Bull. Astr. Soc. India (1985) 13, 217–224

Spacetime curvature, rotation and the pulse profile of fast pulsars*

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Received 1985 June 5

Abstract. Propagation of photons in the spectime exterior to a rapidly rotating neutron star is investigated, using a rotationally perturbed spherical metric, to determine the pulse profile of fast pulsars. We find that spacetime curvature produces a substantial amount of divergence in the pulse beam width accompained by a reduction in the pulse intensity. Effects due to rotation are comparatively smaller, but rotation has the qualitatively important feature of producing a tilt of the pulse cone in the direction of rotation and a deformation of the cone. The asymmetry in the pulse profile so caused introduces a time delay in the arrival of photons emitted within the cone, which, in turn, affects the duty cycle of the pulsar.

Key words: spacetime curvature—rotation—neutron star—fast pulsars—pulse profile

1. Introduction

The discovery of fast pulsars (Backer *et al.* 1982; Boriakoff *et al.* 1983) has opened up new vistas for theoretical investigations of the structure of rapidly rotating neutron stars (Harding 1983; Shapiro *et al.* 1983; Ray & Datta 1984; Friedman *et al.* 1984) as well as their radiation characteristics (Arons 1983; Ray & Chitre 1983; Kapoor & Datta 1984). Large rotation rates (surface velocity as much as 15% of the speed of light) was hitherto unecountered in objects of this class, and a question that naturally arises is whether the dragging of inertial frames induced by rotation, coupled with a large spacetime curvature, can lead to any observable consequences (Kapoor & Datta 1984). In this essay we illustrate some of these effects on the radiation characteristics (specifically, the pulse profile), and point out the significant differences from a simple Schwarzschild treatment. For the purpose of illustration we choose the pulsar PSR 1937 + 214, which has a period of 1.56 ms. Since this pulsar is close to the point of secular rotational instability (Ray & Datta 1984), the results reported here will be indicative of the maximal effects of rotation.

*Received 'honourable mention' at the 1985 Gravity Research Foundation essay competition.

There is no general consensus at present regarding the pulsars' pulse emission mechanism. Here we have chosen one of the widely studied models, namely, the one in which the emission is suggested to take place in the neighbourhood of the surface of the neutron star in the form of a narrow conical beacon (angular width $\lesssim 10°$), directed away from the surface and corotating with the star (Radhakrishnan & Cooke 1969). We take the initial pulse profile to be a steep Gaussian curve with the total beam width equal to 10°. Our theoretical framework, however, is general (since it involves the calculation of trajectories of massless particles in the exterior of a rotating mass); it is also frequency independent and will be applicable to any initial pulse shape and any emission mechanism.

A rotationally perturbed interior spherical metric of the following form (that matches at the surface to an external metric) :

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$$

$$= e^{2\nu} dt^2 - e^{2\lambda} dr^2 - e^{2\mu} d\theta^2 - e^{2\psi}(d\phi - \omega dt)^2 \qquad \ldots(1)$$

is sufficient for our purpose (Ray & Datta 1984). Here ν, μ, λ, ψ and ω are all functions of r. The metric (1) is valid for strong gravitational fields and rotation rates smaller than that corresponding to centrifugal break-up. Neutron star models rotating at the secular instability limit and relevant in the context of fast pulsars are within this bound, provided the star is assumed to be homogeneous. Such models have been numerically constructed by Ray & Datta (1984) for a representative choice of the equation of state for neutron star matter. The structure parameters and the rotationally induced deformations thereof can be used in the external form of the metric (1) to derive photon trajectories (Thorne 1971).

2. Photon trajectories

Consider a photon emitted from a point whose radial location is $r = r_e \geqslant R'$ where R' is the radius of the rotating neutron star, and received by a remote observer at the point $r = D$ (measured from the centre of the star). Spacetime curvature will bend the null geodesic and rotation will drag it away from the original direction of its emission in the direction of the rotation. The net azimuthal angle of bending will be given by

