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Variations in the rotational velocity of sunspot groups during their lifetimes

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Abstract. We determine the mean values, the r.m.s. deviations, and the standard errors of the mean, for differences between the 'initial' (ω_i), 'overall' ($\bar{\omega}$) and 'final' (ω_f) angular velocities of the rotation of sunspot groups whichoccurred during the solar cycles 1923–1333, 1934–1944 and 1945–1954 Sets of spot groups having their maximum areas (A_*) in the intervals 300–500, 500–1000 and > 1000 millionths of the solar hemisphere, and occurring in different cycles, are considered separately and in aggregates defined by various combinations. We find a general trend that spot groups are accelerated during the early phase of their lifetime, and decelerated in the late phase, yielding a net deceleration over the whole lifetime.

The low levels of confidence associated with the trends are probably due to the presence of accelerated as well as decelerated spot groups in the corresponding aggregates rather than due to absence of appreciable acceleration or deceleration during the lifetime of spot groups. The scatters in the values of the differences ($\omega_i - \bar{\omega}$) *etc.* are caused to a larger extent by fluctuations in the rotation rates than by errors in the position measurements.

Key words : solar rotation—sunspot groups

1. Introduction

The question whether the rotation of sunspot groups about the sun's axis is accelerated or decelerated during their lifetimes is of considerable importance in studying the rotation of the solar plasma and magnetic structures underneath the photosphere. From statistical analysis of the angular rotational velocities derived from Greenwich Photoheliographic Data, some authors have concluded that the rotation of sunspot groups is decelerated during their lifetimes (*e.g.* Godoli & Mazzucconi 1979; Tuominen & Kyrolainen 1982; Balthasser *et al.* 1982).

In the first two of these references the samples of spot groups used for averaging the rotational velocity during the early phases of spot groups' lifetimes are not the same as those used for averaging the rotational velocity during the later phases. For example, Godoli & Mazzucconi found that the mean rotational velocity corresponding to the first transit of spot groups across the solar disc is larger than that corresponding to the second transit, and so on. However, a careful examination of their figure 6 reveals that except at high latitudes (where spot groups are much fewer) the spot groups which last only for a smaller number of transits rotate faster than those making a larger number of transits. Hence the former (relatively short-lived but faster rotating) variety of spot groups contributes, in large numbers, to the mean rotational velocity computed for the earlier transits but not to that computed for later transits. Therefor it is not certain whether the lower value of the mean rotation velocity determined from the later transits is due to the smaller representation of the faster rotating spot groups in the corresponding sample, or due to a real rotational deceleration of individual spot groups.

Similar difficulty also arises in the interpretation of the results derived by Tuominen & Kyrolainen. Therefore it is imperative that the mean rotation velocity corresponding to the earlier and the later phases of the spot group's life be determined from *identical* samples of spot groups. This condition was satisfied, for example, in the analysis by Balthasser *et al.* (1982). However, their rigid selection criteria, aimed at minimizing the sources of errors, yielded samples of order $\lesssim 24$. Efforts to determine the rotational acceleration or deceleration of sunspots groups from larger samples are, therefore, necessary.

Here we present the results of our attempt to determine the changes in the angular velocity of rotation during the spot group's life by using samples which are larger and also identical for the purpose of comparing the average rotation rates during the earlier and the later phases of the spot group's life. In this study we consider, separately and in combinations, the spot groups which occurred during the solar cycles 1923–1933; 1934–1944 and 1945–1954 and which had their maximum areas A_* within the intervals 300–500, 500–1000 and > 1000 millionths of solar hemisphere. We determine their 'initial', 'final' and 'overall' angular velocities ω_i, ω_f and $\bar{\omega}$ as defined in section 2. We find that the mean values m of the differences $(\omega_i - \bar{\omega})$, $(\bar{\omega} - \omega_f)$ and $(\omega_i - \omega_f)$ often exceed their standard errors Δm but rarely in the order of magnitude. The signs of the mean values of the differences are sometimes positive and sometimes negative. Thus, aggregates of spot groups do show statistically significant accelerations and decelerations during their lifetimes; but the confidence levels of statements regarding acceleration or deceleration remain rather low. There is a general trend that spot groups are accelerated during the early phase and decelerated during the late phase of their lifetimes, leading to an overall deceleration. Yet no uniformly valid statement can be made about the rotational behaviour of spot groups even from the larger samples used by us. It is argued that this indefiniteness may be caused by mixtures of various effects on rotational acceleration or deceleration rather than by absence of acceleration or deceleration.

