Bull. Astr. Soc. India (1984) 12, 152–161

Estimation of the amount of heating for the solar coronal loops and kernels

Suresh Chandra *Institut für Theoretische Physik, Johann Wolfgang Goethe-Universität, Robert-Mayer-Str. 10, D 6000 Frankfurt/Main, F.R.G.*

Received 1983 September 9; accepted 1984 January 10

Abstract. We have investigated the amount of heating due to an external source for some coronal loops and kernels in the line-dipole and the point-dipole geometries under nonstatic conditions. Two forms for the external source are considered. It is found that the amount of heating for the line-dipole geometry is larger than that for the point-dipole geometry. Further, the amount of heating is larger when the source is extended one (large value of γ). The heating for the second form of the source is found larger than that for the first form in the line-dipole geometry, whereas in the point-dipole geometry the reverse is true.

Key words : heating—coronal loops—kernels—the sun

Introduction

Energy equilibrium in coronal magnetic loop has been investigated by various authors. For example, Antiochos & Sturrock (1976) considered the energy equilibrium given by

$$\frac{\partial}{\partial t}(3nkT) = \frac{1}{A}\frac{\partial}{\partial s}\left(Ak\frac{\partial T}{\partial s}\right), \qquad \ldots(1)$$

where n is the electron density, T the kinetic temperature, A the area of cross-section of the loop, $k(=\alpha T^{2.5};\ \alpha \approx 10^{-6})$ the coefficient of thermal conductivity, t the time, and s the length measured along the loop. Equation (1) is based on the assumptions :

(i) $n = n(s)$,

(ii) $T = T(s, t)$,

(iii) $p = p(t)$, ...(2)

(iv) $v = 0$ (static plasma),

(v) $H = 0$ (no heating due to external source), and

(vi) the losses due to the plasma expansion and the radiation are negligible.

Equation (1) along with the heating due to external source expressed in the form

$$H = Q(t) \exp(-s^2/\gamma^2 R^2), \qquad ...(3)$$

where γ is the extension of the source and R is the vertical height of the loop, has been investigated by Elwert & Narain (1980) and Chandra & Narain (1982). Elwert & Narain (1980) discussed the results for the plane-parallel and the line-dipole geometries, and assumed that the electron density at the base of the loop was infinite. Obviously, the kinetic temperature at the base of the loop was assumed to be zero, which did not seem to be realistic. This condition was removed by Chandra & Narain (1982) who took finite values for the electron density and the kinetic temperature. Furthermore, Chandra & Narain (1982) considered the line-dipole and the point-dipole geometries.

For a nonstatic plasma ($v \neq 0$), Antiochos & Sturrock (1978) expressed the energy balance in the form

$$-2.5\frac{p_0}{n}\left(\frac{\partial n}{\partial t} + v\frac{\partial n}{\partial s}\right) = \frac{1}{A}\frac{\partial}{\partial s}\left(Ak\frac{\partial T}{\partial s}\right), \qquad ...(4)$$

where the first four conditions in equation (2) were modified as :

(i) $n = n(s, t)$ (ii) $T = T(s, t)$

(iii) $p = p_0$ (constant pressure) (iv) $v \neq 0$. ...(5)

Recently, Narain (1983) has calculated the temperature distribution in a flare kernel of 1973 September 1 by considering the plane-parallel and the line-dipole geometries. The physical conditions and the energy equation adopted by Narain (1983) are equivalent to the use of equations (3) through (5). Narain (1983) again assumed the zero value of the kinetic temperature at the base of the kernel. Further, in such an investigation, one defines the temperatures at the base and the apex of the event (loop or kernel). Hence, naturally, the temperature would always vary smoothly in between the two end-values prescribed in the model. Therefore, it does not seem worthwhile to plot the temperature distribution along the event-length. What may be worthwhile is to calculate the amount of heating due to external source, which leads to the base and the apex temperatures assumed in the model.

