

UBV photometry of the eclipsing binary R Canis Majoris

K. R. Radhakrishnan, M. B. K. Sarma and K. D. Abhyankar

Centre of Advanced Study in Astronomy, Osmania University, Hyderabad 500 007

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Abstract. Photoelectric *UBV* light curves of the eclipsing binary R CMa obtained with the 1.2 m reflecting telescope of the Japal-Rangapur observatory during 1980–82 have been used for deriving the eclipse elements of the binary. In combination with the spectroscopic elements, absolute dimensions of the system are obtained. The solution of the system has suggested the primary eclipse to be transit and the secondary to be occultation. The *B* and *V* curves gave a unified solution while the *U* curve gave a different solution. The values of : $k = 0.68$, $r_{g,h} = 0.319$, $r_{s,c} = 0.217$, $\theta_e = 30^\circ.96$, $j = 79^\circ.93$, $R_h = 1.73 \pm 0.04 R_\odot$, $R_c = 1.18 \pm 0.02 R_\odot$, $m_h = 1.52 M_\odot$, $m_c = 0.199 \pm 0.004 M_\odot$, $L_h^{\text{bol}} = 6.64 \pm 0.29 L_\odot$ and $L_c^{\text{bol}} = 0.86 \pm 0.03 L_\odot$ are adopted.

The primary component is found to be a normal F 2 V star while the secondary component is found to be a G 8 IV–V star contracting towards a helium white dwarf stage along Hayashi evolutionary track after losing much of its original mass.

Key words : Algol type eclipsing binaries—photometric elements—stellar evolution

1. Introduction

R Canis Majoris (BD — 16° 1898, HD 57167, HR 2788) was put on the observing program of Japal-Rangapur observatory on account of its low mass function, reported peculiarities in its light curve and changes in its period. Detailed accounts of the study of its period changes and spectroscopic observations have been published earlier by Radhakrishnan *et al.* (1984b, c), the main results being the evidence for a third component of mass $0.5 M_\odot$ and confirmation of the low mass function of $0.00251 \pm 0.00014 M_\odot$ for the binary. We now present here the details of our photometric study of the system. (See also Radhakrishnan *et al.* 1984a).

R CMa had earlier been photometrically observed by Wendel (1909), Pickering (1904), Dugan (1924), Wood (1946), Koch (1960), Kitamura & Takahashi (1962), Knipe (1963), Sato (1971) and Guinan (1977). Their light curves indicated a circular orbit, but showed some asymmetry both inside and outside eclipses. Some light curves showed humps shortly after the primary minimum which were present in some light curves and missing in others. Our observations do not show such peculiarities; consequently we have been able to obtain definitive eclipse elements for the system.

2: Observations

R CMa was observed photoelectrically with the 1.2 m telescope of the Japal-Rangapur observatory using an unrefrigerated EMI 6256 B photomultiplier and standard (Johnson) *UBV* filters during 1980-81 and 1981-82 observing seasons. The stars BD - 15°1734 and BD - 15°1732 were used as the comparison and check stars, respectively. These stars were also used by the earlier workers and were found to be constant in brightness. The instrumental system was standardized by observing a number of standard stars. The actual observations in the standard system are given by Radhakrishnan & Sarma (1982) and plotted in figures 1, 2 and 3. In calculating the phases, the motion of R CMa in the third body orbit was taken into account (Radhakrishnan *et al.* 1984b) which gave the following ephemeris :

$$\text{HJD Pri. Min } 244,4648.3283 + 1^d13593853 E.$$

It can be seen from the figures that the light curves in all the three colours are free from any peculiarities reported earlier.

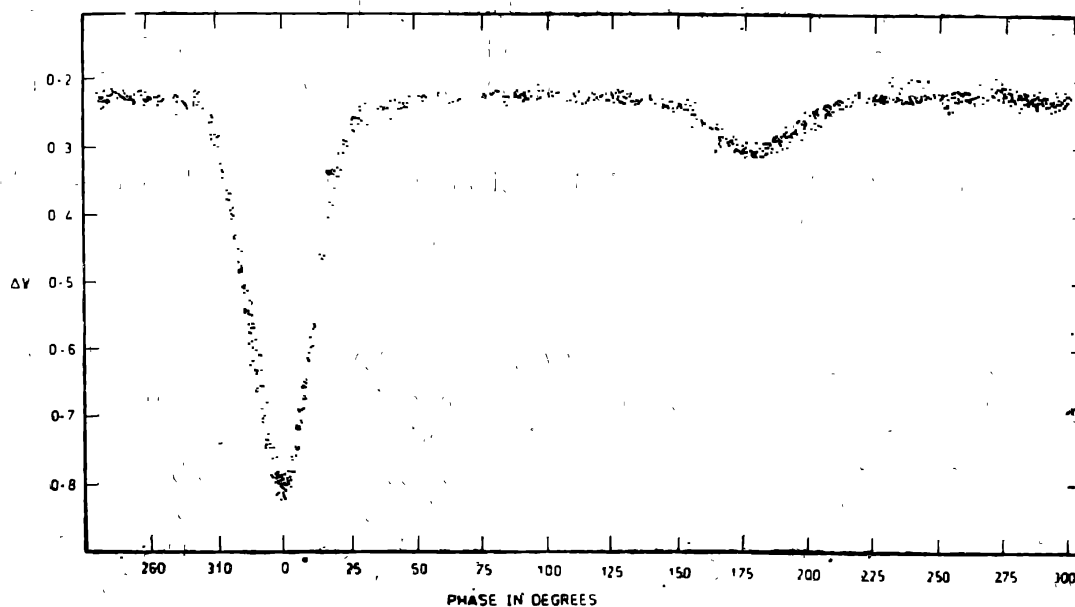


Figure 1. R CMa : Observed light curve in yellow.

