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Energetics of a Kerr blackhole in electromagnetic fields : Role of the relative angular velocity of charged particles

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Abstract. We consider a rotating blackhole in an external dipole magnetic field. By defining the relative angular velocity parameter $\bar{\Omega} = \Omega - \omega$ where Ω is the angular velocity of the particle and $\omega = -g_{t\phi}/g_{\phi\phi}$, we analyse the various contributions due to electromagnetic fields and rotation to the effective potential V. It turns out that the magnetic field contribution is in general related to $\bar{\Omega}$. An interesting case of prescribing the orbit for a locally nonrotating observer (LNRO) by a particle of appropriate charge and angular momentum parameters is considered. For such an orbit $\bar{\Omega} = 0$, stationarity is built in and is ensured by the Lorentz force in contrast to the usual LNRO orbit where it has to be maintained by externally providing proper outward radial acceleration.

Key words : Blackhole energetics—electromagnetic fields on blackholes—Penrose process—active galactic nuclei—star clusters

1. Introduction

The active galactic centres are believed to be possible locales for blackholes. The astronomical observations indicate violent releases of energy in various parts of the universe including the galactic nuclei. Therefore, it is reasonable to ask whether the mass-energy stored in a blackhole might not in certain circumstances be released to the outside world. The idea of extracting energy from a rotating blackhole was first put forward by Penrose (1969) by utilising the curious feature of the Kerr geometry that negative energy orbits exist in the ergosphere. The ergosphere is the region lying between the event horizon (where the timelike vector $\frac{\partial}{\partial t} + \omega \frac{\partial}{\partial \phi}$ becomes null) and the static surface (where the Killing vector $\frac{\partial}{\partial t}$ becomes null). The process envisages that a particle falls onto a rotating blackhole

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and splits into two fragments one of which attains the negative energy orbit and gets captured by the blackhole while the other comes out with energy larger than the incident particle. It results in decreasing the hole's energy. It is shown by Christodoulou (1970) through the concept of irreducible mass that it is the rotational energy which alone can be extracted out. However, the area of a blackhole will increase in accordance with the Hawking-Penrose area nondecrease theorem (Misner *et al.* 1973).

The ergosphere is bounded by the surfaces given by $g_{tt} = 0$ and $g_{tt} + \omega g_{t\phi} = 0$. It is known that the timelike Killing vector $\boldsymbol{\xi} = \frac{\partial}{\partial t}$ is spacelike in the ergosphere. The total energy of a particle relative to infinity $E = -\mathbf{p}.\boldsymbol{\xi}$ (signature + 2), where $\mathbf{p}$ is the four momentum of a particle, can become negative in this region because $\boldsymbol{\xi}$ is spacelike. So there exists negative energy orbits in the ergosphere. This is a necessary condition for energy extraction by the Penrose process.

Here, we consider a Kerr blackhole immersed in external electromagnetic field due to a current loop placed symmetrically around the hole in the equatorial plane. Such fields have been considered by Chitre & Vishveshwara (1975) and by Petterson (1975). The field is taken as perturbative, i.e. it does not appreciably alter the background geometry whereas it would significantly influence the motion of charged particles. A comprehensive analysis of negative energy states for a Kerr blackhole in uniform and dipole magnetic fields has been done in a series of papers by Prasanna & Dadhich (1983), and Dhurandhar & Dadhich (1984a,b) (henceforth referred to as papers I and II). This investigation is in continuation of these two papers. It turns out that the presence of an appropriate magnetic field favours the energy extraction process by (i) extending the region of negative energy orbits (the generalized ergosphere, Denardo & Ruffini 1973) beyond the ergosphere $r = 2m$, in the equatorial plane; (ii) allowing for greater negative values for E; and (iii) removing the objection of relativistic splitting (Wagh *et al.* 1985). The first implies that the negative energy orbits are available at convenient radii from the blackhole, i.e. the incident particle does not have to have a grazing orbit, the second lets the captured orbit fragment to have large negative energy resulting into a greater energy gain thereby enhancing the efficiency of the Penrose process. This feature is being considered elsewhere (Parthasarathy, Dhurandhar & Dadhich 1985, in preparation; Bhat *et al.* 1985).

It was shown by Bardeen *et al.* (1972) and by Wald (1974) that the process could not be astrophysically viable as it required relative velocity between the fragments to be greater than $c/2$. There is no known physical process which can accelerate the fragments almost instantaneously to such large relative velocities. The presence of electromagnetic field around the blackhole overcomes this objection (Wagh *et al.* 1985). Thus the blackhole in a magnetic field is a very conducive setting for this process.

In this work, we attempt to correlate the behaviour of effective potential with the behaviour of $\bar{\Omega} = \Omega - \omega$, where $\Omega = d\phi/dt$ is the angular velocity of the charged test particle and $\omega = -g_{t\phi}/g_{\phi\phi}$ is the nonzero angular velocity of zero-angular momentum particle as observed by the observer at infinity. This is the angular velocity of a locally nonrotating observer (LNRO). For motion in the Kerr geometry, co/counter rotating orbit is determined by $L \gtrless 0$, the angular momentum parameter of the particle. Recently Dadhich (1985) has argued that co/counter rotation is instead determined by $\Omega - \omega \gtrless 0$ when electromagnetic interactions are involved. It reduces to the former criterion when electromagnetic interactions are absent. In the latter case, one is forced to use the new criterion as there exist cases where $\Omega < \omega$ for $L > 0$ and yet admitting negative energy states (paper I). The only relevant question for co/counter rotating orbit is that whether the particle goes ahead or lags behind an LNRO at a given r.

In sections 2 and 3 we take up from the paper I expressions for the 4-potential of the electromagnetic field and the effective potential for radial motion in the equatorial plane. The relative angular velocity parameter $\bar{\Omega} = \Omega - \omega$ is defined. In section 4 a detailed analysis of various factors, indicating various interactions, causing negative energy states (NES) is carried out with reference to the role played by $\bar{\Omega}$. Section 5 considers an interesting case of an orbit for an LNRO without having to worry about balancing radial acceleration which is taken care of by the Lorentz force.