In the present investigation we have calculated the amount of heating due to external source for coronal loops and kernels in the line-dipole and the point-dipole geometries with the finite values of the density and the temperature.

2. Energy equilibrium

When conduction dominates the radiation mechanism, the magnetic field geometry plays an important role, and conduction across the magnetic field lines is negligible. We assume that the magnetic field is current-free (potential field) and therefore, the lines of magnetic field are along the loop. Observationally it is found that the magnetic field at the base of the loop is much larger than that at the apex of the loop. Hence, to conserve the magnetic flux, the cross-section of the flux-tube changes along the length. According to the loop model of Antiochos & Sturrock (1976) a current-free magnetic field above the chromosphere is produced either by a horizontal line-dipole (L) or by a horizontal point-dipole (P) situated below the chromosphere. A detailed discussion about the line-dipole and the point-dipole geometries is given by Chandra & Narain (1982).

For a general form of the heating source

$$H = Q(t)\, f(s), \qquad \text{...(6)}$$

and the nonstatic condition in the loop, the basic equations for the present investigation are (4) through (6). These equations with the help of the equation of continuity can he rearranged in the form

$$2.5\frac{p_0}{T}\frac{\partial T}{\partial t} + \frac{k}{T}\left(\frac{\partial T}{\partial s}\right)^2 + \frac{Q(t)}{AT}\frac{\partial T}{\partial s}\int_0^s Af(s)\,ds$$

$$= \frac{1}{A}\frac{\partial}{\partial s}\left(Ak\frac{\partial T}{\partial s}\right) + Q(t)\, f(s). \qquad \text{...(7)}$$

By expressing

$$T = T_{00} T_t(t)\, T_s(s)$$

$$Q(t) = Q_{\max}\, T_t^{3.5}$$

$$\Psi = T_s^{3.5}, \qquad \text{...(8)}$$

such that $T_t(t = 0) = 1$, $T_s(s = 0) = 1$, one can separate the two independent variables s and t :

$$2.5\frac{p_0}{\alpha T_M^{3.5}}\frac{d}{dt}(T_t^{-3.5}) = \frac{2}{7\psi}\left(\frac{d\psi}{ds}\right)^2 + \frac{Q_{\max}}{\alpha T_M^{3.5}}\frac{1}{A\psi}\frac{d\psi}{ds}\int_0^s Af(s)\,ds$$

$$-\frac{1}{A}\frac{d}{ds}\left(A\frac{d\psi}{ds}\right) - \frac{3.5 Q_{\max}}{\alpha T_M^{3.5}} f(s) = k_c^2, \qquad \text{...(9)}$$

where k_c^2 is a constant for the separation of variables. The left-hand side of equation (9) has a solution

$$T_t = (1 + t/\tau)^{-2/7}, \qquad \text{...(10)}$$

where $\tau(= 2.5\, p_0/\alpha T_M^{3.5}\, k_c^2)$ is the characteristic time of decay. The right-hand side of equation (9) for the line-dipole (L) and the point-dipole (P) geometries can be expressed as

$$\frac{d^2\psi}{d\theta^2} = \frac{2}{7\psi}\left(\frac{d\psi}{d\theta}\right)^2 + G_1 \tan\theta\,\frac{d\psi}{d\theta} + \frac{Q_{\max}R^2}{\alpha T_M^{3.5}}\frac{G_2}{\psi}\frac{d\psi}{d\theta} I_1(\gamma\theta)$$

$$-\left[\frac{3.5\, Q_{\max}R^2}{\alpha T_M^{3.5}} f(\theta) + k_c^2 R^2\right] G_3, \qquad \text{...(11)}$$

where

$$G_1 = 2, \quad \text{...(12L)}$$

$$G_1 = (11 + 9 \sin^2 \theta)/(1 + 3 \sin^2 \theta), \quad \text{...(12P)}$$

$$G_2 = \sec^2 \theta, \quad \text{...(13L)}$$

$$G_2 = (1 + 3 \sin^2 \theta) \sec^5 \theta, \quad \text{...(13P)}$$

$$G_3 = 1, \quad \text{...(14L)}$$

$$G_3 = \cos^2 \theta (1 + 3 \sin^2 \theta), \quad \text{...(14P)}$$

$$I_1(\gamma\theta) = \int_0^\theta f(\theta) \cos^2 \theta \, d\theta, \quad \text{...(15L)}$$

$$I_1(\gamma\theta) = \int_0^\theta f(\theta) \cos^7 \theta \, d\theta. \quad \text{...(15P)}$$

