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Thermal gravitational radiation from stellar objects and its possible detection*

C. Sivaram *Indian Institute of Astrophysics, Bangalore 560 034*

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Abstract. There has been a lot of current interest in attempting to detect or indirectly infer the presence of gravitational radiation especially from compact objects like pulsars. However, applications of general relativity lead us to explore the possibility of compact stellar objects generating high frequency (10^{16}–10^{21} Hz) thermal gravitational radiation which in the case of very young neutron stars could be rather high. Also white dwarfs and main sequence stars can generate such radiation from plasma-Coulomb collisions. Moreover models of the earliest Planck phase of the universe would predict a thermal gravitational wave background whose detection (at frequencies $\sim 10^{11}$ Hz) would enable us to make a choice between various cosmological models and particle physics models that attempt to unify fundamental interactions.

Key words : gravitational waves—thermal background

1. Introduction

Recently there has been a renewal of interest in attempting to understand the emission of gravitational radiation from astronomical sources, especially after the evidence that the orbit of the binary radio pulsar PSR 1913 + 16 decays precisely as predicted by general relativity (Taylor & Weisberg 1982). The recent discovery of the ultrarapid millisecond pulsar PSR 1937 + 214 has further motivated suggestions that it could slow down by emission of gravitational radiation (Sivaram & Kochhar 1984) and stimulated search for such radiation at about 1280 Hz from this pulsar (Hough *et al.* 1983). More recently it was even speculated that the intense gamma ray source Geminga (suspected to be a neutron star hardly hundred parsec away) might be producing sufficiently intense gravitational waves to excite the solar 160-min oscillations; and a spate of recent papers has even put stringent limits on the amount of such gravitational radiation (*e.g.* Anderson *et al.* 1984). For instance, the Pioneer 10 search for a gravitational wave amplitude of a possible

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sinusoidal variation no more than 3×10^{-14} in fractional Doppler frequency translates, for the direction of Geminga, into an upper bound of 2×10^{-14} for the polar component of the gravitational spatial strain which is potentially detectable (Carroll *et al.* 1984). However, in all these attempts to estimate and detect gravitational radiation from mainly compact objects the gravitational waves are of rather low frequency (at most $\sim$ 1 kHz).

What about the possibility of gravitational radiation of high frequency (several gigahertz or more)? One possible source of such radiation is a thermal background of gravitational waves (with a temperature of a few degrees K) which like the well-studied 3 K microwave electromagnetic background radiation would be the fossil remnant of the hottest and earliest phase of the big bang universe. Indeed such gravitational background thermal radiation if it exists—and if one assumes the standard big bang to be a correct description there is no reason to doubt its existence—would have originated in the earliest era, *i.e.* the Planck epoch when the particle energies were as high as 10^{19} GeV $\sim (\hbar c^5/G)^{1/2}$ and the gravitational interaction was a strong interaction with a dimensionless coupling constant $GE_p^2/\hbar c^5 \sim 1 (E_p \sim 10^{19}$ GeV). Note that unlike the electromagnetic case where $e^2/\hbar c \sim 1/137$ is independent of energy, in the gravitational case the coupling rises as E^2. At this stage, the gravitational interaction is strong enough to maintain thermal equilibrium between gravitons and relativistic particle pairs, just as at a later stage during the radiation era photons are in equilibrium with the particles. As the expansion proceeds after the Planck era $\sim 10^{-42}$ s, the extremely high energy gravitons decouple quickly (as the strength of the gravitational coupling weakens with falling energy) and form a noninteracting background whose temperature is just redshifted with the expansion. A similar thing happens at later epochs for neutrinos and photons.

In section 2, we review the factors which determine the present temperature (and hence frequency) of this graviton background with its possible manifestations and see that it would provide the best diagnosis of what went on in the earliest phase of the universe. In section 3 we consider the possible ways by which stellar objects in various stages of evolution might emit fairly copious amounts of thermal high-frequency gravitational radiation (10^{16}–10^{21} Hz), the radiation coming from plasma-Coulomb collisions, in main-sequence stars, and from fermion collisions in dense degenerate matter in more evolved objects such as white dwarfs and neutron stars. In section 4, we give some estimates of thermal gravitational radiation and consider its interactions with matter.

2. Thermal gravitational background radiation

As already stated, the gravitational interaction at the Planck epoch $t \sim 10^{-43}$ s was strong enough to maintain thermal equilibrium between gravitons and other particles. In fact other kinds of interactions between the particles were much weaker at that epoch (the GUTs coupling being $\sim 10^{-2}$ or less at that epoch). However, the gravitational interactions rapidly weakened owing to the expansion of the universe and the gravitons decoupled from the rest of the particles; the universe soon became transparent to gravitons. However, it is not straightforward to estimate the present background temperature of these gravitons.