2. Data analysis

From Greenwich Photoheliographic results, we selected for our analysis those sunspot groups during the solar cycles 1923–1933, 1934–1944 and 1945–1954 whose observed area had reached a maixmum value A_* exceeding 300 units (1 unit =

one millionth of the solar hemisphere). These were classified into three maximum area intervals 300–500, 500–1000 and > 1000 units according to the value of A_* and further into 16 latitude zones between 40°N and 40°S each of width 5°, according to the value of their mean solar latitude. For each spot group in each area interval and latitude zone, the initial, the overall and the final angular velocities of rotation respectively were determined according to the following definitions :

$$\omega_i = (l_2 - l_1)/(t_2 - t_1),$$

$$\bar{\omega} = (l_n - l_1)/(t_n - t_1)$$

and

$$\omega_f = (l_n - l_{n-1})/(t_n - t_{n-1}).$$

Here l_1, l_2, l_{n-1} and l_n are the longitudes of the spot group's centre-of-area from the central meridian on the first, the second, the last-but-one (*i.e.* $(n - 1)$th) and the last (nth) day of the spot group's observed life; and t_1, t_2, t_{n-1} and t_n are the epochs of the corresponding observations (measured in days and fractions from the beginning of the calender year). It may be noted here that the number (n) of days of the observed life is not the same even for spot groups belonging to the same A_* interval. However this does not matter for our purpose, *viz.*, the detection of rotational acceleration or deceleration, if any, during the observed lifetimes of the spot groups.

Since all the spot groups considered here have $A_* > 300$, many of them, especially those with $A_* > 1000$, were so long-lived that either l_1, or l_n, or both, exceed 80° causing relatively large errors in the determination of ω_i, $\bar{\omega}$ and ω_f. Inclusion of such spot groups in an earlier analysis had led us to an erroneous conclusion that spot groups of all the three A_*- intervals are accelerated during their life time. It was later found that this spurious result was due to the inequality of the samples affecting the mean values of ω_i and ω_f. For this reason we omitted all such spot groups in the revised analysis, thereby ensuring identical samples for determining average of ω_i, $\bar{\omega}$ and ω_f for each A_*-interval and each latitude zone. We then determined the mean value m, the r.m.s. deviation from the mean σ, and the standard error of the mean, $\Delta m = \sigma/\sqrt{N - 1}$, corresponding to each sample of spot groups defined by the A_*- interval, the latitude zone and the solar cycle N being the number of spot groups in the sample. These samples were aggregated over all the latitude zones to obtain nine sets of spot groups each defined by a specific A_*-interval and a specific solar cycle. Such an aggregation over all latitude zones is meaningful, inspite of the latitude dependence of rotation, because we are interested in the differences $\omega_i - \bar{\omega}$ *etc.* in the angular velocities at different phases of the lifetimes of individual spot groups; and very few of the latter are likely to cross from one latitude zone to another by latitudinal drift. Further, these sets of spot groups were combined into supersets of spot groups aggregating over all A_*-intervals for each solar cycle and, separately, over all the three cycles for each A_*-interval. Finally a single grand set was obtained by aggregating over all A_*-intervals and all the three solar cycles. Values of m, σ and Δm corresponding to the differences $(\omega_i - \bar{\omega})$, $(\bar{\omega} - \omega_f)$ and $(\omega_i - \omega_f)$ were determined for each aggregate set superset or the grand set). These values are shown in tables 1(a)–(d).

Table 1. Values of the mean m, the r.m.s. deviation/σ and the standard error of the mean Δm (underlined) of the differences $(\omega_i - \bar{\omega})$, $(\bar{\omega} - \omega_f)$ and $(\omega_i - \omega_f)$ for various aggregates of spotgroups. Conclusions regarding the trends of behaviour (A = acceleration, D = deceleration) are also given along with the percentage levels of confidence noted in brackets (Dash indicates conclusion not possible with confidence $\gtrsim$ 60 %)