The boundary conditions for the equation (11) are

$$\left.\begin{array}{l} \text{at the apex } \theta = 0, \quad \psi = 1, \quad d\psi/d\theta = 0 \\ \text{at the base } \theta = \theta_b, \quad \psi = \beta^{3.5}, \quad \beta < 1. \end{array}\right\} \quad \text{...(16)}$$

That is, the temperature at the base of the loop is β times that at the apex of the loop.

3. Parameters

In the present investigation we have accounted for four events, namely, flare loop (1973 June 15), flare loop (1973 November 26), flare kernel (1973 August 9) and flare kernel (1973 September 1). The basic parameters for these events are the same as in Chandra & Narain (1982). Therefore, we need not discuss them in detail here. The values of the parameters, the length of the event (l), the temperature at the apex (T_{00}), the electron density at the apex (n_0), the maximum compression factor (Γ_{max}), the characteristic time of decay (τ), the value of R, and the angle θ_b at the base of the event are given in table 1.

Table 1. Values of the parameters

Event (1973)	l (10^9 cm)	T_{00} (10^6 K)	n_0 (10^9 cm^{-3})	Γ_{max}	τ (s)	R (cm) L	R (cm) P	θ_b (degrees) L	θ_b (degrees) P
Flare loop (June 15)	3.1	6.5	15	77	6.4(2)*	1.1(9)	1.4(9)	83.46	57.76
Flare loop (November 26)	2	5.8	13	88	2.1(3)	6.8(8)	9.1(8)	83.88	58.52
Flare kernel (August 9)	0.36	9.5	400	12	1.3(2)	1.4(8)	2.1(8)	73.22	44.54
Flare kernel (September 1)	0.15	8	200	19	4.5(2)	5.6(7)	8.1(7)	76.74	48.36

*The numbers in the brackets are the powers of ten; *e.g.* 6.4(2) = 6.4 × 10^2.

4. Results and discussion

We have first solved equation (11) numerically in the absence of the heating due to external source ($Q_{max} = 0$). The value of the parameter β which is the ratio of the temperature at the base to that at the apex is obtained and is given in table 2. Table 2 shows that, in the absence of external heating, the variation of temperature in the flare loop of 1973 June 15 is larger than that in the other three events.

Table 2. Values of β in the absence of the heating due to external source ($Q_{max} = 0$)

Event (1973)	L	P
Flare loop (June 15)	0.4874	0.2666
Flare loop (November 26)	0.9506	0.9343
Flare kernel (August 9)	0.9017	0.8920
Flare kernel (September 1)	0.9957	0.9951

If we assume the heating due to external source given by equation (3), the expression for $f(\theta)$ is

$$f(\theta) = \exp [- (F(\theta)/\gamma)^2], \quad \ldots(17)$$

where

$$F(\theta) = \theta, \quad \ldots(18L)$$

$$F(\theta) = \tfrac{1}{2} \sin \theta \, (1 + 3 \sin^2 \theta)^{1/2} + \frac{1}{2\sqrt{3}} \ln [\sqrt{3} \sin \theta + (1 + 3 \sin^2 \theta)^{1/2}]. \quad (18P)$$

With the help of equations (11) through (18), the variation of the parameter Q_{max} *versus* the parameter β is calculated and is shown in figures 1 and 2 for $\gamma = 1$, and in figures 3 and 4 for $\gamma = 0.5$ for the two geometries. Comparison of figures shows that, for a given value of β, for the last three events, the value of Q_{max} for the line-dipole geometry is larger than that for the point-dipole geometry. Further, the value of Q_{max} is larger for the extended source (large value of γ).