2. Superposed electromagnetic fields

The Kerr line-element in the Boyer-Lindquist coordinates is given by

$$ds^2 = g_{tt}\, dt^2 + 2g_{t\phi}\, dt\, d\phi + g_{rr}\, dr^2 + g_{\theta\theta}\, d\theta^2 + g_{\phi\phi}\, d\phi^2, \qquad \ldots(1)$$

where

$$g_{tt} = -\left(1 - \frac{2mr}{\Sigma}\right),$$

$$g_{t\phi} = -\frac{2mar}{\Sigma}\sin^2\theta,$$

$$g_{\phi\phi} = \frac{B}{\Sigma}\sin^2\theta,$$

$$g_{rr} = \frac{\Sigma}{\Delta},$$

$$g_{\theta\theta} = \Sigma,$$

and

$$B = (r^2 + a^2)^2 - \Delta a^2 \sin^2\theta,$$

$$\Sigma = r^2 + a^2\cos^2\theta,$$

$$\Delta = r^2 - 2mr + a^2. \qquad \ldots(2)$$

Here m is the mass and a the angular momentum per unit mass of the blackhole. We have employed the geometrized units $G = c = 1$.

An axisymmetric, static and perturbative electromagnetic field is superposed on the background Kerr geometry. The electromagnetic field then satisfies the covariant Maxwell's equations. The general solutions for electromagnetic field due to a current loop placed symmetrically in the equatorial plane around a Kerr blackhole was obtained by Chitre & Vishveshwara (1975) and by Petterson (1975). Dhurandhar & Dadhich (1984a,b) have done a detailed analysis of energy extraction processes from a Kerr blackhole in a dipole magnetic field by using Petterson's solutions. In paper I, they have discussed the questions involving second law of blackhole physics and conservation of energy in relation to existence of negative energy states for charged particles, whereas in paper II they have worked out a general formalism for the selection of suitable parameters of the participating particles in the energy extraction process. In our analysis here we shall directly use the results obtained in these two papers.

For the $l = 1$ mode in the general solution for the 4-potential obtained by Petterson, set $\beta_1^r = Q = 0$ and $\beta_1^1 = -(3\mu/2)(m^2 - a^2)^{-1}$. This will asymptotically correspond to a magnetic field due to a dipole of strength μ oriented parallel to the axis of rotation of the blackhole. For the field to be regular both at infinity and at the horizon it is necessary to match Petterson's solutions for inside and for outside of the current loop placed at $r = b > r_+ = m + \sqrt{m^2 - a^2}$. The field in the region $r < b$ $(r > b)$ is termed as interior (exterior). Since we shall consider motion of charged particles in the equatorial plane only, we set $\theta = \pi/2$ in the expression for the 4-potential $A_i = (A_t, 0, 0, A_\phi)$:

(i) *Interior field* $(r_+ \leqslant r \leqslant b)$

$$A_t = \frac{\alpha_1^1 a}{\Upsilon}\left(1 - \frac{m}{r}\right),$$

$$A_\phi = -\frac{\alpha_1^1}{2\Upsilon}\left(r^2 + a^2 - \frac{2ma^2}{r}\right) + \alpha_\phi, \qquad \ldots(3)$$

where

$$\alpha_1^1 = \frac{-3\mu}{4\Upsilon^2}\ln\left(\frac{b - r_-}{b - r_+}\right) + \frac{3\mu}{2\Upsilon(b - m)},$$

$$\alpha_\phi = \frac{3\mu}{4(b - m)}. \qquad \ldots(4)$$

(ii) *Exterior field* $(r \geqslant b)$

$$A_t = \frac{-3a\mu}{4\Upsilon^3}\left\{\left(1 - \frac{m}{r}\right)\ln\left(\frac{r - r_-}{r - r_+}\right) - \frac{2\Upsilon}{r}\right\}, \qquad \ldots(5)$$

$$A_\phi = \frac{-3\mu}{8\Upsilon^3}\left\{2\Upsilon\left(r + m + \frac{2a^2}{r}\right) - \left(r^2 + a^2 - \frac{2ma^2}{\Upsilon}\right) \times \ln\left(\frac{r - r_-}{r - r_+}\right)\right\}.$$

where

$$\gamma = (m^2 - a^2)^{1/2},$$
$$r_{\pm} = m \pm \gamma.$$

For convenience, we now define the dimensionless quantities

$$\left.\begin{array}{l} \rho = r/m,\ \alpha = a/m,\ \bar{b} = b/m,\ \bar{L} = L/mM_0, \\ \bar{\gamma} = \gamma/m,\ \alpha_1 = (e/M_0)\ \alpha_1^1,\ \bar{\alpha}_\phi = \alpha_\phi/m, \\ \bar{E} = E/M_0,\ \bar{A}_\phi = eA_\phi/mM_0,\ \bar{A}_t = eA_t/M_0,\ \lambda = e\mu/M_0 m^2, \end{array}\right\} \quad \ldots(6)$$

where the parameters M_0, L, E, and e denote the rest mass, the angular momentum, the energy, and the charge of a test particle respectively. Henceforth, we shall drop overhead bars from the quantities.

In dimensionless units A_t and A_ϕ read as follows :

(i) *Interior field* ($\rho_+ \leqslant \rho \leqslant b$)

$$A_t = (\alpha\,\alpha_1/\gamma)\,(1 - \rho^{-1}), \quad \ldots(7)$$
$$A_\phi = -(\alpha_1/2\gamma)\,(\rho^2 + \alpha^2 - (2\alpha^2/\rho)) + \alpha_\phi.$$

(ii) *Exterior field* ($\rho \geqslant b$)

$$A_t = -\frac{3\alpha\lambda}{4\gamma^3}\left\{\left(1 - \frac{1}{\rho}\right)\ln\left(\frac{\rho - \rho_-}{\rho - \rho_+}\right) - \frac{2\gamma}{\rho}\right\}, \quad \ldots(8)$$
$$A_\phi = -\frac{3\lambda}{8\gamma^3}\left\{2\gamma\left(1 + \rho + \frac{2\alpha^2}{\rho}\right) - \left(\rho^2 + \alpha^2 - \frac{2\alpha^2}{\rho}\right) \times \ln\left(\frac{\rho - \rho_-}{\rho - \rho_+}\right)\right\},$$

where

$$\alpha_1 = -\frac{3\lambda}{4\gamma^2}\ln\left(\frac{b - \rho_-}{b - \rho_+}\right) + \frac{3}{2}\frac{\lambda}{\gamma(b - 1)},$$
$$\alpha_\phi = \frac{3\lambda}{4(b - 1)},$$

and

$$\gamma = (1 - \alpha^2)^{1/2},\ \rho_\pm = 1 \pm \gamma. \quad \ldots(9)$$

The plots for interior and exterior fields for $\lambda = 1$ and $\alpha = 0.9$ are given in figure 1. It is to be noted that $A_t < 0$ and $A_\phi > 0$ throughout the range of $\rho \geqslant \rho_+$.