If we assume that the total number N of the kinds of particles which were in equilibrium with the gravitons is finite and $N \gg 1$, then the total energy density of these particles (which formed an ultrarelativistic gas) is $N \cdot 4aT^4/3$, and the corresponding graviton entropy density is $4aT_g^3/3$. All the heavy particles and antiparticles are annihilated except for relatively few protons (nucleons). At temperatures of a few MeV one has only (i) electron-positron pairs (each with an energy density $\epsilon = (7/8)\,aT^4$, where a = Stefan-Boltzmann constant); (ii) neutrinos and antineutrinos of different (N_ν) types (possibly three) each with $\epsilon = (7/16)\,aT^4$; and of course (iii) photons with $\epsilon = aT^4$; *i.e.* all leptons. The entropy of the annihilating heavy particles is inherited by the leptons so that the neutrino temperature T_ν and the graviton temperature T_g are related by $(1 + \frac{7}{4} + N_\nu \times \frac{7}{8})\,\frac{4}{3}\,aT_\nu^3 = N \cdot \frac{4}{3} \cdot aT_g^3$, on the assumption that during the expansion, the graviton entropy per unit comoving volume is conserved. Moreover we note that photons inherit the entropy of the electron-positron annihilation. The radiation temperature T_r and the neutrino temperature T_ν are then related as $T_r = (11/4)^{1/3}\,T_\nu$; and the graviton and photon background temperatures are finally connected as

$$T_g = \left(\frac{7}{22}N_\nu + 1\right)^{1/3} \frac{1}{N^{1/3}}\,T_r$$

$$= (43/22N)^{1/3}\,T_r \text{ (for } N_\nu = 3).$$

If the initial number of relativistic particles present N is $\sim$ 30 and as $T_{r0} = 3K$, we get $T_{g0} \approx 1$K for the present-day temperature of the thermal graviton background. It is interesting to note in the context of GUTs that N depends on the unification group used. In most of the unification groups used N is about 30 (e.g. SU(5) has 24 generators) and could be much larger in larger groups like in extended supergravity. Thus we can expect T_{g0} to be significantly smaller than the photon temperature of 3K, T_g going as $N^{-1/3}$.

Thus indirectly T_{g0} can confirm ideas about the smallest group structure, which would unify all the fundamental interactions at the Planck epoch. Essentially the ratio of radiation to graviton background temperature T_r/T_g depends on the way the universe expanded between the two epochs when it became transparent to gravitons and photons respectively. Thus if the universe became transparent to gravitons at temperature T_2 (when the radius was R_2) and later to photons at temperature T_1 (radius R_1), then $T_r/T_g = R_1T_1/R_2T_2$, so that the way RT behaved between T_2 and T_1 determines the ratio of present-day temperatures of the two backgrounds.

The significance of this is as follows : in some of the recent modifications of the big bang we have models like the inflationary universe where after a GUTs phase transition the universe passes through a desitter phase which is a state of exponential expansion, $R \sim \exp(\Lambda^{1/2}t)$, where the expansion is driven by the dominant energy of the vacuum giving an effective Λ-term. In such models the entropy accumulates over a time much longer than Planck time which considerably reduces the ratio T_g/T_r (owing to the exponential expansion before the radiation era) so that the graviton background would have vanishingly small present-day energy density and temperature. Thus in some of the modifications of the big bang proposed recently we would have the radiation photon background as usual (including helium and deuterium) but we would not have any thermal graviton background, since it would be of vanishingly low temperature ($\ll 1$K).

Again in some of the other particle-physics models like the statistical bootstrap and Hagedorn models, the number of particle types grows exponentially following a power law, making N very large and thus making T_{g0} vanish. Some of these models also have a limiting maximum temperature much less than the Planck temperature. So in all these cases we would not expect any thermal gravitational background to be present. It is, therefore, of great significance if a thermal graviton background $T_{g0} \sim 1\text{K}$ is present; it would help choose between models (like the inflationary models) of what happened at the very earliest epochs of the universe. Ironically it would even tell us what particle-physics models are appropriate for the unification of all the fundamental interactions at the highest Planck energies!

It has been claimed that the microwave photon background or even helium can be generated independently of the big bang by invoking population III objects or otherwise (Rees 1978). However, there would be no way of generating a thermal graviton background except by having the universe pass through the superhot superdense Planck epoch. We shall see in the next section how much of thermal gravitational radiation can be generated by stellar objects. Thus the discovery of a thermal gravitational radiation background (or its absence) may provide unique irrefutable information on the way the universe began and the way it evolves and perhaps the way particle physics should unify forces. The frequency of this background would be $\sim 10^{11}$ Hz, so that a possible way of detection would be to convert it into electromagnetic waves of the same frequency, *i.e.* microwaves.

