The Diffraction of Light and its Relation to the Performance of Telescopes.

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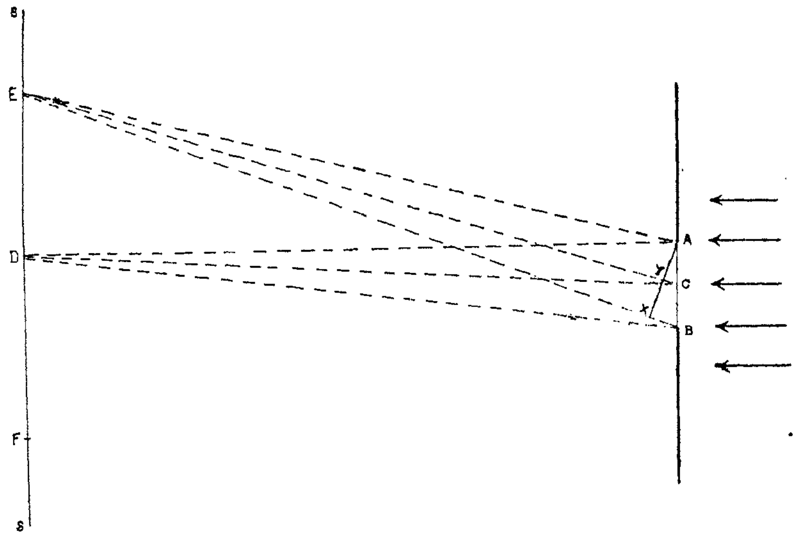
I propose in the present paper to discuss (with a few illustrations from my own work on the subject) some phenomena of the diffraction of light, and the fundamental principles by the aid of which they can be rendered intelligible.

I shall also discuss and emphasise the importance of the part played by diffraction in telescopic work, though here I have necessarily to follow largely the lines laid down by pioneer investigators like Lord Rayleigh.

To begin with, we may consider the case of a reflecting telescope which is directed towards a star at a sufficient altitude and let us put aside for a moment all trouble due to atmospheric conditions. We have coming in into the telescope a stream of light from the star in one definite direction, within very close limits. The text-books say that if the figure of the mirror is a paraboloid of revolution with its axis in the direction of the star, the light passing into the telescope is condensed into a point at the focus of the mirror. This seems evident from the principle of the reflexion of light, since at each point at which the light falls upon the mirror the normal to its surface bisects the angle between the join of that point with the focus and a line parallel to the axis of the mirror. Now the question which both the physicist and the practical astronomer will ask is this : Is all the light really condensed into a point ? A little consideration of physical principles will show that it cannot be so, even if the figuring of the mirror were theoretically perfect. Such a condensation would obviously involve a sudden transition from a very large illumination at the focus to zero illumination at immediately contiguous points. Such a state of things seems *a priori* extremely unlikely on any physical theory of the propagation of light. On the analogy of sound-waves, which as we know can go round or over a brick wall of moderate size without entirely ceasing to be audible, it seems evident that a certain amount of bending or spreading out is inevitable. In the case of a telescope, the entering beam of light is ordinarily limited by a circular aperture and what we get at the focal plane of the telescope as the image of a point source is a diffraction pattern which consists (*vide* Fig. 1 in the plate) of a central bright disc followed by successive dark and bright circular rings of greatly reduced intensity. In actual astronomical work, the second and third bright rings cannot ordinarily be seen because of their excessive faintness. They can, however, be observed in laboratory work, and if white light is used the rings are coloured.

It is not difficult to make out in a general way why we should get these rings. It is well, however, to begin with a simpler case, *i.e.*, when instead of a circular aperture we have a long narrow rectangular slit limiting the beam of light. Let A B in the diagram represent the width of the rectangular slit. A parallel beam of light is incident normally on one side of the plate in which the slit is cut, and of this beam all



except the portion that can pass through the slit is cut off by the plate. We can now consider the effect produced by such of the light as actually gets through at a screen S S placed at a great distance from the slit (this is shown in the diagram much too near the screen for the sake of space and clearness). If there were no diffraction, we would evidently have on the screen merely a narrow bright strip of light identically similar to the slit in width and length. What we actually get is a broadened central bright band parallel to the slit with alternate dark and bright bands of diminishing intensity situated symmetrically on either side of it. We may explain the formation of these bands in the following way. Let C be the mid-point of the aperture A B. We may conceive that the two halves of the aperture A C, C B are further divided up in the same way into a very large number of equal *elements*. We may properly assume that each of these small elements acts as a source and sends out waves on its own account in all directions into the region behind the aperture. To find the net result at any place we have to add up the effects of these individual waves and strike a balance. In working this out it is convenient to consider the elements in pairs, *i.e.*, the first one in A C and the first on C B, and so on. The waves sent out by the two elements of a pair intersect all over the field. The effect of these two sets is somewhat analogous to what we should have on the surface of mercury in a trough if we had two needles attached side by side dipping into the liquid and moved rapidly up and down by an attachment to the prong of a vibrating tuning fork. Both needles would act as centres of disturbance sending out circular ripples on the surface of the mercury and by their criss-crossing we would have a regular interference pattern on the surface. In certain regions the crests of one set of ripples would always coincide with the troughs of the 2nd set and the troughs of the former would coincide with the crests of the latter. The mercury surface would remain practically quiescent in these regions. In other regions the crests of one set of ripples would coincide with the crests of the 2nd set and the troughs of the former with the troughs of the latter, and we would have ripples of double amplitude travelling along these regions. We have an analogous effect with the light waves.