$$-\phi_0(\delta) = \int_{r_e}^{D} f(r, q_e)\, dr, \qquad \ldots(2)$$

$$f(r, q_e) = \frac{d\phi/d\Gamma}{dr/d\Gamma}. \qquad \ldots(3)$$

Here, Γ is an affine parameter and δ is the azimuthal angle of emission made by the photon's direction of motion with the radius vector of the source through the origin of the coordinate system, as seen in the local rest frame of the star. ($\delta = 0$ will thus correspond to radially outward photon and $\delta = -\pi/2$ to a tangentially forward photon). The quantity q_e is the impact parameter of the photon, evaluated at $r = r_e$. The impact parameter is given by the ratio of the orbital angular momentum to the energy of the photon as measured at infinity, Equation (2) neglects polar bending. This will be small because the spacetime chosen is a perturbed spherical one. The polar angle of emission, θ_s, made by the line of sight through the centre of the star with respect to the axis of rotation can, in this approximation, be taken to be the angle of inclination according to a distant observer. The right-hand side of equation (2) can be evaluated from a knowledge of equations of motion of the photon in the spacetime described by the external form of the metric (1). This gives

$$f(r, q_e) = \frac{\omega(1 + \omega q_e) - q_e e^{2\nu - 2\psi}}{e^{\nu - \lambda}[(1 + \omega q_e)^2 - q_e^2 e^{2\nu - 2\psi}]^{1/2}} \quad \ldots(4)$$

$$q_e \equiv q(r = r_e) = \left.\frac{e^{\psi - \nu}(V_s + \sin \delta)}{1 + e^{\psi - \nu}(\omega V_s + \Omega \sin \delta)}\right|_{r = r_e}. \quad \ldots(5)$$

Here Ω is the angular velocity of the star as seen by a distant observer, and

$$V_s = e^{\psi - \nu}(\Omega - \omega),$$

$$\omega = 2J/r^3,$$

J being the angular momentum of the rotating neutron star.

The null geodesic is now fully determined. Figure 1 schematically illustrates the bending. For the pulse cone, bending of photons implies an effective overall divergence in the beam. If we define

$$\delta_{new} = \delta + \phi_0(\delta) + \alpha, \quad \ldots(6)$$

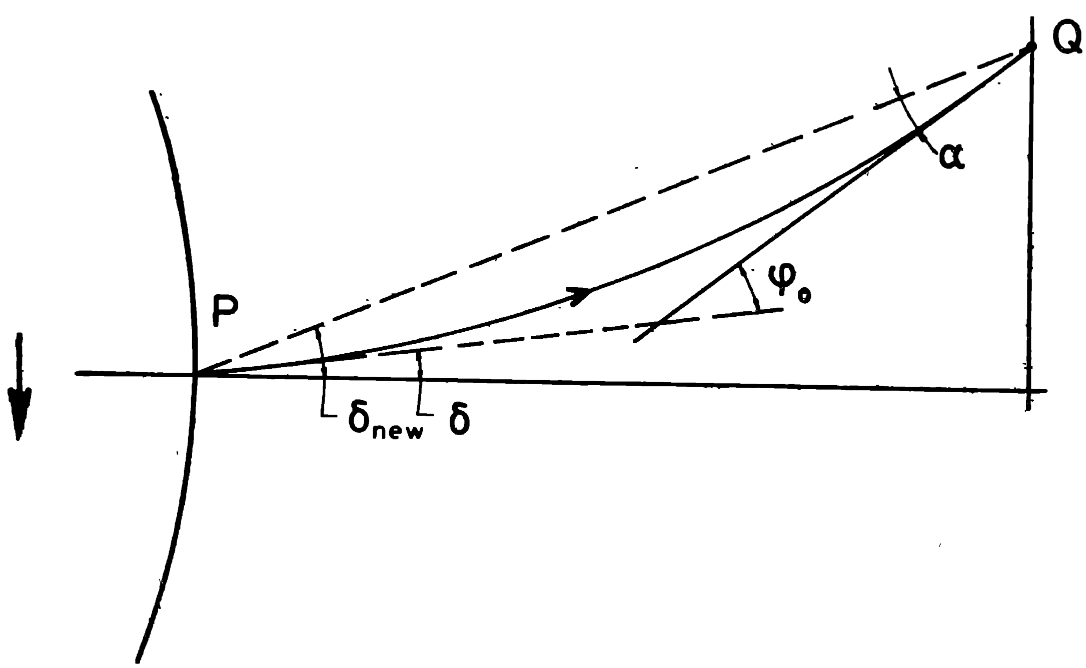

Figure 1. Schematic illustration of photon trajectory in terms of the angles δ, δ_{new} and ϕ_0.