To effect such a conversion one should have an electromagnetic system with a time varying quadrupole term, as gravitational radiation is quadrupolar. For instance, when an electromagnetic wave propagates through a constant magnetic field such a term appears; *i.e.*, a wave with an amplitude H_y in a constant field H_0 can produce a quadrupole stress term, $T_{yy} = H_y H_0 \cos(kx - \omega t)$ and this gives rise to gravitational waves through the linear Einstein equation $\Box h_{yy} = kT_{yy}$, $k = 16\pi G/c^4$, (Zeldovich & Novikov 1983). Alternatively a weak gravitational wave h_{yy} upon propagation through a magnetic field strength H_0 gives rise to magnetic field perturbations through the equation $\Box H_y = \omega^2 h_{yy} H_0$. The fraction of the gravitational wave energy converted into electromagnetic waves of the frequency ω is shown to be

$$f = kH_0^2 d^2, \qquad \text{...(1)}$$

where d is the spatial extent of H_0.

Interestingly enough equation (1) can be used to put a limit on any primordial intergalactic magnetic field. For instance if $H_0 = 0.1$ Gauss at recombination (when redshift $z \sim 10^3$), then $f \sim 10^{-3}$. Larger values of f would cause anisotropies in the photon background larger than measured and a noticeable weakening of a single polarization of the microwaves would result. This already limits present day H_0 to less than 10^{-6} Gauss in intergalactic space, or less than 10^{-4} Gauss in interstellar space. The number density of the graviton background is of $\sim 10^3\ \text{cm}^{-3}$ so that their flow through a 1 km^2 area amounts to $10^{24}\ \text{s}^{-1}$.

3. Thermal gravitational radiation from stellar objects

In main-sequence stars like the sun, thermal gravitational radiation from Coulomb collisions in the plasma at the core can be generated, whereas in white dwarfs and neutron stars such radiation can result from fermion collisions in dense degenerate Fermi gas. If n_1, n_2 are number densities of gas particles undergoing collision with a $d\sigma_{12}/d\Omega$ differential scattering cross-section, with relative velocity V_{12}, and reduced mass μ_{12}, then the usual quadrupole formula gives for the power per unit volume per unit frequency interval summed over different pairs :

$$d\rho/d\omega = \sum_{1,2} \frac{32G}{5c^5} \mu_{12}^2 n_1 n_2 V_{12}^5 \int \frac{d\sigma_{12}}{d\Omega} \sin^2\theta \, d\Omega. \qquad ...(2)$$

For charges e_1, e_2 interacting via a potential $\phi \propto \gamma^{-n}$,

$$\int \frac{d\sigma_{12}}{d\Omega} \sin^2\theta \, d\Omega \simeq \frac{4\pi e_1^2 e_2^2 \ln \Lambda}{\mu_{12}^2 V_{12}^{4/n}}.$$

For $n = 1$, *i.e.* Coulomb collisions, we have the usual scattering formula in a plasma. Now $V_{12} = (2kT_c/\mu_{12})^{1/2}$; core temperature, $T_c = 10^7$ K; $n_1 = n_2 = 5 \times 10^{25}$ cm^{-3} at the solar core; $\ln \Lambda \approx 10$. We now multiply $d\rho/d\omega$ by the volume of 10^{32} cm^3 in the solar core. Since much of the radiation is at the frequency $kT_c/\hbar \approx 10^{18}$ s^{-1} (the collision frequency is $\sim 10^{16}$ Hz) we multiply by the frequency, and sum over electron-electron and electron-proton collisions. This gives total power output of the sun in thermal gravitational radiation, of frequency centred around 10^{17} Hz, as $\sim 10^9$ Watt (*i.e.* 10^{16} erg s^{-1}). The energy falling on the earth would be about 0.5 Watt at the same frequency.

As the absorption is negligible, all this radiation will come right off the solar core, and so if detected would be another diagnostic in addition to the neutrinos in determining conditions in the solar core. Regarding the detection it may be noted that the total 21-cm radio radiation from neutral hydrogen falling on the earth from all over the universe is also only 0.5 Watt (with of course no problems regarding its detection).

In the case of the degenerate cores of red giant stars, the velocity V_{12} in equation (2) would be the Fermi velocity $\hbar n^{1/3}/m$ (n = number density), and the temperature and the number densities would be an order of magnitude or more higher. The thermal gravitational radiation from such stars would be $\sim 10^{22}$ erg s^{-1} at a frequency of about 10^{18} Hz. We can similarly estimate the radiation from white dwarfs and neutron stars. In general, the radiation intensity by transition from initial states P_1, P_2 of the colliding particles to final momenta P_1', P_2' is given by

$$I_{P_1', P_2'; P_1, P_2} = \frac{2}{45} \frac{G}{c^5} \left| \langle P_1', P_2' \left| \dddot{\varphi}_{12} \right| P_1, P_2 \rangle \right|^2,$$

$$\text{with } \dddot{\varphi}_{12} = \frac{2\omega^3}{\mu_{12}} (3P_1P_2 - P^2 J_{12}); \; P = P_1 - P_2. \qquad ...(3)$$