If D is the point on the screen exactly opposite the slit, and on the assumption that the former is at a sufficiently great distance from the latter, it is evident that the length C D differs from A D by a quantity which is too small to be appreciable, and it is clear that the elements at A and C produce practically identical results at the point D. This is also the case in respect of all the other elements in the aperture A B, and as the result we have bright illumination at the point D. If, however, we consider the effect at a point removed to one side, *i.e.*, say at E, the case is different. The distance C E is greater than A E by the length C Y. The crests of the waves from the element at C therefore lag behind those of the waves from A, and we would have appreciably less illumination than at D. If the angle D C E is of such magnitude that the distance C Y is half a wave-length, *i.e.*, the distance between a crest and a trough, the wave from the element at C just annuls the wave from the element at A, and this is also obviously true of each pair of elements in the two halves of the aperture and we would have complete darkness. E would then be the position of the 1st dark band. Similarly on the other side of D, F would be the position of the 1st dark band if D E = D F. If we go further out on either side beyond E or F, the elements would cease to annul each other and we would have a restoration of the light, but in greatly diminished intensity, since now the different pairs of elements do not all work together. The formation of the successive dark and bright bands can be traced in this way. The angular width $2\,\theta$ of the central bright band can easily be calculated since $CY = \frac{\lambda}{2}$ where λ represents the full wave-length. Putting $AB = a$, the equation is

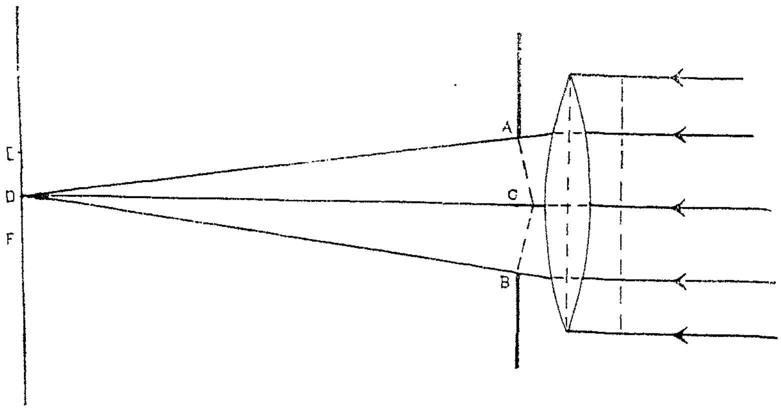
$$\sin\theta = \frac{\lambda}{a}.$$

If we now substitute a circular aperture of diameter a for the rectangular slit of the same width, it is evident from considerations of symmetry that we would have circular rings instead of parallel bands on the screen, but the angular width of the 1st dark ring is somewhat greater than that given by the formula for the rectangular slit. The reason for this is easily surmised. For the width of the bands increases when the aperture is decreased and a is only the *maximum* width of the circular aperture as measured on a diameter. Along parallel chords the width is less. A full mathematical treatment shows that in the case of the circular aperture the angular radius of the 1st dark ring is given by the formula

$$\sin\theta = 1{\cdot}22\frac{\lambda}{a}.$$

In the discussion given above it is assumed that the screen is at a sufficiently great distance to give us these rings in perfection. When, however, the aperture is large, the distance at which the screen would have to be held would be unmanageable and the simplest thing would be to put in a lens just behind the aperture to focus the diffraction pattern on to the screen, which should then be placed in the focal plane of the lens. We may regard the object-glass or reflector of a telescope as serving this purpose and the angular diameter of the rings seen in the focal plane would be determined by precisely the same formulæ.

Another instructive way of regarding this question would be to commence with considering the effect of the object-glass. The function of an object-glass is evidently to convert the plane parallel waves arriving at it into converging spherical waves, which come to a focus at their centre, as shown in the diagram.