3. Effective potential and $\bar{\Omega}$

3.1. *The effective potential*

The axial symmetry and the stationary character of the gravitational and the electromagnetic fields readily yield the following two constants of motion :

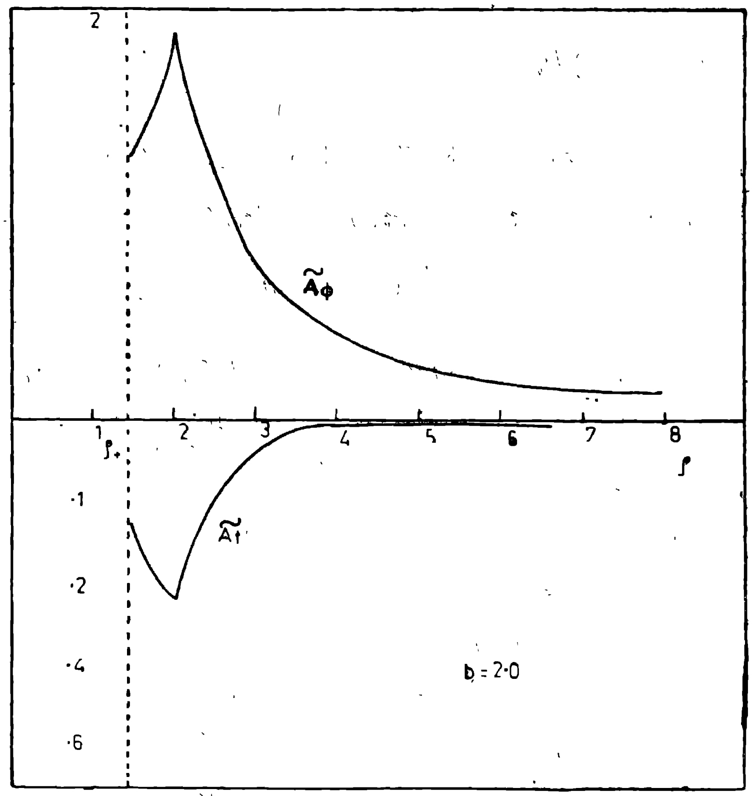

Figure 1. $\tilde{A}_t$ and $\tilde{A}_\phi$ are plotted in the range $\rho > \rho_+$.

$$U_t = -(E + A_t) = -\bar{E}, \qquad \ldots(10)$$

$$U_\phi = L - A_\phi = \bar{L} \qquad \ldots(11)$$

where U represents the four-velocity of the particle. The parameters energy, angular momentum and charge of the particle appear here in dimensionless form as given in equation (6).

The motion of a partcile in the equatorial plane is characterized by $\theta = \pi/2$ and $\dot{\theta} = 0$ initially. The equation of motion for this particular values of θ ensures $\dot{\theta} = 0$ all through so that the particle continues to move in the equatorial plane. In this case, the motion is completely determined by λ, L and E. By substituting equations (10) and (11) in the metric (1) we obtain the quadratic energy equation as

$$\frac{g_{\phi\phi}}{\psi}\bar{E}^2 + \frac{2g_{t\phi}}{\psi}\bar{E}\bar{L} + \frac{g_{tt}}{\psi}\bar{L}^2 + g^{rr}p_r^2 + g^{\theta\theta}p_\theta^2 + 1 = 0, \qquad \ldots(12)$$

which yields

$$E = -A_t + \omega\bar{L} + (-\psi)^{1/2}\left(\frac{\bar{L}^2}{g_{\phi\phi}} + D\right)^{1/2}, \qquad \ldots(13)$$

where

$$\psi = g_{tt} + \omega g_{t\phi} < 0 \ \text{ for } \rho > \rho_+,$$

$$D = g^{rr}p_r^2 + g^{\theta\theta}p_\theta^2 + 1,$$

$$\bar{E} = E + A_t,$$

$$\bar{L} = L - A_\phi. \qquad \ldots(14)$$

The effective potential for radial motion in the equatorial plane is obtained by putting $p_r = 0 = p_\theta$ in equation (12) :

$$V = -A_t + K/R, \qquad \ldots(15)$$

where

$$K = 2\alpha(L - A_\phi) + \Delta^{1/2}\{\rho^2(L - A_\phi)^2 + \rho R\}^{1/2},$$

$$R = \rho^3 + \alpha^2\rho + 2\alpha^2,$$

$$\Delta = \rho^2 - 2\rho + \alpha^2. \qquad \ldots(16)$$

Alternatively by writing $D = 1$ in equation (13) we obtain

$$V = -A_t + \omega\bar{L} + \left[(-\psi)\left(\frac{\bar{L}^2}{g_{\phi\phi}} + 1\right)\right]^{1/2} \qquad \ldots(17)$$

It is clear from the above expression that V can assume negative values for suitable choice of the particle parameters, i.e. a particle (future timelike 4-momentum) can have negative energy *relative to infinity* though the energy relative to a local observer will always be non-negative.

3 2. *Relative angular velocity* $\bar{\Omega}$

Since Ω is the angular velocity of the particle while ω is that of an LNRO, $\bar{\Omega}$ represents angular velocity of the particle relative to an LNRO. It turns out that $\bar{\Omega}$ is a dynamically significant parameter. As shown in the papers I and II, a particle may have $L > 0$ and yet $\bar{\Omega}$ can be negative. It will have a counter-rotating orbit for energetics ($V < 0$) and dynamical considerations. In particular, we shall demonstrate that a particle may have $\bar{\Omega} = 0$, i.e. it is rotating with the same angular velocity as that of an LNRO, yet the former in contrast to the latter has nonzero angular momentum.