(where $\alpha \simeq 0$ because of small δ and $D \gg R'$), then a measure of the divergence effect can be quantitatively expressed by means of the parameter Δ defined as

$$\Delta = \frac{d\delta_{\text{new}}}{d\delta} \qquad \ldots(7)$$

$$\simeq \left(1 + \frac{d\phi_0}{d\delta}\right)_{r=r_e} . \qquad \ldots(8)$$

It may be noticed that as a consequence of rotation

$$|\,\phi_0(+\delta)\,| \neq |\,\phi_0(-\delta)\,|,$$

and so

$$\Delta(\delta, r_e) \neq \Delta(-\delta, r_e).$$

This implies that the divergence in the beam width will be asymmetric in the azimuthal angle (assuming the beam to be initially symmetric). Furthermore, the diverged pulse will suffer a deamplification in its intensity. This follows from the requirement of conservation of energy flux. The deamplification may be expressed through the ratio

$$\epsilon = \frac{I(\delta_{\text{new}})}{I(\delta)} = \frac{1}{\Delta} \frac{\sin \delta}{\sin \delta_{\text{new}}}. \qquad \ldots(9)$$

Note that in the limit $\delta \to 0$ (*i.e.* corresponding to radially outward photon), $\epsilon \to \Delta^{-2}$.

As a consequence of rotation, photons emitted in the forward direction (*i.e.* with $\delta < 0$) will be blueshifted and those emitted in the backward direction (*i.e.* with $\delta > 0$) redshifted. Following Liouville's theorem, forward radiation in the pulse beacon will then get Doppler-boosted while backward radiation will get Doppler-diminished. These can be evaluated from a knowledge of how the redshift factor varies with respect to δ (Kapoor & Datta 1984). The magnitudes of these corrections are small but nontrivial and will affect the functional dependence of ϵ on δ, so that $\epsilon \to \epsilon'$ (see Datta & Kapoor 1985).

The times of emission, t_0, and reception, T, of a photon can be related through an integral similar to equation (3):

$$T - t_0 = \int_{r_e}^{D} \frac{dt/d\Gamma}{dr/d\Gamma}\, dr$$

$$= \int_{r_e}^{D} g(r, q_e) dr, \qquad \ldots(10)$$

$$g(r, q_e) = \frac{1 + \omega q_e}{e^{2\nu}[(1 + \omega q_e)^2 - q_e^2 e^{2\nu - 2\psi}]^{1/2}}. \qquad \ldots(11)$$

For the range of δ that we consider here, namely $5° \geq \delta \geq -5°$, the amount of bending $|\,\phi_0(\delta)\,|$ turns out to be small. Nonetheless, a finite time delay will be introduced between two neighbouring null geodesics emitted at azimuthal angles δ and $\delta + d\delta$. This will have important implications for pulsar physics, and is discussed in the following section.

3. Results and discussion

An analysis of binary pulsar data indicates the neutron star mass to be in the range $(1.4 \pm 0.2)\, M_{\odot}$ (Joss & Rappaport 1984). So, although no reliable mass estimates are available for the recently discovered fast pulsars, we adopt here the above value as the representative mass in order to illustrate the effects of spacetime curvature and rotation on the pulse profile. For this purpose, we take the rotating neutron star model for PSR 1937 + 214 with the Friedman-Pandharipande (1981) model for the equation of state as reported in Ray & Datta (1984), and use it in the external form of the metric (1) to compute photon trajectories for the rotating as well as Schwarzschild configurations, as prescribed by Throne (1971). The results of our calculations are summarized below.

The main effect of spacetime curvature is to widen the pulse cone by a factor $\Delta \approx 2$. This will effectively make the pulsar act as a diverging lens for its own radiation. The difference over the Schwarzschild values of Δ introduced by rotation is however small. Hence, the rotationally induced asymmetry in the pulse profile is barely graphically noticeable (see figure 2). The asymmetry is brought out more explicitly in table 1. From the table, it may be noticed that rotation tends to compress the pulse beam for backward emission ($\delta > 0$) and strech it for forward emission ($\delta < 0$).