This the object-glass effects by its varying thickness from point to point, the central parts of the glass retarding the progress of the waves to a greater extent than the marginal areas with the result that on emergence they are spherical and convergent. The distances D A, D C, D B are equal, D being the centre of the convergent wave. To find the effect at any point on a screen, held in the focal plane, we have now to divide up the spherical surface A C B into little elements and consider the interference of the wavelets proceeding from the different elements, which depends as before on the difference of their distances from the point D and the further treatment is on much the same lines as that given before. We have the ring or band system round the point D as centre, this being the position where the waves from all the elements conspire and produce the largest effect.

A numerical example would be useful here. The reflecting telescope presented by Dr. Harrison to the Society has an aperture of 4 inches. The angular radius of the 1st dark ring in this case is given by the formula

$$\theta = 1{\cdot}2\,\frac{\lambda}{a}$$

λ for yellow light is about $\frac{1}{50{,}000}$ inch, and a is 4 inches

$$\theta = 1{\cdot}2 \times \frac{60 \times 60 \times 180 \times 7}{22 \times 200{,}000} \text{ seconds of arc}$$
$$= 1{\cdot}2''$$

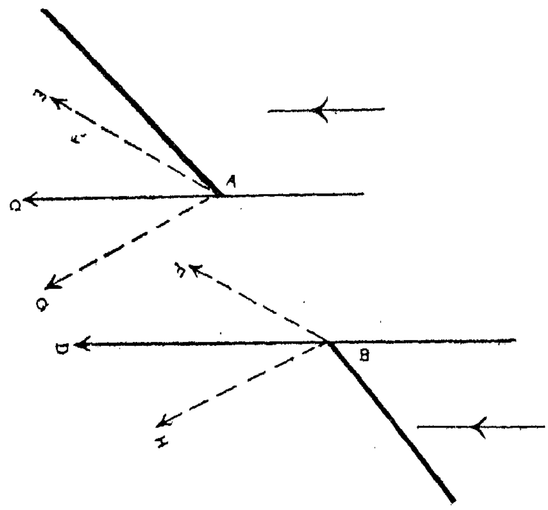
The diameter of the central disc is double this, *i.e.*, 2·4″

In the cases that have been considered in the foregoing, the apertures were taken to be held in a position normal to the direction of the beam of light whose width they restricted.

It is evident that it is also possible for the incident beam of light to be restricted in width by an obliquely-held aperture and such cases are fairly common in spectroscopic work. In the case of the rectangular aperture, the first effect of inclining it is to increase the width of the bands while their general character and symmetry remain unaltered. The reason for this is clear. With an obliquely-held slit, the effective width of the beam of light entering the telescope is less than when the aperture is held normally and the width of the bands is therefore necessarily increased in inverse proportion.

When the obliquity is very considerable, the character of the diffraction bands undergoes certain modifications, which are not only interesting in themselves but throw much light upon some matters of fundamental principle. One effect is that the diffraction pattern becomes unsymmetrical, the bands on one side of the system becoming much broader than those on the other side. This is well shown by Figs. 5 and 6 in the plate. In taking these photographs, the diffraction pattern was obtained by reflecting the beam of light into the telescope by a rectangular mirror, obliquely held, the source of light being a distant vertical slit, the direct image of which broadened by photographic halation also appears in the photographs. These unsymmetrical bands can, of course, also be obtained by transmission through an obliquely-held aperture, the two arrangements being equivalent in theory.

The asymmetry can be explained in the following simple manner :—

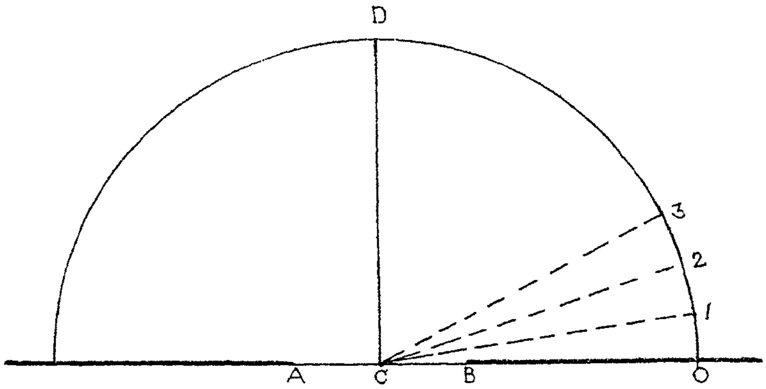


In the diagram A B is the aperture on which the light is obliquely incident. I have already explained that in the direction of the incident beam (*i.e.*, A C or B D in the diagram) we get the maximum light in the diffraction pattern.