From the relations (10) and (11), we obtain

$$\Omega = -\frac{g_{t\phi} + kg_{tt}}{g_{\phi\phi} + kg_{t\phi}}$$

$$= \omega\left(\frac{1 + kg_{tt}/g_{t\phi}}{1 - k\omega}\right),$$

where

$$k = \bar{L}/\bar{E}.$$

It can be solved to give $\bar{\Omega}$:

$$\bar{\Omega} = \frac{k\omega}{1 - k\omega} (\psi/g_{t\phi})$$

$$= \frac{(\psi/g_{t\phi})\, \omega\bar{L}}{\bar{E} - \omega\bar{L}}$$

$$= \frac{\bar{L}}{\left[(-\psi)\left(\frac{\bar{L}^2}{g_{\phi\phi}} + D\right)\right]^{1/2}} (\psi/g_{t\phi}), \qquad \ldots(18)$$

where equation (13) has been used.

It is clear that the sign of $\bar{\Omega}$ depends on the sign of $\bar{L}$ as the remaining terms are positive definite. From the above equation we can also write

$$E = -A_t + \omega\bar{L} + (\psi/g_{t\phi}) \frac{\omega\bar{L}}{\bar{\Omega}}. \qquad \ldots(19)$$

It is to be noted that the last term on the right-hand side is positive definite. Consequently, E can become negative only if the first two terms together are sufficiently negative to dominate over the last term. This can be achieved by a judicious choice of L and λ. In the next section we analyse the occurrence of negative energy states in relation to $\bar{\Omega}$ (or $\bar{L}$).

4. Negative energy states (NES)

Setting $A_\phi = \lambda\tilde{A}_\phi$ and $A_t = \lambda\tilde{A}_t$, equation (17) becomes.

$$V = -\lambda\tilde{A}_t + \omega(L - \tilde{\lambda}A_\phi) + \left[(-\psi)\left(\frac{(L - \lambda\tilde{A}_\phi)^2}{g_{\phi\phi}} + 1\right)\right]^{\frac{1}{2}},$$

$$= \lambda\Phi + \omega L + \left[(-\psi)\left(\frac{(L - \lambda\tilde{A}_\phi)^2}{g_{\phi\phi}} + 1\right)\right]^{\frac{1}{2}}, \qquad \ldots(20)$$

where

$$\Phi = -\tilde{A}_1\eta^1 = -(\tilde{A}_t + \omega\tilde{A}_\phi), \qquad \ldots(21)$$

$$\eta^1 = (1, 0, 0, \omega).$$

Here Φ is the measure of electrostatic potential as measuied by an LNRO. The first term $\lambda\Phi$ represents the electromagnetic contribution while the second term ωL gives the rotational contribution to the effective potential. In figure 2, Φ is plotted against ρ, which shows that

$$\Phi \leqslant 0 \text{ for } \rho \leqslant 3 \text{ and } \Phi \geqslant 0 \text{ for } \rho \geqslant 3.$$

Let us consider the following cases :

(*I*) $\lambda > 0$

(i) $L > 0$ which will imply in view of equations (17) and (20) that V can only be negative if $\bar{L} < 0$ and $\rho < 3$. For $V < 0$, Φ has to be negative. In this case NES

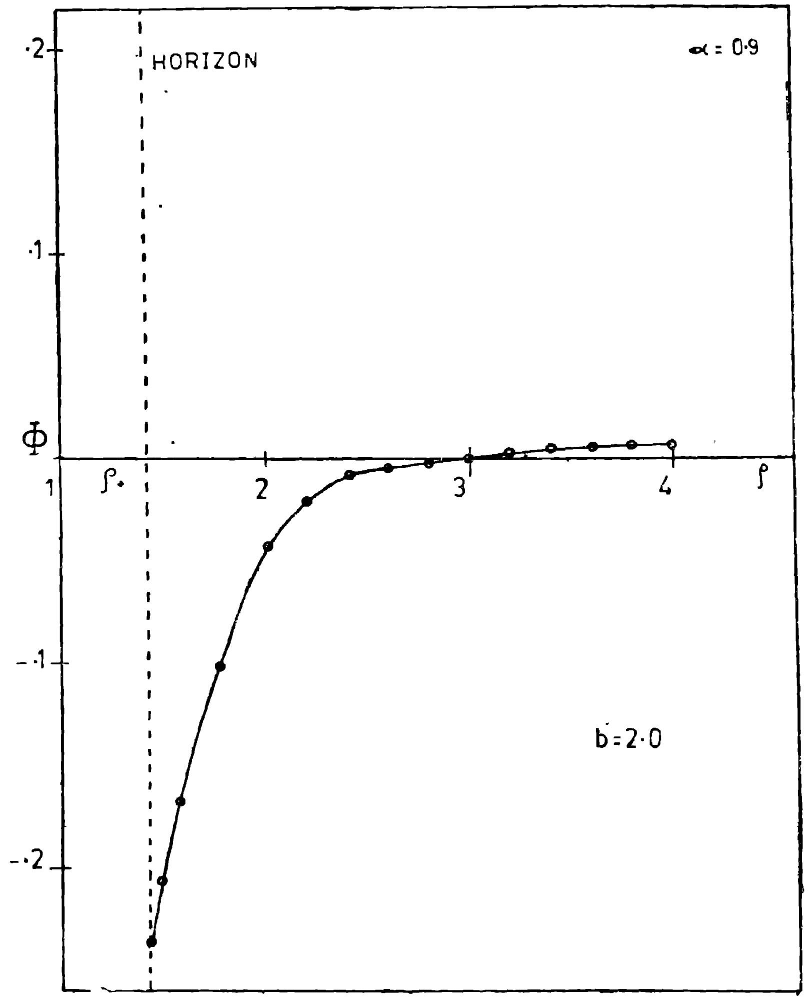

Figure 2. Φ is plotted against $\rho \geqslant \rho_+$.

will be caused by the electromagnetic field (see figure 4). In the paper I this case has been discussed in great detail as it elusively threatened the area nondecrease theorem in the first instance.

(ii) $L < 0 \Rightarrow \bar{L} < 0$. Then electromagnetic field and rotation collude together to make V negative (figure 5). However, Φ will become positive for $\rho > 3$. Then NES can only be caused by ωL which one would not expect to occur for $\rho > 2$ (see appendix *A*.)

(*II*) $\lambda < 0$

(iii) $L > 0$, clearly NES cannot occur for $\rho < 3$. It would be later shown that they do not occur for any $\rho > \rho_+$ (appendix *B*).