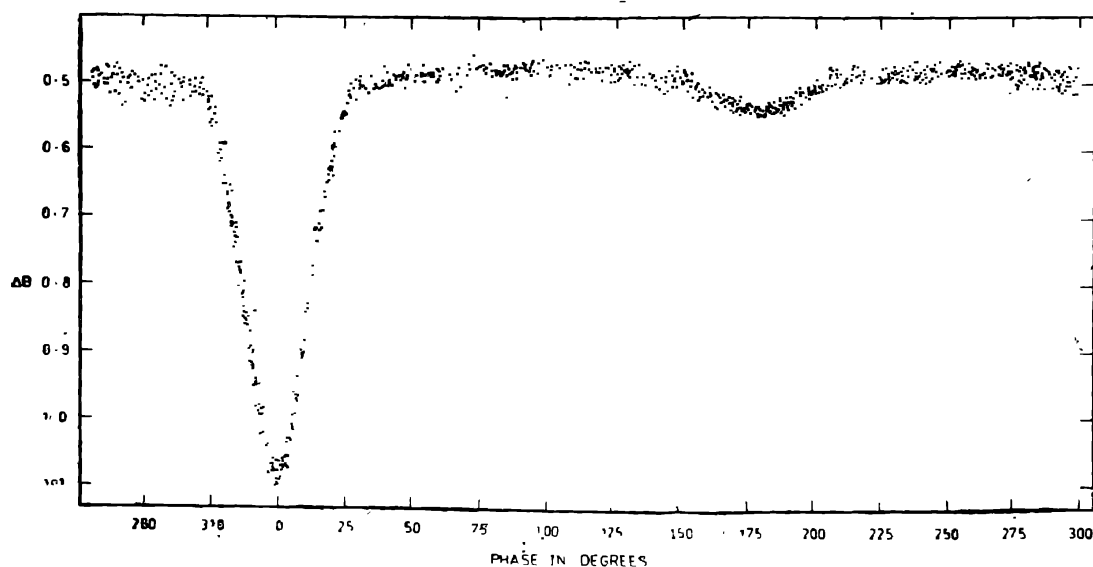


Figure 2. R CMa : Observed light curve in blue.

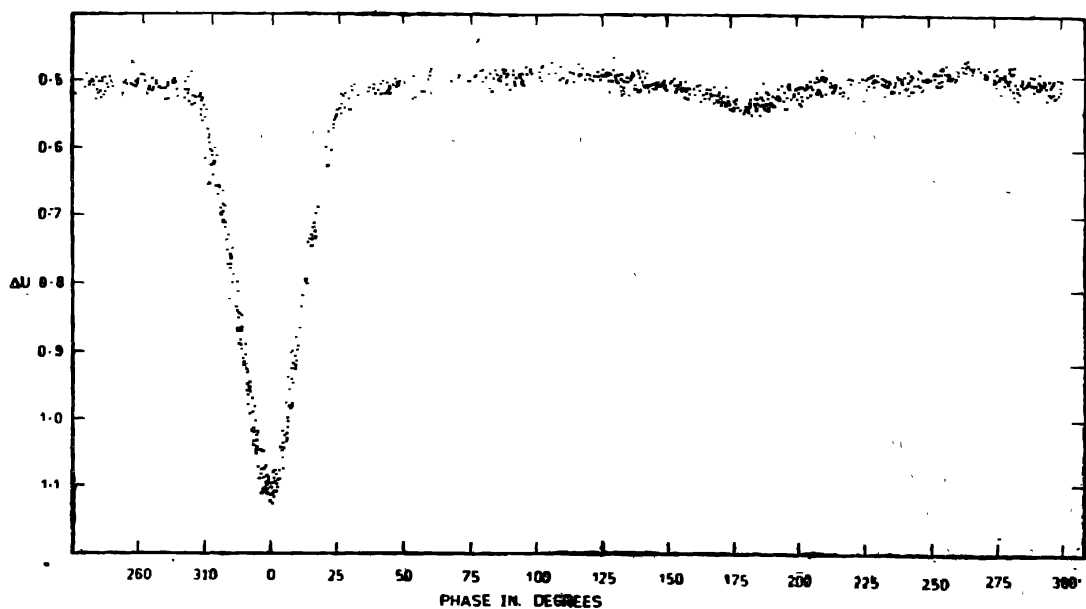


Figure 3. R CMa : Observed light curve in ultraviolet.

3. Results

The light curves were rectified using the method of Russell & Merrill (1952). The derived Fourier and rectification coefficients are given in table 1. In rectifying the light, all the sine and cosine terms with $n = 3$ and 4 were treated as distortion terms and subtracted from the observed luminosity. The adopted limb darkening and gravity brightening coefficients and N values used for rectifying the light curves are given in table 2.