To get to the first dark band on either side, the angle turned through must be such that a wavelet from an element at B gains or loses a wave-length over a wavelet from A. From the figure it is obvious that to gain a wave-length on the side B H a smaller angle need be turned through than is necessary on the side B F to lose a wave-length, and the angular width of the bands on the right-hand side is therefore less than the width on the left. The number of bands on the latter side is also limited.

With a circular aperture held at a moderate obliquity we get a system of elliptical bands (Fig. 2 in the plate), since the projection of the aperture, *i.e.*, its effective shape itself becomes an ellipse. At very oblique incidences the bands become unsymmetrical, *i.e.*, are elongated on one side in preference to the other. This is clearly shown in Fig. 3 in the plate. Theory shows that in this case, the dark and bright rings take the shape of Cartesian Ovals.

Figs. 5 and 6 in the plate which relate to diffraction at very oblique incidences show another effect that is really of fundamental importance. It will be observed by comparing corresponding bands on either side of the pattern that they are less in intensity on the side on which they are broader than on the side where they are narrow. In order to explain this effect, it is necessary, as I have shown elsewhere, to consider the peculiar character of the waves which in the preceding treatment we have assumed the *elements* of the aperture to send out and which by their interference produce the observed diffraction pattern. In the foregoing discussion I said we may reasonably assume that the elements send out waves (or wavelets rather) in all directions into the region behind the aperture. But do they send out wavelets with equal *strength* in all such directions ? This point, important as it is, had never been worked at from an experimental point of view. Experiments on normally held apertures cannot throw any light on the subject. For in such cases, the diffraction pattern is formed at regions contiguous to the apex of the hemispherical wavelets sent out by the elements in the direction C D in the diagram.



The intensity at D due to any one element is obviously a maximum and at any neighbouring points it cannot differ very appreciably. If, however, we work at very oblique incidences so that the diffraction pattern is formed in the region marked 0, 1, 2, 3, we should get some "obliquity" effects as I have called them. For, at the point 0 the amplitude must be zero and at any one of the points 1, 2, 3 it must be finite and increase as we go up. A diffraction pattern formed at such an incidence should obviously show a progressive increase in brightness from one side to the other superposed on the fluctuations caused by the interference of the wavelets. That the effects actually observed are due to such a cause, I have shown by photometric determination of the relative intensity of corresponding bands on either side of the pattern (using the well-known method of "revolving sectored-disc"), and the mathematical law of obliquity proposed by me has been fully verified.

The diffraction of light is of great importance in practical telescopic work. If, as we have seen above, the image of a mathematical point is not itself a point but a diffraction pattern, it is evident that the telescopic image cannot be an exact representation of the object viewed. Much detail is necessarily obliterated. The simplest case is that of a double star. A photograph of the diffraction pattern due to two adjacent point sources as seen through a circular aperture is shown in Fig. 4 in the plate. It is seen that the two discs have run together into a slightly elongated patch and, except probably under the most favourable atmospheric conditions, it would be impossible to detect that an object of this kind was a double star, or a triple star or one star, by itself. Herein lies the principal advantage of telescopes of large aperture. As the angular diameter of the diffraction disc due to a point source decreases in inverse proportion to the aperture, the resolving power increases *pari passu* provided that the figuring of the mirror or lenses continues perfect.

The same principle applies also in planetary work. Other things being the same, the larger the aperture the finer the detail that can be revealed by the instrument. This point is easily verifiable in laboratory experiments in which a disc with alternate white and black strips ruled on it is observed or photographed through an aperture with adjustable jaws. As the width of the aperture is gradually reduced, the white strips become fuzzy and broaden out, and after a certain stage completely obliterate the black areas in spite of the fact that the lens performs best with the smallest apertures.

On a small scale, this experiment can be made by the readers of this Journal with very simple apparatus and without any telescope at all. A piece of wire gauze and a cardboard in which two holes have been pierced with a pin are all that is required. One of the holes in the card should be larger than the other. The piece of gauze should be placed against a window so as to be backed by the sky, or in front of a lamp provided with a ground glass or opal globe. You then look at the gauze through the pinholes. Using the smaller pinhole and gradually drawing back from the gauze, you find that you lose definition and ultimately all sight of the wires, though there is light enough for the purpose. The distance at which this will happen depends upon the fineness of the gauze and the size of the pinhole: 5 or 6 feet will probably be sufficient. If, when looking through the smaller hole you have just lost the wires, you shift the card so as to bring the larger hole into operation, you will see the wires again perfectly.

In closing I should mention that Prof. Lowell holds that in planetary work there is no use increasing your apertures beyond a certain point, *i.e.*, say 20 or 25 inches, the reason advanced by him being the trouble from atmospheric conditions. If this point could be attacked and decided mathematically there would be good work done, I think.