The Observers' Hand-Book for 1916 published by the Royal Astronomical Society of Canada.

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Government of India, Meteorological Department, India Weather Review, Annual Summary for 1914.

Gazette Astronomique for 1914, Nos. 9 and 10.

The election by the Council of the following gentlemen as members of the Society was confirmed :—

Mr. Edelji S. Olpad ...	... Calcutta.
Mr. G. R. Kaye ...	.. Simla.
Captain Leo F. Bodkin	... Gyantsa, Tibet.

The papers read at the meeting are published in this issue.

A few considerations concerning the Chemical and Physical State of Comets.

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THESE visitants to the solar system were originally regarded as omens of evil, though the great comet of 1066 seems to have been regarded as a favourable omen by William the Conqueror. In the time of Aristotle they were considered to be of terrestrial origin and so did not come within the ken of the astronomer. In 1577, however, Tycho Brahe pointed out that the observations of the position of the comet of that year with reference to other stars taken at his own observatory at Uranienburg in Denmark were sensibly identical with those of an observer at Prag; and that hence the comet must have been much more distant than the Moon. Since that time these bodies have proved a constant puzzle to astronomers, more especially as regards their physical condition, which forms the subject of this paper.

The physical condition of these bodies presents many curious paradoxes. They are subject to the laws of gravitation, yet parts of them are very strongly repelled from the Sun. They shine partly by reflected light and are partly self-luminous. They are of huge extent in space, yet their mass is exceedingly small.

Before proceeding further let us recount the physical properties of comets as we know them. Practically all comets consist of three parts—the coma, or nebulosity about the head; the nucleus, a bright point which appears near the centre of the coma as it approaches the Sun; and finally the tail, which is a stream of light following the comet as it approaches the Sun and preceding it as it recedes from that luminary, after passing its perihelion. Sometimes also the nucleus seems to throw off jets of light and sometimes bright envelopes which seem to move outwards through the coma in concentric rings.

In size comets vary very considerably, the coma being from 40,000 to more than 1,000,000 miles in diameter; but as the comet approaches the Sun the coma contracts very considerably, only to expand again on the comet's recession. The nucleus varies from 100 to 8,000 miles in diameter, while the tail may extend for something exceeding 100,000,000 miles behind the comet. Many estimates as to the mass of comets have been made, but the only thing we know is that compared with that of the Earth the masses must be extremely small, and hence that their density must be infinitesimal.

From observations by means of the polariscope, spectroscope, etc., it appears that the light of comets is partly reflected and partly intrinsic. For while they fade away, being yet in full view, owing to their becoming too faint to be seen, and while their light shows traces of polarisation as it should do if reflected, yet it also shows a spectrum which is not the solar spectrum, and consequently must be due to the comet's possessing a certain amount of intrinsic light. Moreover while a comet's brightness varies, as a rule, much as it would do if due to reflected sunlight; yet there are many instances of sudden, and so far unaccounted for, variations in the brightness of comets.

The part of a comet which always attracts attention on account of its curious and unwonted appearance is the tail. Why should a mass of matter, even if very attenuated, develope such an appendage on approaching the Sun? The researches of Bessel, Norton, and Bredichin seem to show that the tail consists of matter expelled from the nucleus and repelled both by the comet and by the Sun. Such a supposition accounts for the position, motions, curvature, etc., of the tail.

But what is this repulsive force? Comets consist of matter, and by the law of gravitation all parts of them must be attracted not repelled by the Sun. The explanations which have been advanced to reconcile these two apparently contradictory facts are three in number:—Firstly, if we accept Maxwell's Electro-Magnetic Theory of Light, it follows that light itself must exercise a very minute pressure on all surfaces upon which it impinges; and this pressure, although very small for objects of practically appreciable magnitude, becomes greater than the gravitational attraction in the case of small particles of the order of magnitude of light-waves. Secondly, it is established that bodies at a high temperature send off rays and particles such as those projected under normal circumstances from radio-active bodies. Thirdly, Zenker has suggested that there might be a greater evaporation or sublimation from the sunward surfaces of the particles composing a comet than from the opposite side and that this would cause a definite recoil of the particle away from the Sun.

It will be my object in this paper to enquire into these theories and to see what we can really learn of the physical state of comets from a consideration of their motions and the observed phenomena; and I must ask the indulgence of my readers if, in the choice of letters to represent certain constants, I select those which are familiar to me as a chemist, rather than those usually found in treatises dealing with the matter from a mathematical point of view.

