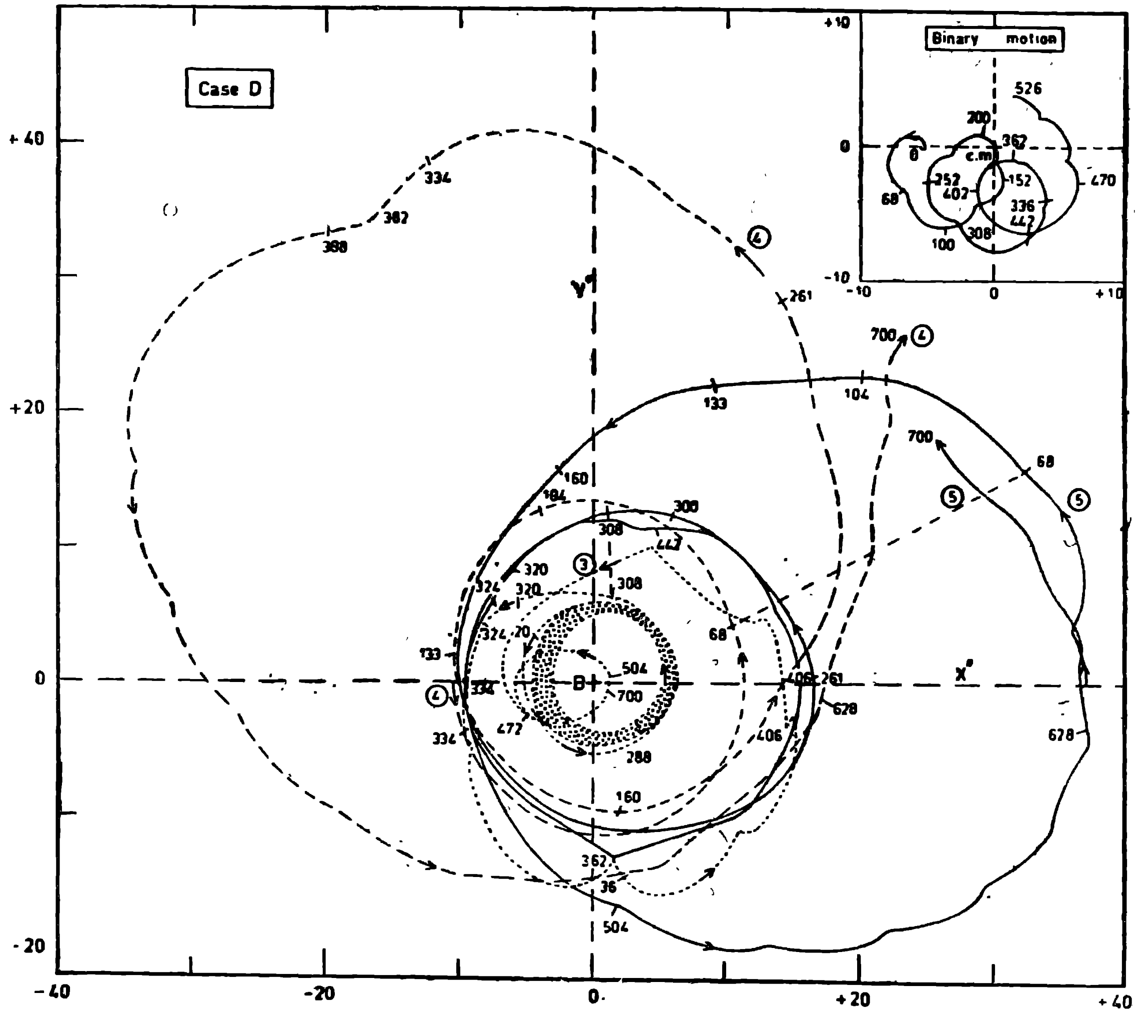

Figure 8. Trajectories for case D with respect to the binary B. Inset shows the motion of the binary with respect to the centre of mass.

In fact Abhyankar & Panchatsaram (1984) have shown that with a new ephemeris

$$\text{Hel. Primary Min.} = JD\ 2432341.8859 + 2.80685833\ (E - 3075) \quad \dots(10)$$

the minimum times of TW Draconis after 1947 can be well represented by a single sinusoid corresponding to the motion of a single body of mass of about 0.8 $M_\odot$, moving in a circular orbit of period 24.2 yr around the binary. But their figure 2 shows wiggles around the mean curve which most probably indicates the presence of another small star in a closer orbit around the binary system, star 3 of the previous sections. TW Draconis could thus be a quadruple if not a quintet systems.

Acknowledgement

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