The rectified light curves were analysed for the elements using Wellmann's (1953) modified method (Sarma & Abhyankar 1979). Further, (α, p) tables were generated

Table 1. Fourier and rectification coefficients of R CMa

	<i>V</i>	<i>B</i>	<i>U</i>
A_0	0.9902 ±.0008	0.9885 ±.0005	0.9911 ±.0009
A_1	-.0045 ±.0005	-.0061 ±.0012	—
A_2	-.0124 ±.0013	-.0120 ±.0007	-.0065 ±.0017
A_3	—	0.0011 ±.0008	0.0048 ±.0005
A_4	-.0029 ±.0008	—	0.0034 ±.0009
B_1	-.0018 ±.0003	0.0028 ±.0004	0.0010 ±.0004
B_2	-.0007 ±.0004	0.0026 ±.0004	-.0020 ±.0005
B_3	-.0015 ±.0004	—	0.0019 ±.0005
B_4	-.0013 ±.0004	—	-.0019 ±.0006
C_0	.0060	.0103	0
C_1	.0045	.0061	0
C_2	.0020	.0034	0
Z	.021	.021	.021

Table 2. Adopted values for x , y and N for the U , B , V colours of R CMa

Coefficient	Component	<i>V</i>	<i>B</i>	<i>U</i>
x	Primary	0.60	0.78	0.76
	Secondary	0.70	0.85	0.96
y	Primary	0.948	1.159	1.358
	Secondary	1.277	1.562	1.765
N	Primary	2.532	3.069	3.318
	Secondary	3.109	3.777	4.326

for the exact limb darkening coefficients using the equations of Merrill (1950) and Jurkevich (1970). A solution could be obtained with the primary eclipse as a transit and secondary as an occultation. The solution of the three light curves gave an average $k = 0.68 \pm 0.003$. Using the elements obtained from each solution, the correction to the depths of the hand drawn curves were derived (Vivekananda Rao & Sarma 1981). Adjustments of the depths within the limits of the calculated corrections were made to the minima in the three colours so as to make the geometrical depth p_0 equal in all the colours. With a correction of $-0^m.002$ in the primary depth and $-0^m.003$ in the secondary depth of yellow, and $+0^m.002$ in primary depth and $+0^m.002$ in secondary depth of blue, the geometrical depth p_0 could be made the same *i.e.* -0.6725 for the two colours. In order to have the same geometrical depth for UV also a correction in depth of $-0^m.02$ is required which the observations do not permit, and hence, the solution obtained from the analysis of the U curve was not considered. With the corrected depths and common p_0 , the V and B light curve analysis gave identical orbital elements. These elements were adopted for the system. The elements obtained from the light curve analysis of the

Table 3. Adopted photometric elements of R CMa

Element	V	B	U ⁺	U
α_0^{tr}	.8135	.8019	.8032	.8448
α_0^{oc}	.8776	.8869	.8946	.9261
$1 - I_0^{tr}$.388	.404	.409	.433
$1 - I_0^{oc}$.052	.036	.024	.028
L_h	.941	.959	.973	.973
L_c	.059	.041	.027	.027
L_h''	.964	.976	.986	
L_c''	.036	.024	.014	
θ_e		30°.96		32°.06
P_0		-.6725		-0.7466
k		.680		0.680
j		79°.93		80°.02
r_h		.319		.326
r_c		.217		.222

three colours are given in table 3. The parameters given under the column U^+ in this table refer to those derived theoretically using the elements obtained from V and B curve analysis. Theoretical curves (rectified) computed for all the three colours using the adopted elements are shown in figures 4, 5 and 6 by solid lines. The fit of the observations to the theoretical curve is satisfactory for V and B but poor for the U colour. The fit to the observations obtained from the U curve solution alone is shown in figure 6 by a dashed line. The continuous curve can be brought closer to the dashed curve by increasing the limb darkening coefficient for U from 0.76 to 1.00 and by increasing r_g .

Using L_h and L_c values given in table 3, L_h'' and L_c'' were calculated using the equations (Koch *et al.* 1970):

$$L_h'' = L_h - 0.8 L_c(a b)_h E_h/E_c,$$

$$L_c'' = L_c - 0.8 L_h(a b)_c E_c/E_h.$$

The values of E_h and E_c were taken from Cester's (1969) tables. Normalized values of L_h'' and L_c'' are given in table 3.

4. Absolute elements

(a) Mass ratio

Since R CMa is a single lined spectroscopic binary, we can get only the mass function. The determination of the absolute elements of the system requires a knowledge of the mass ratio m_2/m_1 . This was estimated by the following three methods:

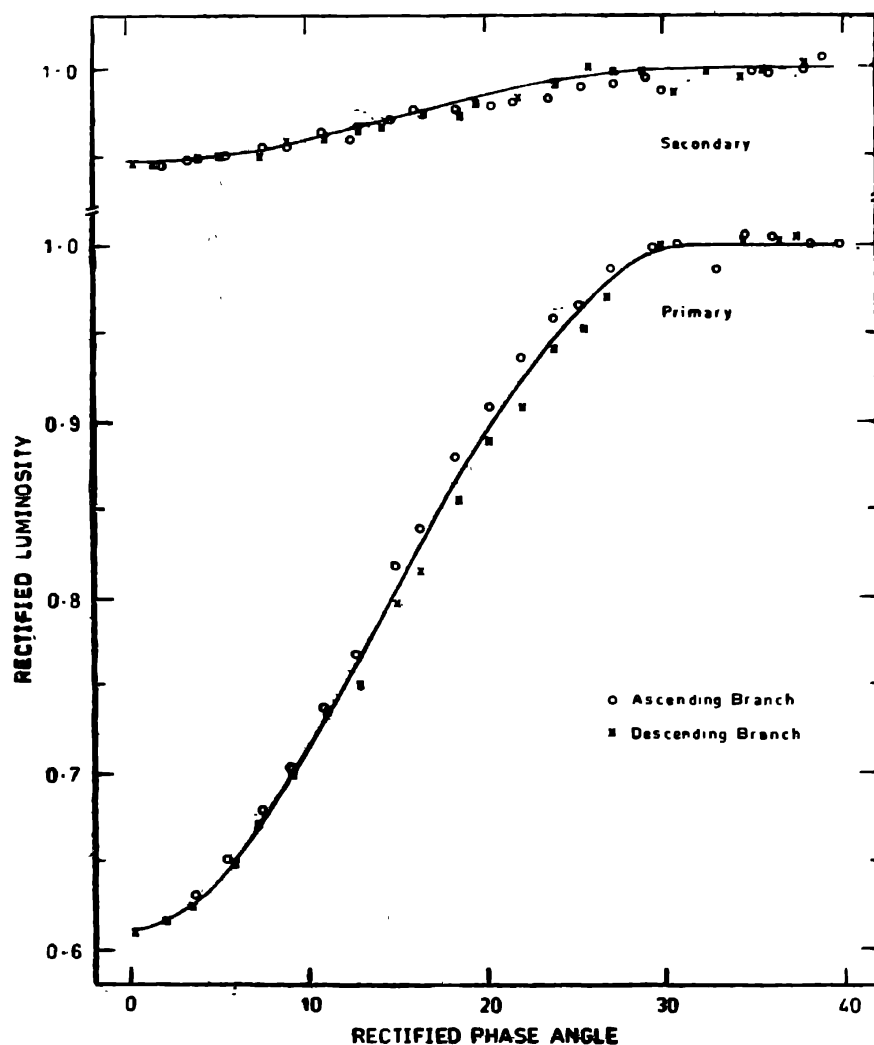


Figure 4. R CMa : Rectified normal points in yellow. The continuous line represents the theoretical curve obtained using the adopted elements from the V and B light curves.

(i) *By assuming the primary to be a main sequence star* : Taking the spectral type of the primary to be $F 2 V$, its mass comes out to be $1.52 M_{\odot}$ (Allen 1976). Then taking the adopted value of $i = 79^{\circ}.93$ in the mass function, $f(m) = 0.0025$, obtained from the spectroscopic solution (Radhakrishnan et al. 1984), we obtain a mass ratio $m_2/m_1 = 0.13$.

(ii) *By assuming that the secondary fills its Roche lobe* : R CMa is a semidetached system. In most of such systems the evolved, less massive secondary component is found to fill its Roche lobe. Assuming a similar situation for this system also, we can determine the mass ratio from the Roche lobe dimensions. The semi-axes of the Roche lobes for various mass ratios have been tabulated by Plavec & Kratochvil (1964). The semi-axes listed are those for the axis b of the ellipsoid of semi-axes a , b and c . To a first approximation, this may be taken to be the fractional radius of the spherical star as derived from the solution of the light curves.

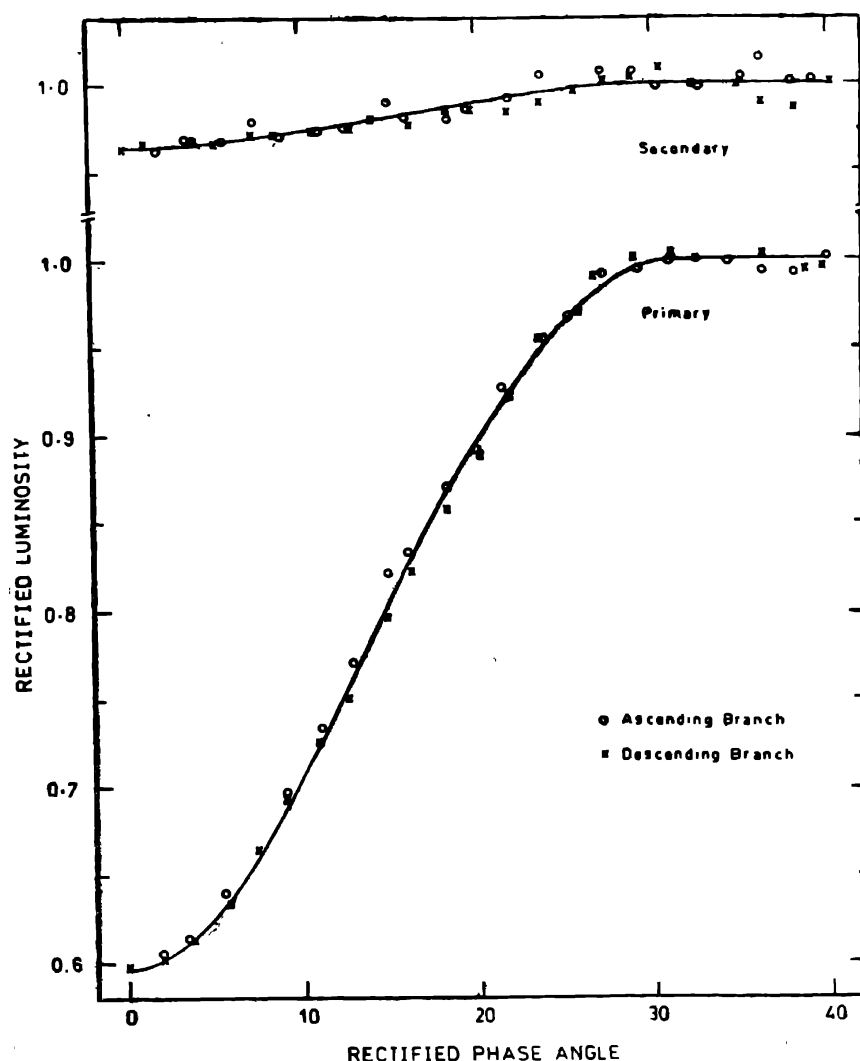


Figure 5. R CMa : Rectified normal points in blue. The continuous line represents the theoretical curve obtained using the adopted elements from the *V* and *B* light curves.

Thus using the adopted value of $r_B = 0.217$ for the fractional radius of the secondary, we obtained a mass ratio of 0.14.