Maxwell showed that on his Electro-Magnetic Theory, sunlight, at the distance of the Earth from the Sun, should exert a pressure of about 0·4 dynes per square metre of a perfectly absorbent surface, or about 0·8 dynes per square metre of a perfectly reflecting surface; or generally that the mechanical pressure per unit surface, due to a parallel pencil of light, normally incident, is equal to the energy per unit volume of the luminiferous ether near that surface. Despite the almost insuperable difficulties of experiment Lebedew has succeeded in proving that in practice "light does exercise a true pressure on a surface on which it is incident; the pressure being twice as great for a perfectly reflecting as for a perfectly absorbent surface." He found, moreover, that the absolute magnitude of the pressure was equal to that predicted by Maxwell. We have, therefore, some right to assume that calculations based on this theory are calculations dealing with perfectly definite and known magnitudes.

According to Maxwell's Theory, since the energy per unit volume of a perfectly elastic solid transmitting simple harmonic waves is $\frac{1}{2}\rho\frac{(2\pi a)^2}{\tau^2}$ obviously the pressure exerted on unit surface will be proportional to this, and will in fact be equal to the amount of energy transmitted per second through a surface of unit area $=\frac{1}{2}\rho\nu\frac{(2\pi a)^2}{\tau^2}$, where P is the density of the perfectly elastic solid, V the velocity of propagation of the waves, *a* their amplitude, and T their period. By assuming the ether to act as a perfectly elastic solid, it is found that results are obtained by calculation which accord in all particulars with those of experiment, and therefore the assumption that it does so is justified until the contrary be proved.

From the above formula it is clear that the mechanical pressure of light per unit area must vary directly as the square of the amplitude of the vibrations, provided that ρ, V and T are constant. That is to say, that the density of the ether, the velocity of propagation of light waves through it, and the period of the waves themselves are constant. The first two of these conditions are dependent, for if the density of the ether were not constant, then the velocity of propagation of light waves through it would also vary; but as far as observation goes, this velocity seems to be constant, and we may, therefore, assume the density of the ether to be constant also. T will obviously be constant if monochromatic light only be considered.

We come, therefore, to the conclusion that for any ray of monochromatic light coming from the Sun the pressure will be proportional only to the square of the amplitude of the vibrations of which it consists, and, therefore, since sunlight may be considered as consisting of a number of infinitely thin monochromatic pencils of various colours in constant proportion; the pressure exerted by a ray of sunlight will be proportional only to the square of the amplitude of the vibrations of which it consists. Hence, since the intensity of illumination also varies as the square of the amplitude of the vibrations, provided that the velocity of propagation and the density of the medium are constant that, therefore, the pressure of light per unit area will be directly proportional to the intensity, and will, therefore, vary inversely as the square of the distance from a point source of light. Thus we have that the repulsion of light varies as the attraction of gravitation for this also varies inversely as the square of the distance from a point source. Hence distance from the Sun cannot affect the ratio of the forces of attraction and repulsion.

Since the repulsive force varies as the area of a cross section of a particle, while that of gravitation varies, in the case of the same substance, as the volume of a particle; it follows that in the case of particles of the same substance assumed to be perfectly reflecting and spherical, the particle will remain at rest if $f\rho v = f^1 s$.

Where f is the solar attraction of unit volume of a substance of unit density, ρ is the density of the substance in terms of that taken as the unit in the calculation of the quantity f, v is the volume of the particle, f^1 is the solar repulsion on unit surface and s is the area of the cross section of a particle. Substituting v and s in terms of r the radius of a particle we have :—

$$f\rho\,\tfrac{4}{3}\,\pi r^3 = f^1\,\pi r^2 \text{ whence } f\rho\,\tfrac{4}{3}\,r^3 = f^1 r^2 = o$$

$$\text{and } \left(r^2\,\frac{4f\rho}{3f^1}\,r = 1\right) = o \quad \text{whence we have}$$

$$r = 0 \text{ or } \frac{3f^1}{4f\rho}$$

$r = 0$, simply expresses the fact that a point in space is neither repelled nor attracted for it has neither cross section nor mass. The value of r will be obtained by substituting any known values in $r \frac{3f^1}{4f\rho}$ for $\frac{3f^1}{4f}$ is a constant, equal to the radius of a particle of unit density which would be neither attracted nor repelled.