(a) For sets of spot groups defined by separate A_*-intervals and solar cycles

		$300 < A_* \leqslant 500$				$500 < A_* \leqslant 1000$				$A_* > 1000$			
Solar cycle	Difference	N	m	σ and Δm	Concl. & Conf.	N	m	σ and Δm	Concl. & Conf.	N	m	σ and Δm	Concl. & Conf.
No. 16 (1923–33)	$\omega_i - \bar{\omega}$	63	−0.19	1.36 0.17	A(75)	42	−0.02	0.96 0.15	—	10	+0.39	0.83 0.26	D(80)
	$\bar{\omega} - \omega_f$		+0.03	0.96 0.12	—		−0.31	1.38 0.21	A(80)		+0.27	0.44 0.14	D(90)
	$\omega_i - \omega_f$		−0.16	1.86 0.23	A(70)		−0.33	1.59 0.25	A(80)		+0.65	1.00 0.31	D(92)
No. 17 (1934–44)	$\omega_i - \bar{\omega}$		−0.00	1.18 0.11	—		−0.03	1.39 0.16	—		−0.52	1.01 0.26	A(90)
	$\bar{\omega} - \omega_f$	120	+0.32	1.39 0.13	D(95)	73	+0.15	1.36 0.16	D(75)	15	+0.00	0.98 0.25	—
	$\omega_i - \omega_f$		+0.32	1.94 0.18	D(90)		+0.11	2.17 0.25	—		−0.51	1.46 0.38	A(90)
No. 18 (1945–54)	$\omega_i - \bar{\omega}$		−0.05	1.21 0.07	A(70)		−0.02	1.24 0.09	—		−0.13	1.12 0.18	A(70)
	$\bar{\omega} - \omega_f$	133	+0.18	1.30 0.07	D(95)	196	+0.07	1.47 0.10	D(70)	39	+0.30	0 86 0.14	D(95)
	$\omega_i - \omega_f$		+0.13	1.90 0.11	D(80)		+0 06	2.02 0.14	—		+0.17	1.53 0.25	D(70)

(b) For supersets obtained by combining sets of spot groups during all the three solar cycles

		$300 < A_* \leqslant 500$				$500 < A_* \leqslant 1000$				$A_* > 1000$			
Period	difference	N	m	σ and Δm	Concl. & Conf.	N	m	σ and Δm	Concl. & Conf.	N	m	σ and Δm	Concl. & Conf.
All three cycles (1923–54)	$\omega_i - \bar{\omega}$		−0.05	1.21 0.07	A(70)		−0.01	1.24 0.09	—		−0.13	1.12 0.18	A(70)
	$\bar{\omega} - \omega_f$	316	+0.18	1.30 0.07	D(95)	196	+0.07	1.47 0.11	D(70)	39	+0.31	0.86 0.14	D(95)
	$\omega_i - \omega_f$		+0.13	1.90 0.11	D(80)		+0.06	2.02 0.14	—		+0.17	1.53 0.25	D(70)

(c) For supersets obtained by combining sets of spot groups in the three A_*-intervals

Period	Difference	N	M	σ and Δm	Concl. & Conf.
Cycle No. 16 (1923–1933)	$\omega_i - \bar{\omega}$		−0.08	1.19 0.11	A(70)
	$\bar{\omega} - \omega_f$	115	−0.07	1.10 0.10	A(70)
	$\omega_i - \omega_f$		−0.15	1.71 1.16	A(75)
Cycle No. 17 (1934–1944)	$\omega_i - \bar{\omega}$		−0 05	1 25 0.09	A(60)
	$\bar{\omega} - \omega_f$	208	+0.24	1.35 0 09	D(95)
	$\omega_i - \omega_f$		+0.19	1.99 0.14	D(80)
Cycle No. 18 (1945–1954)	$\omega_i - \bar{\omega}$		−0.02	1.20 0.08	A(60)
	$\bar{\omega} - \omega_f$	228	+0.18	1.44 0.09	D(90)
	$\omega_i - \omega_f$		+0.16	1.95 0.13	D(80)

Table 1. Continued

(d) For the grand set obtained by combining spot groups in all the three A_*-intervals and all the three cycles

Difference	N	m	σ and Δm	Concl. & Conf.
$\omega_i - \bar{\omega}$		−0.04	1.21 0.05	A(70)
$\bar{\omega} - \omega_f$	551	+0.15	1.34 0.06	D(90)
$\omega_i - \omega_f$		+0.11	1.92 0.08	D(80)

In the case of spotgroups belonging to those aggregates for which $|m| \geqslant \Delta m$, statistically significant conclusions can be drawn regarding their rotational acceleration or deceleration during their *entire* observed life, according as $m\,(\omega_i - \omega_f)$ is negative or positive. Similar conclusions regarding the changes in the rotational velocity during 'early' and 'late' phases of their observed life can be obtained from the signs of $m\,(\omega_i - \bar{\omega})$ and $m\,(\bar{\omega} - \omega_f)$ and their values relative to $\Delta m(\omega_i - \bar{\omega})$ and $\Delta m(\bar{\omega} - \omega_f)$ respectively. For those aggregates and angular velocity differences for which $|m| > \Delta m$ we calculated the confidence levels for the sign of m by using z-test (t-test when the sample size is > 20).