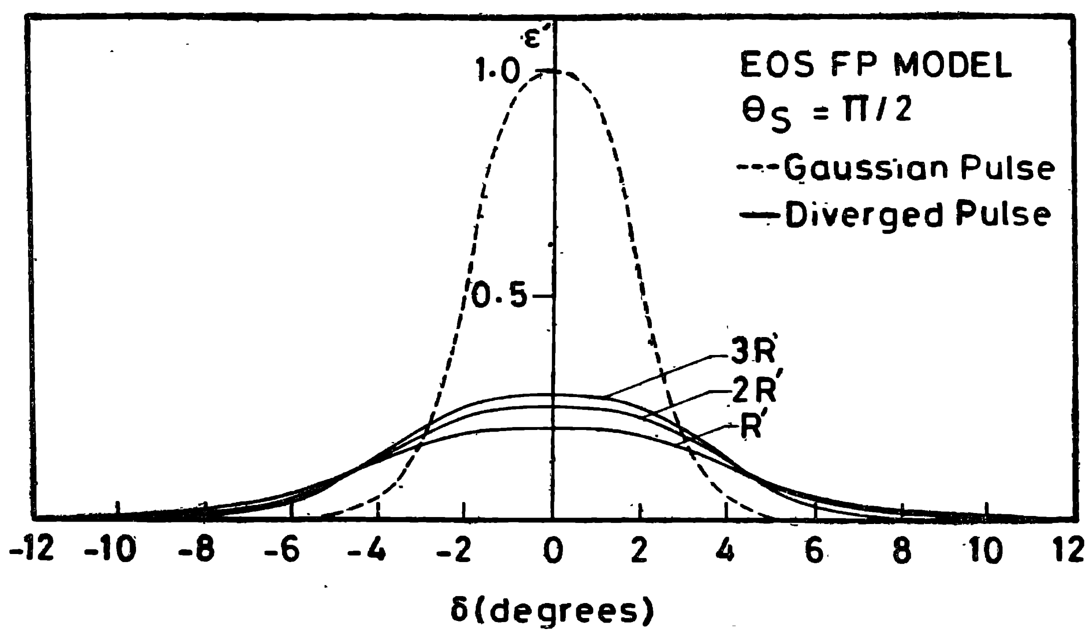

Figure 2. Net intensity deamplification factor versus azimuthal angle of emission for different values of emission location r_e (solid curves). The dashed curve is the initial (Gaussian) profile.

Table 1. Azimuthal bending, divergence index Δ, intensity deamplification factor ϵ, ϵ', and time delay for the rotational case (for $\theta_s = 0.3\pi$, 0.5π) and the Schwarzschild case for several value of r_e

δ	Rotational $\theta_s = 0.3\pi$ δ_{new}		Δ		ϵ'		delay (μs)	Rotational $\theta_s = 0.5\pi$ δ_{new}		Δ		ϵ'		delay (μs)	Schwarzschild $\theta_s = 0.5\pi$ δ_{new}	Δ	ϵ
	+	−	+	−	+	−		+	−	+	−	+	−				
0	0	0	2.512	2.512	.158	.158	0	0	0	2.213	2.213	.204	.204	0	0	2.226	.202
1	2.51	−2.51	2.507	2.517	.158	.160	.2	2.21	−2.21	2.208	2.217	.203	.206	.3	2.23	2.226	.202
2	5.01	−5.03	2.503	2.522	.157	.161	.5	4.42	−4.43	2.203	2.222	.202	.207	.6	4.45	2.226	.202
3	7.52	−7.56	2.498	2.527	.157	.162	.7	6.62	−6.66	2.199	2.227	.201	.209	1.0	6.68	2.227	.202
4	10.01	−10.09	2.494	2.532	.156	.162	1.0	8.81	−8.89	2.194	2.232	.200	.210	1.3	8.91	2.227	.202
5	12.50	−12.62	2.489	2.537	.156	.164	1.2	11.01	−11.12	2.190	2.237	.199	.212	1.5	11.13	2.227	.203
0	0	0	2.279	2.279	.192	.192	0	0	0	2.013	2.013	.247	.247	0	0	2.07	.233
1	2.28	−2.28	2.273	2.285	.191	.194	.8	2.01	−2.02	2.008	2.019	.244	.249	1.0	2.07	2.07	.233
2	4.55	−4.57	2.268	2.291	.189	.196	1.6	4.02	−4.04	2.002	2.025	.242	.252	1.9	4.14	2.07	.233
3	6.81	−6.86	2.262	2.297	.188	.198	2.4	6.01	−6.07	1.997	2.031	.240	.255	2.8	6.21	2.07	.234
4	9.07	−9.16	2.257	2.303	.187	.200	3.1	8.01	−8.10	1.991	2.037	.238	.258	3.9	8.28	2.07	.234
5	11.32	−11.47	2.251	2.309	.186	.202	3.9	10.00	−10.14	1.986	2.043	.236	.260	4.7	10.35	2.07	.234
0	0	0	2.165	2.165	.213	.213	0	0	0	1.899	1.899	.277	.277	0	0	2.02	.246
1	2.16	−2.17	2.158	2.173	.211	.215	1.5	1.90	−1.90	1.892	1.906	.274	.280	1.8	2.02	2.02	.246
2	4.32	−4.35	2.151	2.181	.209	.218	3.1	3.79	−3.81	1.885	1.914	.272	.283	3.6	4.04	2.02	.246
3	6.46	−6.53	2.144	2.188	.208	.220	4.6	5.67	−5.73	1.878	1.921	.269	.287	5.4	6.05	2.02	.246
4	8.61	−8.72	2.137	2.196	.206	.223	6.1	7.54	−7.66	1.872	1.929	.267	.290	7.2	8.07	2.02	.246
5	10.74	−10.92	2.130	2.204	.204	.226	7.7	9.41	−9.59	1.865	1.937	.265	.294	9.1	10.09	2.02	.247