(iii) *By comparing the theoretical and observed Fourier coefficients* : The Fourier coefficients determined from the outside eclipse portion of the light curve are given in table 1. We have considered only the cosine terms which can be calculated theoretically from the formulae given by Merrill (1970). The observed as well as the theoretical Fourier coefficients were normalized to unit luminosity at quadrature. The sum of the squares of the differences between the observed and calculated Fourier coefficients $\Sigma(O - C)^2$ were calculated for various assumed values of the mass ratio m_2/m_1 . From the plot of $\Sigma(O - C)^2$ versus m_2/m_1 , the minimum value corresponding to the best fit was determined. This way a best fit was obtained for the mass ratio of 0.150 in *V* colour, 0.125 in *B* colour and 0.100 in *U* colour yielding a mean of 0.125 ± 0.025 .

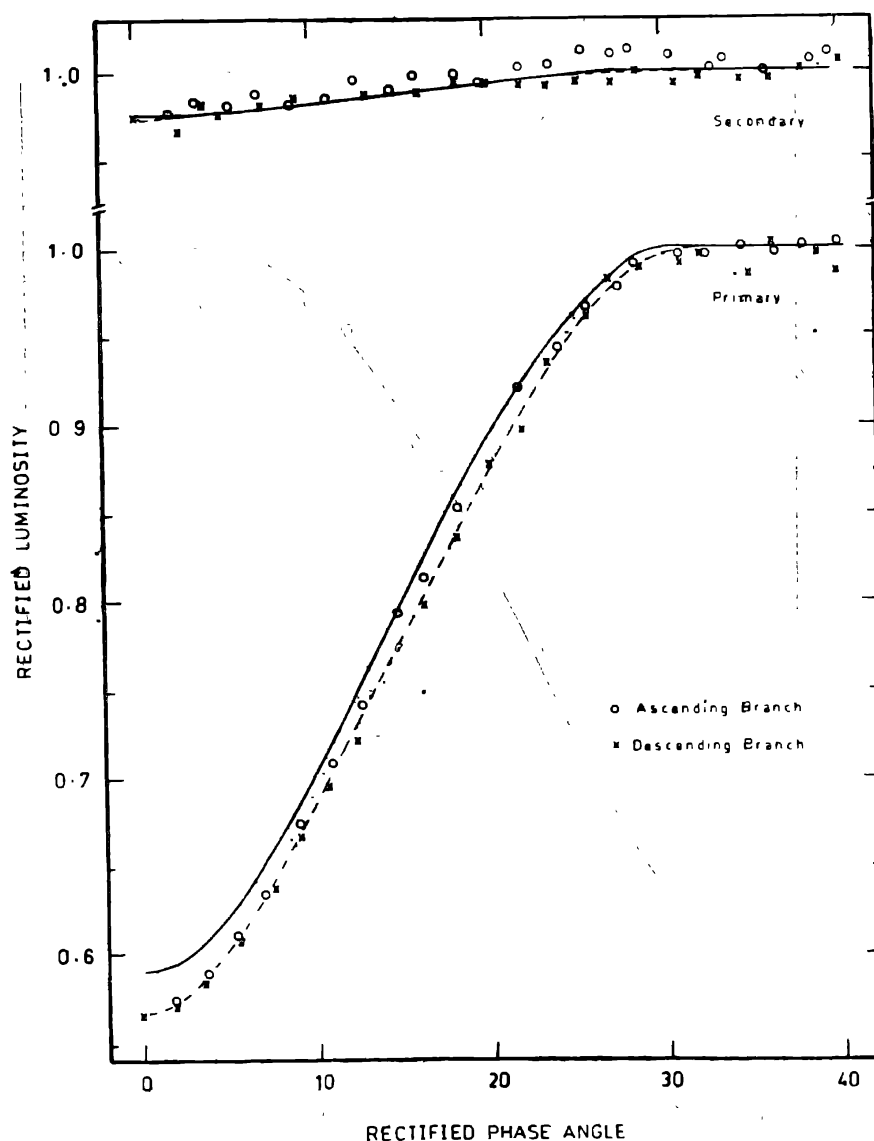


Figure 6. R CMa : Rectified normal points in ultraviolet. The continuous line represents the theoretical curve obtained by using the adopted elements from V and B light curves. The broken line represents the theoretical curve obtained by using the elements derived from the U light curve solution alone. The continuous curve can be brought closer to the broken curve by increasing the limb darkening coefficient for U from 0.76 to 1.00, and by increasing r_g .

From all the above three determinations it is felt that $m_2/m_1 = 0.13 \pm 0.01$ is the best value to be adopted for R CMa.

(b) *Absolute dimensions*

The masses and radii obtained from the adopted mass ratio $m_2/m_1 = 0.13$, $a_1 \sin i = 4.33 \pm 0.18 \times 10^5$ km obtained from the spectroscopic orbital solution (Radhakrishnan *et al.* 1984b) and $i = 79^\circ.93$ obtained from the adopted eclipse elements of the present study, are given in table 4 which also gives the semiaxes of

the triaxial ellipsoid. These axes were determined from the equations (Russell & Merrill 1952)

$$\frac{a-b}{r} = \frac{3}{2} m_2/m_1 r^3(1+2K),$$

$$\frac{b-c}{r} = \frac{m_1+m_2}{2m_1} r^3(1+2K), \quad \dots(2)$$

where K depends upon the central concentration of density and is less than 0.02 (Kopal 1959). Following Merrill (1970), assuming $K = 0.01$ and simplifying equations (2) we get

$$a = r + (0.17 + 1.19 m_2/m_1) r^4,$$

$$b = a - 1.53(m_2/m_1) r^4,$$

$$c = 3r - a - b. \quad \dots(3)$$

The semiaxes b of the Roche lobes of the components for the mass ratio of 0.13 as obtained from the tables of Plavec & Kratochvile (1964) are 0.572 and 0.214. Comparing these with the b axis values obtained for the components, it is seen that the primary component is well within its Roche lobe and the secondary component is just filling its lobe. R CMa is thus seen to be semidetached.