We found that high confidence levels (*e.g.* $\geqslant 90\%$) are possible only for some aggregates and that too for one or two of the three differences $(\omega_i - \bar{\omega})$, $(\bar{\omega} - \omega_f)$ and $(\omega_i - \omega_f)$. In many cases the confidence levels were found to decrease when smaller aggregates were combined into larger ones. Thus it was not possible to improve the confidence levels by combining the aggregates. Therefore we decided to study only the trends in the rotational acceleration or deceleration by lowering the threshold of acceptable confidence level to $\sim 60\%$. For aggregates satisfying this condition, we have marked in tables 1(a)–(d) A for acceleration or D for deceleration, as the conclusion may be, along with the corresponding percentage level of confidence.

3. Conclusions and discussion

From tables 1(a)–(d) we note the following :

(i) Majority of sets and subsets show statistically significant differences between ω_i, $\bar{\omega}$ and ω_f, thereby indicating significant rotational accelerations or decelerations of spot groups during their observed lifetimes, and also during early andl ate phases, on low to medium (and occasionally high) levels of confidence.

(ii) The signs of the differences $(\omega_i - \bar{\omega})$, $(\bar{\omega} - \omega_f)$ and $(\omega_i - \omega_f)$ seem to vary from one A_*-interval to another during any solar cycle and also from one solar cycle to another for any given A_*- interval. These variations are not systematic.

(iii) When spot groups of all the three A_*-intervals during cycle no. 16 (1923–1933) are combined, they show significant acceleration over their whole observed life as well as during the early and the late phases of their life [*cf.* table 1 (c)].

(iv) During each of the cycles 17 and 18 spot groups of all the A_*-intervals combined show acceleration during the early phase, deceleration during the late phase and on overall deceleration during the whole lifetime [table 1(c)].

(v) When spot groups of all the three A_*-intervals and during all the three cycles are combined they show the rotational behaviour similar to that described above for cycles 17 and 18 [table 1(d)]. This is expected in view of the high amplitudes of cycles 17 and 18 compared to that of cycle 16.

All these conclusions are obtained by accepting levels of confidence $\gtrsim 60\%$. This was done with a view to finding out at least the trends in the rotational behaviour of sunspot groups. Clearly further work is necessary to bring out more definite relations, if any, between the rotational behaviour and other parameters like A_*.

We note that conclusions with high confidence levels do emerge at least for some sets and supersets. Therefore the low levels of confidence corresponding to other sets and supersets may not be due to absence of any rotational acceleration or deceleration of spot groups during their lifetimes, but may be, more probably, due to some mixing of accelerated and decelerated spot groups in such aggregates. This conclusion derives support from the fact that confidence levels corresponding to some combined aggregates are lower than those corresponding to the combining aggregates. Within a single cycle the mixing of accelerated and decelerated spot groups is likely, for example, if the rotational behaviour of the spot groups is related to the fast and slow latitudinal zones in the rotation of the photospheric plasma (*cf.* Labonte & Howard 1982). However this cannot account entirely for the values of the scatter σ responsible for the low confidence levels, since the values of σ are larger, by factors up to three, than the angular velocities of the latitude zones relative to one another.

It is interesting to look into some factors contributing to the scatter σ in the differences $(\omega_i - \bar{\omega})$, $(\bar{\omega} - \omega_f)$ and $(\omega_i - \omega_f)$. The values of ω_i, $\bar{\omega}$ and ω_f are subject to random errors upto $\sim 1°$ day $^{-1}$ due to the uncertainties in the positions of the centres of areas of spot groups. Probably they are also subject to random fluctuations $\sim$ 1–2° day $^{-1}$ due to the displacements imposed by supergranulation flows. The r.m.s. values of the errors are expected to increase with increasing A_* whereas those of the fluctuations are expected to decrease with increasing A_*. The values of the r.m.s. variations $\sigma(\omega_i - \bar{\omega})$ *etc.* given in table 1 are $\sim$ 1–2° day $^{-1}$ and are somewhat smaller for $A_* > 1000$ than for the lower A_*-intervals. This suggests that supergranulation flows may be contributing to the fluctuations in the rotation rates of spot groups and the magnitude of this effect may be smaller for large (*e.g.* $A_* > 1000$) spot groups.

In the case of individual sunspots the errors in the position measurements of the centres of areas will be smaller, but the fluctuations due to supergranulation are likely to be larger. It remains to be seen whether a similar analysis of the rotation of individual sunspots yields more definite results or otherwise.

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