(iv) $L < 0$ which admits both $\bar{L} \gtrless 0$. For NES occurring in the region $\rho > 3$, both electromagnetic field and rotation join hands while for those occurring in $\rho < 3$, rotation alone contributes (figures 6, 7).

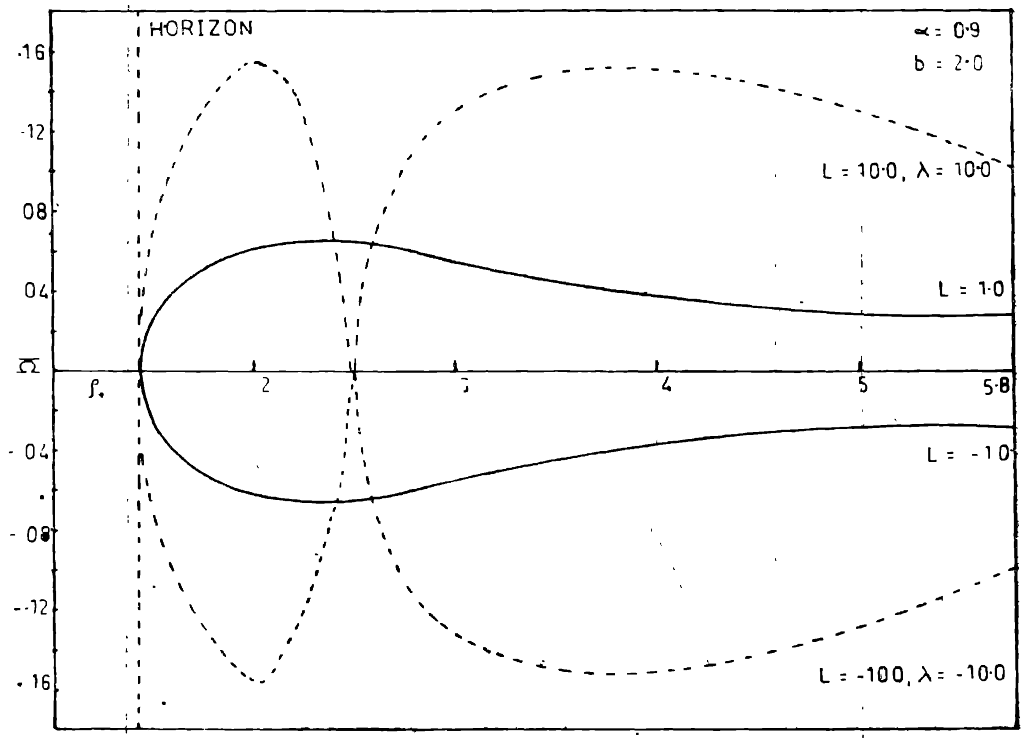

Figure 3. Plots of $\bar{\Omega}$ are shown for $\lambda = 0$, $L = \pm 10$ (solid lines) and for $\lambda = L = \pm 10$ (dashed lines).

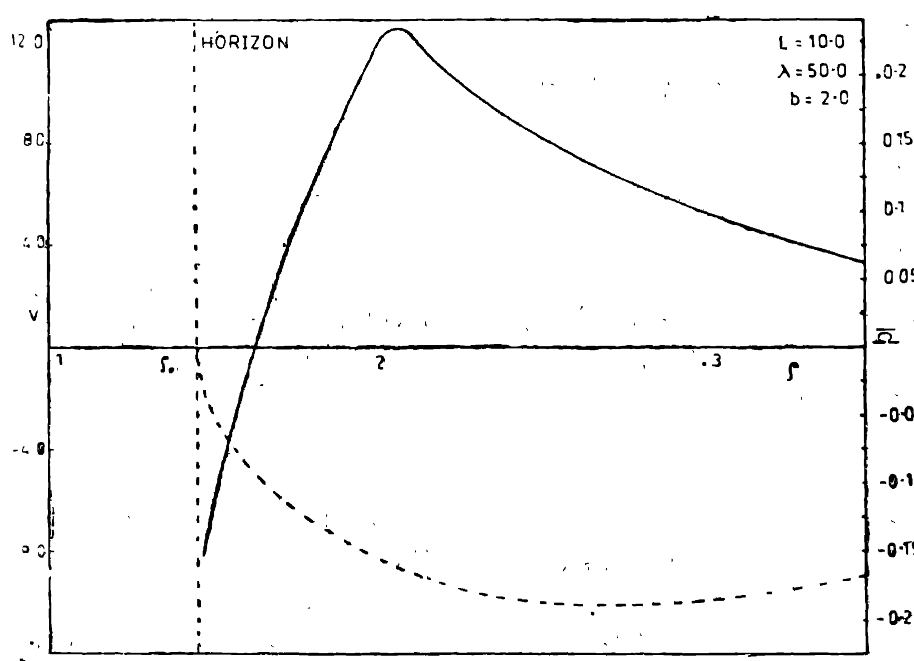

Figure 4. The solid curve shows the V plot while dashed curve shows the $\bar{\Omega}$ plot.

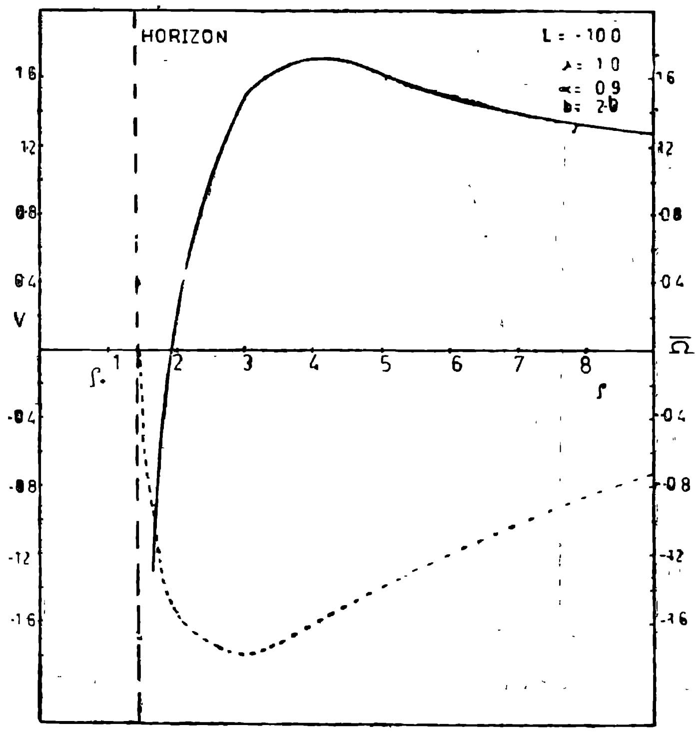

Figure 5. The solid curve shows the *V* plot while dashed curve shows the $\bar{\Omega}$ plot.