Table 4. Absolute parameters of R CMa

Element	Hotter component	Cooler component
Mass	1.52 M_{\odot} (assumed)	0.199 ± 0.004 M_{\odot}
Radius	1.73 ± 0.04 R_{\odot}	1.18 ± 0.02 R_{\odot}
a	0.322	0.218
b	0.320	0.217
c	0.315	0.216
V	5.76 ± 0.01	9.34 ± 0.01
$(B - V)$	+ 0.31 ± 0.01	+ 0.77 ± 0.01
$(U - B)$	+ 0.04 ± 0.02	+ 0.60 ± 0.02
Sp (adopted)	F 2 V	G 8 IV - V

(c) Spectral types

The magnitude and colours of the comparison star C_1 (BD - 15° 1734) after standardization are

$$V = 5^m.48 \pm 0.01,$$

$$(B - V) = + 0.07 \pm 0.01,$$

$$(U - B) = + 0.04 \pm 0.01.$$

Using these values and the differential magnitudes corresponding to unit luminosity at maximum light ($\Delta V = 0.222$, $\Delta B = 0.480$, $\Delta U = 0.490$), we obtain for the variable at quadrature

$$V = 5^m.70 \pm 0.01, \quad B = 6^m.03 \pm 0.01, \quad U = 6^m.08 \pm 0.02.$$

For computing the magnitudes and colours of the individual components, we first calculate $L'_h = L_h - 0.4 L_c(ab)_h E_h/E_c$ and $L'_c = L_c - 0.4 L_h(ab)_c E_c/E_h$ and equate $L'_h + L'_c$ to the magnitudes at quadrature. Then the magnitudes corresponding to unnormalized L''_h and L''_c give the magnitudes and colours for the two components; they are also given in table 4. On the assumption that there is no space reddening at the small distance of 42 pc (Guinan and Ianna 1983), the $(B - V)$ colours correspond to the spectral type of F 1 V for the primary and G 7 V for the secondary component while the $(U - B)$ colours correspond to F 4 V for the primary and K 1 V for the secondary (Allen 1976). Taking the distance modulus of $m - M = + 3^m.10$ as given by Guinan & Ianna (1983), the visual absolute magnitudes of the primary and secondary are obtained as $+2.66 \pm 0.30$ and 6.24 ± 0.30 . These correspond to F 1 ± 2 V for primary and K 1 ± 2 V for the secondary. From these results we have adopted F 2 V and G 8 IV-V as the spectral types of the primary and secondary, respectively. These are in agreement with the classification by others. From spectral classification analysis Fringant (1956) had classified R CMa as F 1 V. The primary was classified as F 4 V by Kitamura & Takahashi (1962), F 1 V by Sato (1971) and F 0 V by Guinan (1977). For the secondary Sato (1971) had obtained a spectral type of G 4 and Guinan (1977) K 3 V.

(d) *Bolometric luminosities*

With the assumed temperature of 7072 K for primary (F 2 V) and 5148 K for the secondary (G 8 IV-V) (Allen 1976) and with the derived radii of $1.73 R_\odot$ and $1.18 R_\odot$ for the two components, Stefan-Boltzmann's law gives the following bolometric luminosities for the two stars

$$L_h^{\text{bol}}/L_\odot = 6.64 \pm 0.29 \quad \text{and} \quad L_c^{\text{bol}}/L_\odot = + 0.86 \pm 0.03,$$

which correspond to bolometric magnitudes of

$$M_h^{\text{bol}} = + 2.69 \pm 0.04 \quad \text{and} \quad M_c^{\text{bol}} = + 4.91 \pm 0.03.$$

If the mass-luminosity relation holds good, the primary with mass $1.52 M_\odot$ should have a bolometric magnitude of $+ 2^m.7$ and the secondary with mass $0.199 M_\odot$ should have a bolometric magnitude of $10^m.3$ (Allen 1976). Comparing these values with the values derived above, it is seen that while the primary component is normal, the secondary is overluminous by about five magnitudes for its mass.

5. Evolutionary status of R CMa

R CMa is a typical semidetached Algol system with a secondary of very low mass. Evolution of such binaries has been interpreted in terms of case B mass transfer (Refsdal & Weigert 1969; Plavec 1973; Hall 1975). We will first consider the case of evolution of R CMa under conservative mass exchange and study its implications. As the present masses of the components of R CMa are $1.52 M_\odot$ for the primary and $0.2 M_\odot$ for the secondary one can start with a system having $m_1^0 = 1.0$ to $1.2 M_\odot$ and $m_2^0 = 0.7$ to $0.5 M_\odot$. In such a case there are two difficulties :

(i) The more massive star of mass 1.0 to 1.2 M_{\odot} would take about 10^{10} yr (Iben 1964) to complete its main sequence life and come to case B mass transfer stage. But Guinan & Ianna (1983) have found the system R CMa to be a high velocity star of age $2-6 \times 10^9$ yr.

(ii) According to theoretical calculations of Neo *et al.* (1977) the mass accreting star cannot absorb a large mass of 0.8 to 1.0 M_{\odot} from its companion. Further with the accretion of such a large mass, the accreting stars are found to move away from the main sequence. Since the mass-accreting component of R CMa (the present primary) is still near the main sequence it could not have started with an initial mass much less than its present mass of 1.52 M_{\odot} .

An assumption of a complete reversal of masses by taking $m_1^0 = 1.5 M_{\odot}$ and $m_2^0 = 0.2 M_{\odot}$ will also not be tenable because, m_2^0 then would still be in the gravitational contraction phase and so would not be in a position to receive the matter lost by m_1^0 . We have therefore to conclude that we are dealing here with non-conservative mass transfer.