Taking the case of water at the Earth's distance from the Sun we have:—

$$r = \frac{3 \times \cdot 8 \times 10^{-4}}{4 \times \cdot 5926 \times 1} \text{ am.} = 1010 \text{ tenth-metres.}$$

So that for all substances we may say that if d is the diametre of the particle which will remain at rest under the action of these two forces, $\frac{2020}{d = \rho}$ tenth-metres where ρ is the specific gravity of the substance.

The question which next arises is, can the particles which compose the tail of a comet be smaller than this critical value? And fortunately physical chemistry is able to answer this question for us. The methods and arguments are not as perfect and rigorous as those one is accustomed to find in astronomy; but results obtained from consideration of the critical equation of condition derived from Van der Waals equation, those derived from the dielectric constant from the mean free path of the molecules, from specific refraction, and from the ionic charge, all agree as regards the order of magnitude of the molecules; and we can say that for simple substances their diameters fall in the units of tenth-metres. Hence they are of a size to be repelled by the light of the Sun much more than they are attracted by the Sun itself. Thus if no other factors intervened we should have the particles of the tail of a comet repelled from the Sun at all parts of the comet's orbit, provided that they were gaseous, and the magnitude of the resultant of the forces of attraction and repulsion would vary inversely as the square of the comet's distance from the Sun. There are, however, factors which interfere with this arrangement.

Even if the attraction of the Sun were greater than the repulsion of its light on the small particles, these would tend to leave the comet and describe hyperbolic orbits round the Sun, and Bredichin has put forward an hypothesis as to the chemical composition of the tails of comets according to the shape which they assume in their passage round the Sun. He divides the tails into three classes, *viz*:—Hydrogen tails, which are almost straight; Hydrocarbon tails, which are more curved; and Iron tails, which are short very much curved brushes. He concludes that these three types of tail consist of the substances named, because he assumes the repulsive force to be a surface action, and that therefore the accelerations produced would vary as the mass, *i.e.*, as the molecular weights in the case of gases.

There seem to me to be two objections to this theory. Firstly, Bredichin finds that the accelerating force in the case of the tails of the Hydrogen type is from twelve to fifteen times that of the gravitational attraction, while in the case of tails of the Hydrocarbon type the repulsive accelerating force varies from 2·2 on the convex edge of the tail to 1·1 times the attraction on the concave edge. Thus, since the molecular weight of Hydrogen is 2, the molecular weights of the Hydrocarbons of which these tails consist must lie between $2 \times \frac{12}{2\cdot2}$ and $2 \times \frac{115}{1\cdot1}$, *i.e.*, between II and 28. Now there is no Hydrocarbon of molecular weight II, the lowest being Methane with molecular weight 16. Consequently we are unable to account for the observations on this hypothesis, unless we assume some amount of dissociation to have taken place, in which case of course the prediction of the consistency of a tail from its form becomes impossible. There are, moreover, only three other Hydrocarbons whose molecular weights fall, even approximately, within the prescribed limits, *viz*:— Acetylene 26, Ethylene 28, and Ethane 30, and these being all round about 28 we are at a loss to account for the central parts of the tails of this type on Bredichin's theory.

The second objection is that if the theory be correct, then all molecules must be of the same size. Now it seems almost inconceivable that molecules of Hydrogen, which consist of two atoms of that element, should be of the same size as molecules of Ethane, which consist of two atoms of Carbon and six atoms of Hydrogen. We can, however, advance more cogent reasons than this by actually calculating the volumes of the atoms of a number of substances.

As this paper is intended for the eyes of students of astronomy rather than of physical chemistry, it may add to the conviction it carries if I indicate first of all the method by which these values are to be calculated.

Van der Waals, by making certain approximate assumptions, was able to give an equation of condition, which agreed much better with the observed phenomena than the ordinary $pv = RT$. The equation of Van der Waals was :—

$$\left(+ \frac{a}{v^2} \right) \left(v - b \right) = RT$$

Where a represents the molecular attraction, and b four times the volume occupied by the molecules themselves, that is the smallest volume the gas could be made to occupy if subjected to infinite pressure.