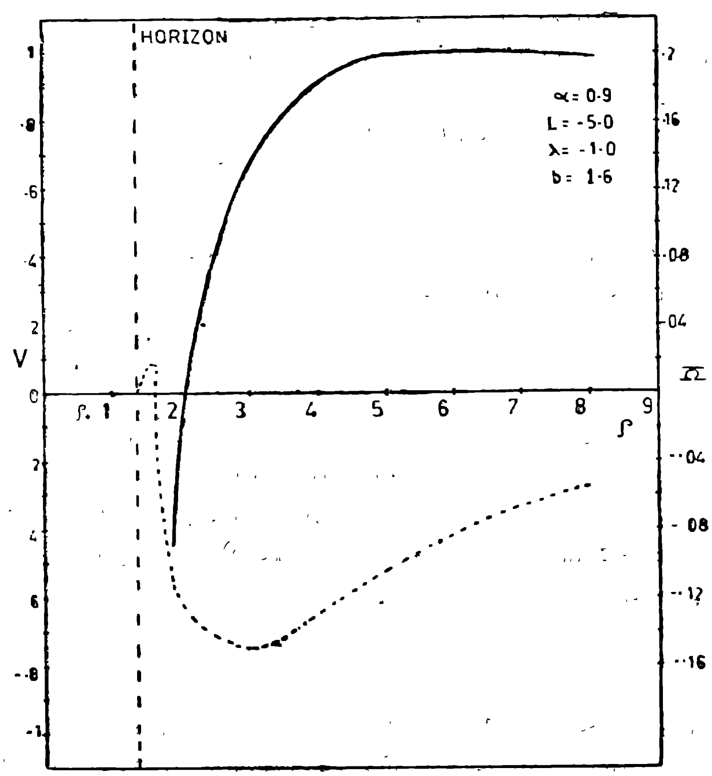

Figure 6. The solid curve shows the *V* plot while dashed curve shows the $\bar{\Omega}$ plot.

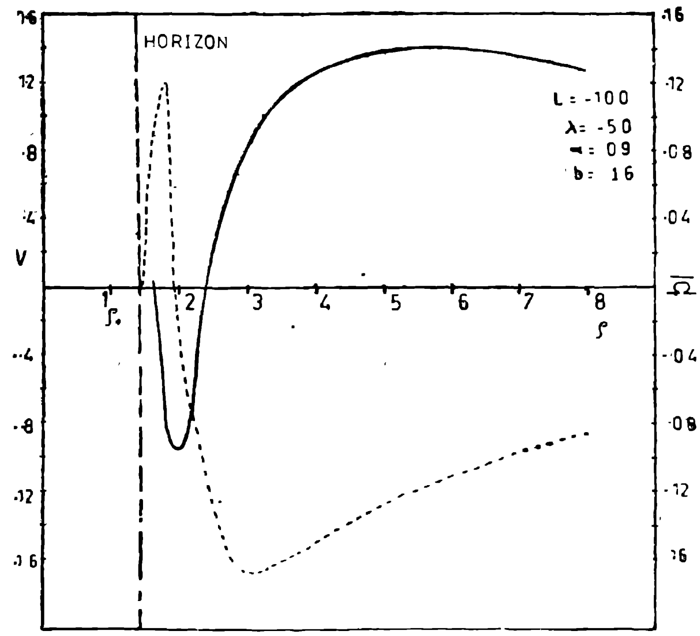

Figure 7. The solid curve shows the V plot while dashed curve shows the $\overline{\Omega}$ plot.

Except in case (iv) whenever electromagnetic field contributes negatively to V we note $L < 0$ which also means $\overline{\Omega} < 0$. The case (iv) gives the double band NES structure (as shown in paper I), i.e. $V < 0$ very close to the horizon and also again for moderately large ρ.

The above analysis could be taken relative to an LNRO. We shall now consider these various cases relative to an asymptotic observer.

In the expression for V in equation (17), $-\lambda\tilde{A}_t$, $-\lambda\tilde{A}_\phi$ and ωL represent the contributions from electric field, magnetic field, and rotation respectively. Again we consider their relative contributions as follows :

(*III*) $\lambda > 0$

(i) $L > 0$ will give $V < 0$ only if $\overline{L} < 0$ and the term $-\lambda\tilde{A}_\phi$ dominates over the other two terms. In this case NES will be caused by the magnetic field (figure 4).

(ii) $L < 0$ which implies $\overline{L} < 0$, then both magnetic field and rotation are responsible for the occurrence of NES (figure 5). In appendix A we show that $V < 0$ only for $\rho < 2$.

(*IV*) $\lambda < 0$

(iii) $L > 0$ implies $\overline{L} > 0$, and NES if they occur will be due to the electric field. However, it turns out that NES do not occur for this case for all $\rho > \rho_+$ (appendix *B*).

(iv) $L < 0$ then $\bar{L} \gtrsim 0$ and the NES will be caused by both electric field and rotation (figures 6,7).

It is worth noting that whenever magnetic field alone or alongwith the other term causes NES, $\bar{L} < 0$ or $\bar{\Omega} < 0$ though vice versa is not true as in the case (iv) above. That is, magnetic field contributes negatively to particle's energy only when it is rotating slower than the LNRO defined at that ρ.