We suggest that the present primary of R CMa would have started with an initial mass of about 1.3 M_{\odot} , slightly less than its present mass of 1.52 M_{\odot} , and the secondary was much more massive with an initial mass of around 1.8 M_{\odot} and the evolution of this pair of stars took place under nonconservative mass transfer. The present positions of the components of R CMa in the H R diagram (zero age main sequence) are shown in figure 7. It is seen that the present primary component of R CMa of 1.52 M_{\odot} , is near the zero age main sequence corresponding to a normal F 2 V star. The secondary of R CMa is also seen to be near the main sequence position of a G 8 star which would correspond to a mass of 0.84 M_{\odot} . But our studies showed that the mass of the present secondary of R CMa is about 0.2 M_{\odot} indicating an abnormality. Since it is overluminous and hotter for its mass its mean molecular weight should be higher than that of a normal hydrogen star.

The theoretical evolutionary track of a star of initial mass 1.8 M_{\odot} in a binary system as computed by Refsdal & Weigert (1969) is shown in figure 7 by the solid line *abcdfgl*. The present primary which is the mass-accreting component in R CMa with an initial mass of 1.3 M_{\odot} would have occupied the position near F 5 as shown. The present secondary which is the mass losing component with the initial mass of 1.8 M_{\odot} would have occupied the position indicated by *a* on the main sequence. Since this was the more massive star in the system initially, it would evolve first and leave the main sequence early along the evolutionary track *ab*. According to the calculations of Refsdal & Weigert (1969), this star after leaving the main sequence reaches the position *c* after 6.876×10^8 yr which position corresponds to the cessation of core hydrogen burning. Beyond *c* the hydrogen in a shell surrounding the core is ignited and the hydrogen shell starts expanding. 8.455×10^8 yr after leaving the main sequence, the star fills its Roche lobe at *d* and starts mass transfer to its companion. The dashed line *dd'* shows the evolutionary track of single stars which at this stage start moving towards the red giant region. During the phase *df*, the star rapidly loses most of its mass under case B mass exchange. According to Refsdal & Weigert (1969), 8.462×10^8 yr after leaving

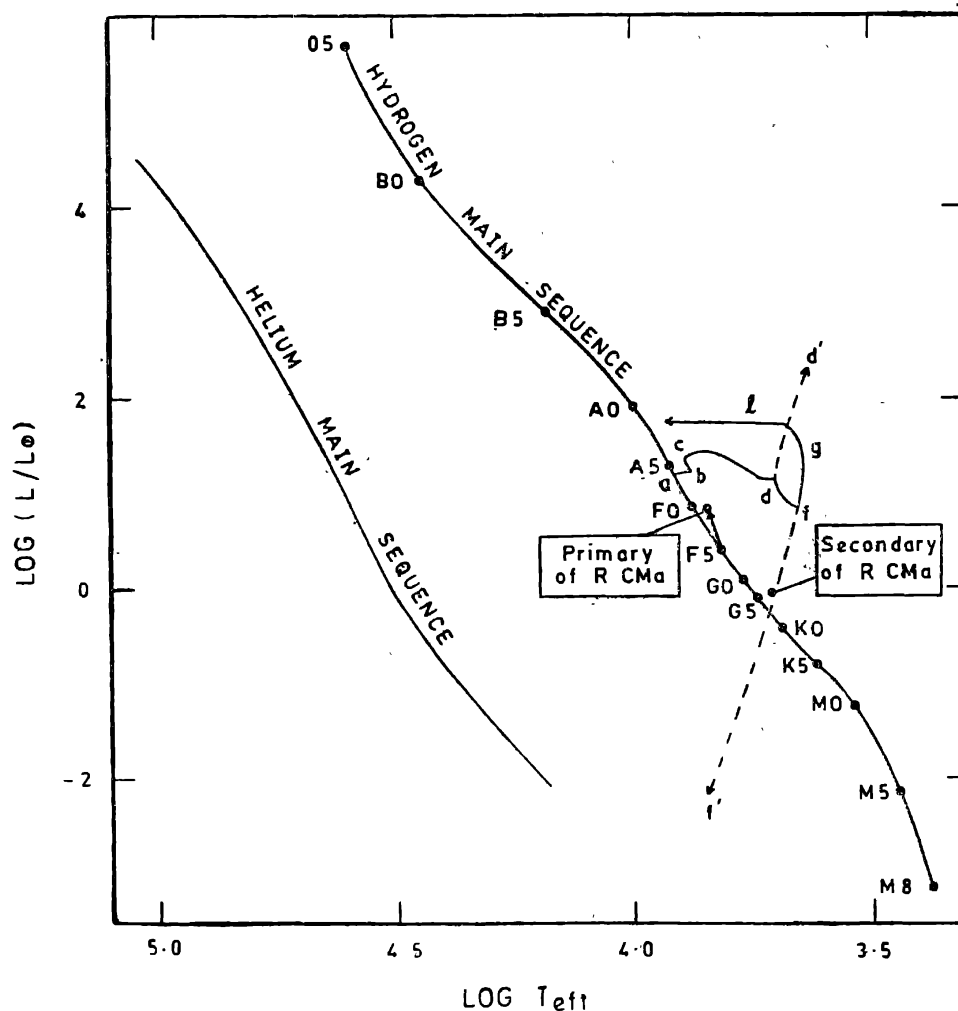


Figure 7 R CMa : Evolutionary tracks of the components in the H R diagram. See text for explanation.

the main sequence, helium is ignited at *f* due to core contraction and the star starts moving up in the H R diagram. The main sequence of helium stars obtained from the theoretical computations of Hansen *et al.* (1972) is shown in figure 7. The star after losing some more mass stops mass transfer at *l* and will finally settle down on the helium main sequence. We find that the secondary of R CMa does not take this route.