It may be mentioned here that $\Delta \to 1$ as $\delta \to \pm \pi/2$. This means that in the relativistic beaming model of pulsed emission from pulsars, divergence effects are minimal.

An interesting feature introduced by rotation is the 'sweeping' of the entire pulse cone, in the direction of rotation (by an amount of $\sim 10^\circ$ if $r_e = R'$ and larger if $r_e > R'$). We term this the tilting of the pulse cone, which is to be understood as the net bending in the direction in the rotation of those photons that are emitted along the axis of the pulse cone *i.e.* with $\delta = 0$. This effect is absent in the Schwarzschild case, where $\phi_0(\delta = 0) = 0$.

Another point of difference from the Schwarzschild spacetime is in the arrival time differences for two photons characterized by azimuthal angles $\pm \delta$. Because of dragging of inertial frames due to rotation, $a + \delta$ photon will take a longer time to reach the remote observer at $r = D$ than $a - \delta$ photon. The difference in their arrival times, which we called time delay, can be calculated from equation (10) for $r_e \geqslant R'$. We find the magnitudes of the time delay to be of the order of a few microseconds (see table 1). If the time delay between photons with maximum δ and minimum δ equals the pulse duration, the complete pulse will be detected by the observer. On the other hand, if this time delay exceeds the pulse duration, photons with δ larger than a critical value will miss being detected by the observer. This will mean that the pulse profile will have a gradual build-up but a steeper fall-off. Our calculations indicate that this can have a constraining effect on the pulsar duty cycle if the pulse cone is wider than 10° and r_e is several times the radius of the neutron star. We emphasize here that the asymmetry will be over and above the asymmetry in the final pulse profile due to dragging of inertial frames and interstellar/interplanetary scintillations, if any.

It may be relevant to mention here that the rotationally induced dragging of inertial frames will bring about a rotation in the plane of polarization of the pulse This is of interest in view of the observed fact that pulsar pulses are, generally speaking, linearly polarized. Now, it is known that an inertial compass near the surface of a rotating body will precess with a certain angular velocity. Therefore, the plane of polarization will also rotate with the same angular velocity (Zeldovich & Novikov 1971). For emission along the polar axis, for instance, the amount by which the plane of polarization will rotate in the direction of the rotation of the pulsar will be given by

$$\chi = \int_{r_e}^{D} \omega \frac{dt/d\Gamma}{dr/d\Gamma}\, dr \simeq \tfrac{1}{2} r_e\, \omega(r_e). \qquad \ldots(12)$$

Its numerical value, however, turns out to be small ($\sim 1°$), and in the absence of a reference is of formal academic significance only.

In view of the main result of our calulations, that the pulse beam divergence and the pulse intensity deamplification are of a similar magnitude be it the rotational or the Schwarzschild case, it may be generally inferred that the pulse will start out from the emission region in the shape of a narrow spike. Observationally speaking, most pulsars have narrow pulse profiles. Therefore, an important conclusion that follows is that the brightness temperature (which is directly proportional to the intensity of radiation) of pulsars in general should be larger by an order of magnitude (in the emitter's frame of reference) than is presently presumed, and this will be valid for the emitter located anywhere between the pulsar surface and several times its radius.

Acknowledgement

B. Datta acknowledges the Indian National Science Academy for the award of Biren Roy Trust Fellowship.

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