It is of interest to separately consider the special case of $\bar{\Omega}$ being zero at some $\rho > \rho_+$. For uncharged particles in the Kerr field $\bar{\Omega}$ never becomes zero for $\rho > \rho_+$. In contrast, for charged particles when electromagnetic fields are present $\bar{\Omega} \propto \bar{L} = L - A_\phi$ which can have zeros for $\rho > \rho_+$ (see figures 6–8). That is, $\bar{\Omega}$ may be zero at some ρ indicating that the particle has the same angular velocity as that of an LNRO at that ρ. When $\bar{L} = 0$ (for some ρ_0) it implies

$$\lambda = \frac{L}{\tilde{A}_\phi(\rho_0)}. \qquad \text{...(22)}$$

Then equation (17) will read as

$$V = \frac{L}{|\tilde{A}_\phi(\rho_0)|} [\, |\tilde{A}_t| + (\tilde{A}_\phi(\rho_0) - \tilde{A}_\phi] + \sqrt{(-\psi)\left(\frac{\bar{L}^2}{g_{\phi\phi}} + 1\right)}. \qquad \text{...(23)}$$

It should be borne in mind that $\bar{L} = 0$ only at $\rho = \rho_0$ and it prescribes the relation (22) between L and λ. The behaviour of V in equation (23) is crucially determined by L because $\tilde{A}_\phi(\rho_0) \geqslant \tilde{A}_\phi(\rho)$ for $\rho \geqslant \rho_0 \geqslant b$ (figure 1). It is clear that V can turn negative only when $L < 0$.

At $\rho = \rho_0$, equation (23) gives

$$V(\rho_0) = \frac{L\, |\tilde{A}_t(\rho_0)|}{|\tilde{A}_\phi(\rho_0)|} + \sqrt{-\psi(\rho_0)}. \qquad \text{...(24)}$$

It then follows that $V(\rho_0) \leqslant 0$ implies

$$L \leqslant -\left|\frac{\tilde{A}_\phi(\rho_0)\sqrt{-\psi(\rho_0)}}{\tilde{A}_t(\rho_0)}\right| \qquad \text{...(25)}$$

or

$$\lambda \leqslant \frac{\sqrt{-\psi(\rho_0)}}{|\tilde{A}_t(\rho_0)|}. \qquad \text{...(26)}$$

Thus if $\bar{L} = 0$ at some $\rho > \rho_+$ and $V < 0$ there, then L and λ are both necessarily negative. This case is included in the case (iv) above. In other words a particle with $\bar{\Omega} = 0$, i.e. having the same angular velocity as that of an LNRO, can have negative energy. This prompts us to consider in the next section an interesting situation of a particle with suitable L and λ values such that it has a circular orbit with $\bar{\Omega} = 0$.

5. Circular orbits with $\overline{\Omega} = 0$: Prescription for an LNRO

An LNRO has a stationary orbit at a fixed ρ having the inherent angular velocity $\omega = -g_{t\phi}/g_{\phi\phi}$. The orbit is nongeodesic, for the radial pull due to the blackhole has to be counter-balanced so as to keep ρ fixed. For charged particles in electromagnetic fields, the Lorentz force can provide the balancing radial force for appropriate values of L and λ. However one would further require that $\overline{L} = 0$, i.e. $\Omega = 0$ and the orbit is circular at that ρ. Then the particle will have the same orbit as that of an LNRO at the given ρ. What it suggests is that an observer carrying appropriate charge and angular momentum (relative to infinity) will behave like an LNRO and we do not have to bother about keeping it stationary. He will be equivalent to a zero-angular-momentum observer with no charge (at given ρ) and appropriate radial balancing force. Further it is interesting to note that our construction also *allows for the LNRO to have negative energy relative* to infinity. This is because the minimum value of V can be negative (see figure 8).

What we then require is $V' = 0$ and $L = 0$ simultaneously at the same ρ. By differentiating equation (17) and using $\overline{L} = 0$, we get (at some $\rho = \rho_0$),

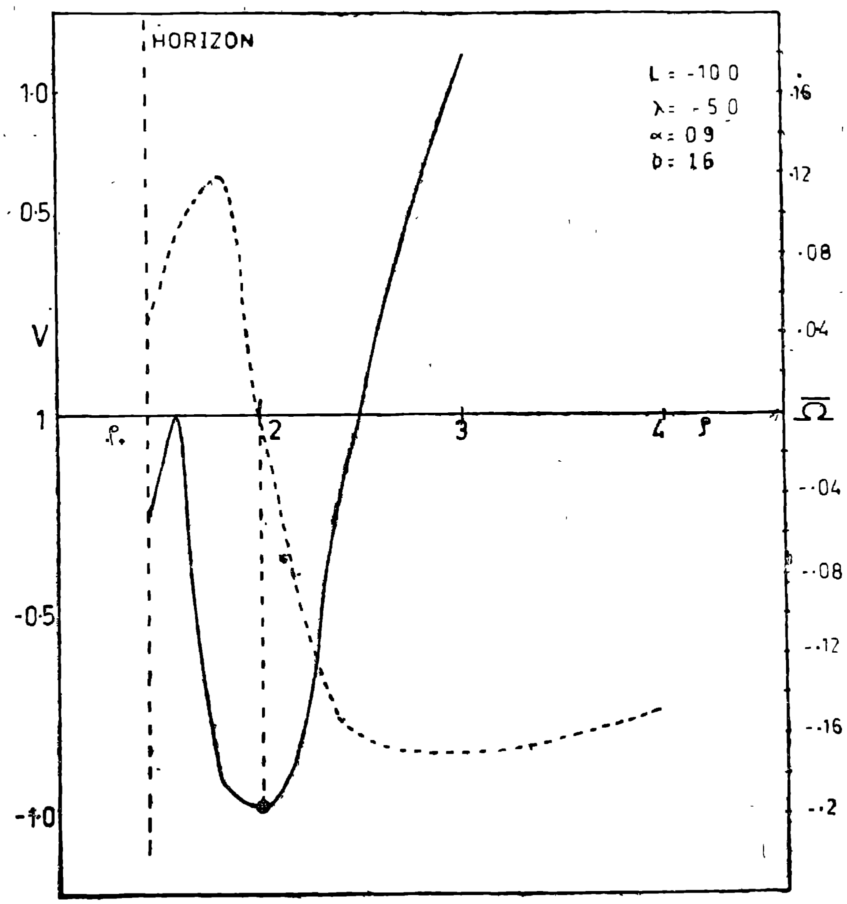

Figure 8. The solid curve shows the V plot and the dashed curve shows the $\overline{\Omega}$ plot

$$V'(\rho_0) = \left[\frac{L}{\tilde{A}_\phi}\left(|\tilde{A}'_t| - \omega A\tilde{A}'_\phi\right) + \frac{(-\psi')}{2\sqrt{-\psi}}\right]_{\rho=\rho_0} = 0. \qquad ...(28)$$

In the LNRO's frame, the first term gives acceleration due to the Lorentz force while the second term represents the gravitational acceleration. The above equation brings about the balancing of the two.