The helium main sequence, unlike the hydrogen main sequence, is double valued with regions of same mass corresponding to high density and low density. Hansen *et al.* (1972) have found that the minimum mass for stable helium configuration is $0.2905 M_{\odot}$ below which the star will not be able to ignite helium. Such stars will evolve down the Hayashi track under contraction with convective cores and will end up as degenerate objects without ever reaching the helium main sequence. The present mass of $0.2 M_{\odot}$ of the secondary of R CMa and its position in the H R diagram shows that during the phase *df* of its evolution, the star should have expanded rapidly and lost most of its hydrogen envelope through the second

Lagrangian point L_2 as well as by Roche lobe overflow because the present primary could have absorbed only a small amount of this matter. During this phase, the present primary, after absorbing about $0.2 M_{\odot}$, has moved slightly upwards and reached the present position. The present secondary having lost a large part of the mass amounting to $1.6 M_{\odot}$ is left with a mass of $0.2 M_{\odot}$. The fact that the star has now apparently moved down from point f instead of going up, indicates that it has lost its hydrogen envelope very rapidly. Its mass of $0.2 M_{\odot}$ being too small to ignite helium, the star contracts further and moves down the Hyashi track to become a helium white dwarf. Similar conclusion about the final stage of evolution of this component is arrived at by Guinan & Ianna (1983).

It has recently been shown by Abhyankar (1984) that evolution of all Algol systems is nonconservative. The present secondaries which were originally the more massive components evolved faster and have lost most of their mass through the outer Lagrangian points. Only a fraction of that mass is captured by the companion. The state of the secondaries is characterized by the fact that they are not only overluminous for their mass but also hotter. This is the result of loss of hydrogen from the envelope and consequent increase in its mean molecular weight. *R Canis Majoris* systems are characterized by the fact that their secondaries have lost a very large fraction of their mass and hence exhibit extreme characteristics of overluminosity and excessive surface temperatures.

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References

- Abhyankar, K. D. (1984) *Ap. Sp. Sci.* **99**, 355.
 Allen, C. W. (1976) *Astrophysical Quantities*, Athlone Press.
 Cester, B. (1969) *Mem. Soc. Astr. Ital.* **11**, 169.
 Dugan, R. S. (1924) *Contr. Prin. Univ. Obs.* **6**, 49.
 Fringant, A. M. (1956) *Contr. Inst. Astr. Paris Ser. A*, No. 216.
 Guinan, E. F. (1977) *Astr. J.* **82**, 51.
 Guinan, E. F. & Ianna, P. A. (1983) *Astr. J.* **88**, 126.
 Hall, D. S. (1975) *Acta. Astr.* **25**, 1.
 Hansen, C. J., Cox, J. P. & Herz, M. A. (1972) *Astr. Ap.* **19**, 144.
 Iben, I. (Jr). (1964) *Ap. J.* **140**, 1631.
 Jurkevich, J. (1970) *Vistas Astr.* **12**, 63.
 Kitamura, M. & Takahashi, C. (1962) *Publ. Astr. Soc. Japan*, **14**, 44.
 Knipe, G. F. G. (1963) *Rep. Obs. Johannesburg. Circ.* **7**, No. 122, 21.
 Koch, R. H. (1960) *Astr. J.* **65**, 326.
 Koch, R. H., Plavec, M. & Wood, F. B. (1970) *Publ. Univ. Penn. Astr. Ser.* **11**, p. 9.
 Kopal, Z. (1959) *Close Binary Systems*, Chapman & Hall.
 Merrill, J. E. (1950) *Contr. Prin. Univ. Obs.* **23**.
 Merrill, J. E. (1970) *Vistas Astr.* **12**, 43.
 Neo, S., Miyaji, S., Nomoto, K. & Sujimoto, D. (1977) *Publ. Astr. Soc. Japan* **29**, 249.
 Pickering, E. C. (1904) *Harvard Annals* **46**, 184.
 Plavec, M. (1973) *Extended Atmospheres and Circumstellar Matter in Spectroscopic Binary Systems* (ed : A. H. Batten) Reidel, p. 216.

- Plavec, M. & Kratochvil, P. (1964) *Bull. Astr. Inst. Czech.* **15**, 165.
- Radhakrishnan, K. R., Abhyankar, K. D. & Sarma, M. B. K. (1984b, c) *Bull. Astr. Soc. India* **12**, 182; 411.
- Radhakrishnan, K. R. & Sarma, M. B. K. (1982) *Contr. Nizamiah & Japa-Rangapur Obs. No.* 16.
- Radhakrishnan, K. R., Sarma, M. B. K. & Abhyankar, K. D. (1984a) *Ap. Sp. Sci.* **99**, 239.
- Refsdal, S. & Weigert, A. (1969) *Astr. Ap.* **1**, 167.
- Russell, H. N. & Merrill, J. E. (1952) *Contr. Prin. Univ. Obs.* **26**.
- Sarma, M. B. K. & Abhyankar, K. D. (1979) *Ap. Sp. Sci.* **65**, 443.
- Sato, K. (1971) *Publ. Astr. Soc. Japan*, **23**, 335.
- Vivekananda Rao, P. & Sarma, M. B. K. (1981) *Acta. Astr.* **31**, 107.
- Wellmann, P. (1953) *Z. Ap.* **32**, 1.
- Wendell, O. C. (1909) *Harv. Ann.* **69**, 66.
- Wood, F. B. (1946) *Contr. Prin. Univ. Obs.* **6**, 49.