We assume collisions of neutrons described by hard sphere fermion model with scattering length $l \sim 5 \times 10^{-14}$ cm. We restrict ourselves to S-wave scattering as de Broglie wavelength is large compared to this length. Multiplying by the density of states in momentum space and mean occupation numbers of Fermi statistics (noting for approximation that only particles within an energy range of $\pm kT$ around the threshold Fermi energy participate in collisions) we finally get the integrated power density as

$$P_{(g)} = \frac{8G}{5c^5}(3\pi^2 n)^{2/3}\, l^2 \left(\frac{M}{m_n}\right)\left(\frac{kT_c}{\hbar^{1/2}}\right)^4, \qquad \ldots(4)$$

where M is the mass of the star, m_n the neutron mass, n the central number density, and T_c the core temperature assumed much smaller than the Debye temperature T_D. Even for newly born hottest neutron stars $T_c = 5 \times 10^{10}$ K $< T_D$ (10^{14} K) (Flower & Itoh 1979). Now assuming $\rho \approx 5 \times 10^{14}$ gm cm^{-3}, $l = 5 \times 10^{-14}$ cm, the thermal gravitational luminosity from a fresh neutron star works out to $P_{(g)} \approx 10^{32}$ erg s^{-1} which shows that it could be important in cooling the star at the earliest stage. For a white dwarf like Sirius B, the corresponding $P_{(g)}$ is 10^{23} erg s^{-1}. Even for the neutron star, calculation shows that only 10^{-18} of the high frequency ($\sim 10^{21}$ Hz) thermal gravitational radiation is absorbed by the star, the rest of it going right through the star. If this neutron star were at a distance of about a kiloparsec from the earth (i.e. near the galactic centre), the flux of such radiation falling on the earth would be about 0.3 Watt, more or less the same as that from the sun. On the contray, suppose it were a nearby source like Geminga (hardly 100 pc away); then the flux on earth could be as high as 100 Watt. Equation (1) shows that such a neutron star with a field of 10^{13} Gauss should be able to convert 10^{22} erg of this radiation into high-energy MeV γ-ray photons. This is somewhat beyound the threshold of existing gamma-ray space telescopes.

4. Detection of the radiation

The high frequency thermal gravitational radiation can induce atomic transitions at a very slow rate. As an example we can estimate the decay rate of the $3d(m = 2)$ (quadrupolar transition) state of the hydrogen atom into the 1s state with emission of a graviton. The decay rate is given by equation (3), with the Hamiltonian of the transition matrix $H = k^{1/2} T_{\mu\nu} h^{\mu\nu}$, $T_{\mu\nu} = P_\mu P_\nu / m_e$ (for electron in hydrogen atom). The lifetime of the transition works out to be $\sim 10^{38}$ s. The frequency of this transition is about 10^{16} Hz which is within the range of thermal gravitons emitted by the sun. About 10^3 gravitons m^{-2} s^{-1} fall on the earth from the sun at around this frequency. Thus there is a finite probability of detecting induced emission with a sufficiently large detector.

This radiation would of course be very penetrating. By a coincidence the lifetime for this transition to take place is the same as the proton decay time ($\sim 10^{31}$ yr $\sim 10^{38}$ s). Proton decay is a major prediction of GUTs and despite the long lifetime is being tested by several experiments, some already functioning and bigger ones planned to detect about an event per year. So it may not altogether be impossible to also observe the effects so high frequency thermal gravitational radiation which can also induce transitions with a lifetime comparable to that of proton decay The absorption rate of 10^{16} Hz gravitons can be estimated as $\sim 10^{-27}$ (for terrestrial detectors), so one requires a detector of several hundred km^2 to detect a few transitions in some decades. Alternatively one can use equation (1) for the conversion of this radiation into electromagnetic waves and to detect minute fluctuations in magnetic fields induced by the thermal gravitational background. With present superconducting quantum interference devices (SQIDs) a change of less than 10^{-12} Gauss is detectable (Sivaram 1984). With a magnetic field of $\sim 10^6$ Gauss maintained over a spatial distance $d \sim 10^3$ cm, the conversion efficiency of thermal gravitational waves of frequency $\sim 10^{11}$ Hz to microwaves of the same frequency is $\sim 10^{-30}$, which means that the SQID would have to detect fluctuations as small as 10^{-14} Gauss to identify the gravitational wave signal. This is just about two orders of magnitude beyond the sensitivity of present SQIDs. It is remarkable that lifetimes of the same order are involved both in the experiments to verify GUTs and its possible unification with quantum gravity.

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