In figure 8 for $L = -10$, $\lambda = -5$ the LNRO orbit occurs at $\rho = 2$. It has energy negative relative to infinity. There do however exist positive energy orbits. The former case is only feasible for both $L, \lambda < 0$ when V is minimum in the negative range while the latter may be possible in almost all situations.

6. Conclusions

On the basis of results of paper I, we have further studied negative energy states (NES) in relation to various factors causing them. It turns out that for $\lambda < 0$ and $L > 0$ there occurs no NES in the entire range of $\rho > \rho_+$. That means that electric field contribution in this case cannot dominate over the other terms. The nature of contribution from magnetic field is largely determined by the behaviour of $\bar{\Omega}$ except in cases (iv) above. Though rotation by itself may not be able to cause NES, particularly in the region $\rho > 2$ (NES in the pure Kerr case occur only for $\rho < 2$), but it can join hands with the electromagnetic interaction for suitable values of L and λ.

The parameter $\bar{\Omega} = \Omega - \omega$ measures the angular velocity of a particle relative to an LNRO at a given ρ. It tells us whether a particle is lagging behind or going ahead of the LNRO. Physically relevant observer in the Kerr geometry is the LNRO and co/counter rotation relative to him is determined by $\bar{\Omega} \gtrless 0$ and not by $L \gtrless 0$ (Dadhich 1985). However, the two criteria agree in the absence of electromagnetic interaction. We have attempted to associate $\bar{\Omega}$ with the electromagnetic field contribution in the effective potential.

One of the interesting offshoots of this investigation is that an LNRO orbit can be prescribed at a given ρ with proper L and λ parameters because the Lorentz force could provide the balancing outward radial acceleration. The prescribed orbit at the given ρ will be absolutely equivalent to the zero angular momentum observer *being held* at a fixed ρ. One has to provide for 'holding up' from outside while in our consideration it is taken care of by the Lorentz force. It is interesting that an LNRO can as well have a negative energy orbit.

The energy extraction process favours high value of λ as the efficiency is directly proportional to λ (Parthasarthy, Dhurandhar & Dadhich 1985, in preparation). To get the feel of the numbers involved we compute λ for the setting of galactic centre. We have $\lambda = e\mu/M_0 m^2 c^3$, $m = MG/c^2$ in cgs units. By expressing the dipole moment μ (Chitre & Vishveshwara 1975) in terms of the magnetic field strength B (measured in gauss), we write

$$\lambda \sim (eb^3/M_0 m^2 c^2)\, B,$$

where b is the radius of the current loop. Now for a typical case, $M \sim 10^8 M_\odot$, $b = 2$m and e and M_0 referring to electronic charge and mass; we therefore get

$$\lambda \sim 10^{10} B.$$

Thus even a very weak magnetic field $B \sim 10^{-5}$ (intergalactic) gauss, allows large value of $\lambda \sim 10^5$. At the galactic centre, $B \sim 10^{10}$ gauss which would keep $\lambda \sim 1$ for a star-like body having one electron charge in excess over 10^{17} baryons. Thus our analysis is applicable in almost all cases involving micro- as well as macro-objects.

Our analysis is also relevant when one attempts to build up a realistic model of extracting blackhole's energy through electromagnetic interactions (Rees *et al.* 1982). The consideration of efficiency of energy extraction process (Bhat *et al.* 1985; Parthasarthy, Dhurendhar & Dadhich 1985, in preparation) will be of prime significance for the mechanism to be astrophysically viable. Then the nature of electromagnetic field and corresponding negative energy states for various cases considered above will determine which set of parameters are favoured and what is the nature of the extracted energy. One believes that blackhole together with electromagnetic field will behave as a synthetic whole. The work is in progress to consider astrophysically viable mechanism of electromagnetic extraction of energy from a rotating blackhole.

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Appendix A

For $\lambda > 0$, $\bar{\bar{L}} < 0$, $L < 0$ and $\rho > 2$, V cannot attain negative values. Since we have

$$V = -\lambda \tilde{A}_t + \omega \bar{L} + \sqrt{(-\psi)\left(\frac{\bar{L}^2}{g_{\phi\phi}} + 1\right)},$$

where $\bar{L}$ alone contributes negatively. But

$$\sqrt{(-\psi)\left(\frac{\bar{L}^2}{g_{\phi\phi}} + 1\right)} = \sqrt{\omega^2 \bar{L}^2 + \frac{g^2{}_{t\phi}}{g_{\phi\phi}} - g_{tt}\left(\frac{\bar{L}^2}{g_{\phi\phi}} + 1\right)}$$

$$> |\, \omega \bar{L}\, |$$

for $\rho > 2$ because $g_{tt} < 0$ in this range. Hence V remains positive for $\rho > 2$.

Appendix B

For $\lambda < 0$, $\bar{L} > 0$ let us consider V for the limiting case of $L = 0$ at $\rho = 2$ which will simply read as

$$V(\rho = 2) = -\lambda(2\omega\tilde{A}_\phi - |\tilde{A}_t|). \tag{B1}$$

For $\alpha \to 1$

$$\tilde{A}_t \sim \frac{1}{2\rho(\rho - 1)^2},$$

$$\tilde{A}_\phi = \frac{1}{4}\frac{(\rho^2 + 1 - 2\rho^{-1})}{(\rho - 1)^3} \tag{B2}$$

$$= \frac{2}{\rho(\rho^2 + 1 + 2\rho^{-1})}$$

with these values $(2\omega\tilde{A}_\phi - |\tilde{A}_t|) > 0$. However, it will also be valid for $\rho > 2$ and $L > 0$. Since $\lambda < 0$, (B1) shows that $V > 0$ for $\rho \geqslant 2$.

For the region $\rho < 2$, let us evaluate V at the horizon where $\psi = 0$. By using relations (7) and (8), we obtain for $L = 0$ after some algebraic manipulations, $V(\rho_+) \doteq -\alpha\phi > 0$ in view of $\lambda < 0$. Thus it is established that there occurs no NES for this case in the entire range of $\rho > \rho_+$.