Now from the above equation we have :—

$$v^3 - \left(b + \frac{RT}{p}\right) v^2 + \frac{a}{p} v - \frac{ab}{p} = 0$$

And if the roots of this are :—

$$\alpha, \ \beta \text{ and } \gamma$$

$$(\nu - \alpha)\ (\nu - \beta)\ (\nu - \gamma) = 0$$

Since the product of the roots is real, if one is imaginary, two must be imaginary, and therefore for one value of p there are one or three of v. The limiting point at which the three roots of the equation are equal may be found from the isotherm of the critical temperature of the gas, that is from the curve obtained by plotting the pressure against the volume for the constant critical temperature of the gas. The value of v found in this way will be the critical volume and the corresponding pressure will be the critical pressure.

Thus $\alpha = \beta = \gamma = \phi_0$

hence $\left(\nu - \phi_0\right)^3 = \nu^3 - \left(b + \frac{R\vartheta_0}{\pi_0}\right) \nu^2 + \frac{a}{\pi_0} \quad \frac{ah}{{}_0} = 0$

where ϕ_0 = The critical volume.

$_0$ = The critical pressure.

θ_0 = The critical temperature.

whence :—since the coefficients of v must be equal.

$$3\phi_0 = b + \frac{R\theta_0}{\pi_0} \text{ and } b = \frac{b\pi_0 + R\vartheta_0}{9\pi_0} \therefore \tfrac{8}{9} b = \frac{R\vartheta_0}{9_0} \therefore b = \tfrac{1}{8} R \frac{\theta_0}{\pi_0}$$

Here R is the gas constant $\cdot 0821 = \frac{22\cdot42}{273}$ whence

$$b = \tfrac{1}{8} \frac{22\cdot42}{273}, \frac{\theta_0}{\pi_0} = \tfrac{1}{8} \frac{\theta_0}{273_0} \text{ if } 22\cdot42 \text{ litres is our volume unit.}$$

Now if we take b as four times the volume of the molecules, which is the value which makes Van der Waals equation represent observed facts most nearly, b being calculated from independent data, we have :—

$$x = \frac{1}{32}, \frac{\theta_0}{273\pi_0}$$

Where x is the fraction of the volume of the gas at normal temperature and pressure occupied by the molecules.

Hence since we know that the number of molecules in I c.c. of any gas at N.T.P. is $3{\cdot}5 \times 10^{19}$ approximately, the volume of a molecule may be calculated.

I append a table showing the volumes of the molecules of various substances of which I have been able to obtain the critical data :—

Substance.	Volume of Molecule.	Ratio.
Hydrogen	$7{\cdot}14 \times 10^{-24}$ c.c.	1·0
Oxygen	$9{\cdot}71 \times 10^{-24}$ c.c.	12·9
Nitrogen	$11{\cdot}71 \times 10^{-24}$ c.c.	10·2
Ethane	$22{\cdot}59 \times 10^{-24}$ c.c.	7·0
Ethylene	$16{\cdot}00 \times 10^{-24}$ c.c.	8·2
Carbon Monoxide	$12{\cdot}29 \times 10^{-24}$ c.c.	9·8
,, Dioxide	$14{\cdot}29 \times 10^{-24}$ c.c.	13·8
,, Tetrachloride	$39{\cdot}14 \times 10^{-24}$ c.c.	25·1
Benzene	$36{\cdot}57 \times 10^{-24}$ c.c.	13·3
Ammonia	$11{\cdot}43 \times 10^{-24}$ c.c.	6·2
Ether	$42{\cdot}57 \times 10^{-24}$ c.c.	11·3
Water	$10{\cdot}57 \times 10^{-24}$ c.c.	7·0
* Methane	$15{\cdot}00 \times 10^{-24}$ c.c.	5·0

These figures enable us to compare the volumes of the molecules but their probable error is large.

* The values for Methane are probably about right, judging from its composition. I have been unable to procure the critical data of the gas.

If the molecules were all of one size the theory of Bredichin would hold, since $\alpha = f/m$. The variation in size, however, causes the factor f to vary as $v^{\frac{2}{3}}$, for f varies as r^2 whereas v varies as r^3 m is in this case the molecular weight of the gas, and is therefore constant. In the third column of the accompanying table the inverse ratio of the repulsion to that of Hydrogen, measured by the accelerations produced has been entered.

These figures are interesting as tending to show that the convex edge of a cometary tail of the second, that is the Hydrocarbon type, consists more probably of carbon monoxide the spectrum of which resembles that of the lower Hydrocarbons.

I leave a discussion of the composition of the central and concave portions to a future occasion.