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Theory of Unstable Equilibrium of Revolving Masses in reference to Double Stars

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It is obvious from purely mathematical considerations that a rotating fluid mass will assume the form of an oblate spheroid, because centrifugal force which is zero at the axis of inertia becomes a maximum in the plane of the spheroid's equator, and hence the meridian will be an ellipse, whilst the equator will be a circle. It is also obvious that the greater the velocity of rotation the greater becomes the oblateness. But what happens when the velocity of rotation exceeds a certain critical limit, and the spheroid must either assume a wholly new shape, or become disintegrated? Here we have a very difficult and complex mathematical problem. In the first place, it is not possible to deal mathematically with bodies other than liquid and homogeneous. Hence the results as applied to heavenly bodies cannot be wholly conclusive, for such bodies are for the most part neither fluid liquid nor homogeneous. The most skilful analysis can therefore only give us an approximation to the truth regarding their evolution. However, the mathematical solution of the various stages in the changing shape of a homogeneous revolving mass is of the utmost practical importance in astronomy as explaining "double stars." Now, when the velocity of a rotating spheroid passes the critical limit, the mass will acquire three unequal axes, instead of two, and not only will the meridians then be elliptical but also the equator. The result is what mathematicians call a "Jacobean" ellipsoid. This shape of rotating mass also possesses in itself stable equilibrium if the velocity be not changed. But in the case of the heavenly bodies, they are continuously cooling, and hence continuously contracting, and hence continuously accelerating their rotational velocity. When this velocity again exceeds a *second* critical limit, the body will lose its symmetry, and become what is called an "apioid." An apioid resembles a pear-shaped body. This form also possesses a conditional stability, until a further critical velocity of rotation is reached. Mr. Jeans in papers sent to the Royal Astronomical Society has shown by rigorous mathematical reasoning, that a certain increase in the rotational velocity of an apioid will cause it to throw off a satellite. And the eminent mathematicians Poincaré and the younger Darwin both agree in the view, that such a process as we have described can fully account for the birth of double stars, in spite of their lack of homogencity. In the case of most of the stars, moreover, their substances (for the purposes of mathematical analysis) do not differ very materially from the liquid. For they are probably gaseous throughout, and consequently homogeneous, and resemble the liquid condition, in that, owing to the law of gravity, the attraction of their particles to the centre of mass varies only as the inverse *square* of the distance from that centre, whereas the law of gaseous expansion varies inversely as the *cube* of the volume; and therefore they, like liquid masses, possess definite limiting bounding surfaces. Beta Lyrae may be taken as an instance of the above process just after its third or apioid stage. For here we have two barely separated ellipsoids, revolving in thirteen days, with two unequal eclipses. They are of a density far less than that of air or about one twelve-hundredth of that of water. Again the pair of stars U Pegasi certainly revolve in absolute contact, presenting the actual apioidal form as defined by Poincaré. The smaller portion of the apioid is eight-tenths of the larger. Here then we have before our very eyes a pear-shaped mass or apioid revolving in the period of its light-changes.