Since the exact nature of the heating source is still unknown, we consider a second form of heating such that

$$f(\theta) = \exp [-F(\theta)/\gamma]. \quad \ldots(19)$$

For this type of heating, the variation of Q_{max} *versus* β is shown in figures 5 and 6 for $\gamma = 1$. Comparison of figures 1 and 2 with 5 and 6 shows that, for a given value of β, the value of Q_{max} for the second form of heating is larger than that for the first form.

Figures 1 through 6 show that in the region of $\beta = 0$ to 0.1, the value of β is very sensitive to the value of Q_{max}. Consequently, the precise value of Q_{max} for a given value of β in this range cannot be calculated. In order to calculate the total amount of heating, we have considered the case of $\beta = 0.2$. The value of Q_{max} for $\beta = 0.2$ is calculated and is given in table 3.

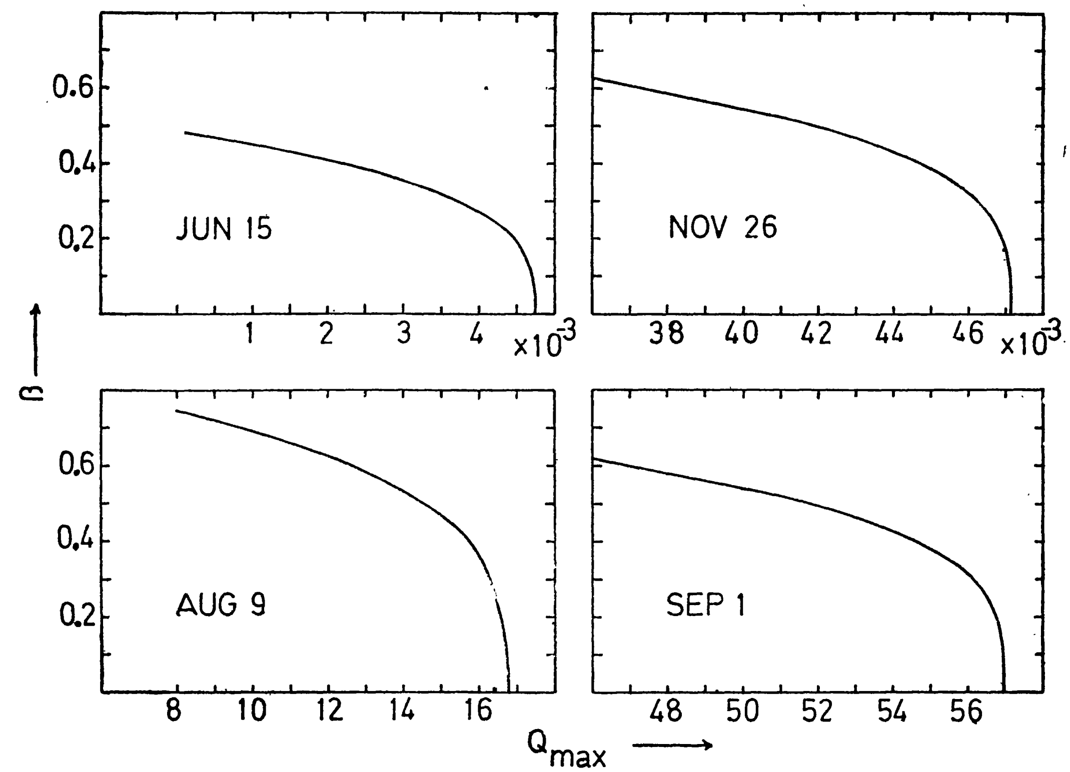

Figure 1. Variation of Q_{max} *versus* β for four events in the line-dipole geometry and for the first form of heating source (equation 17) with $\gamma = 1$.

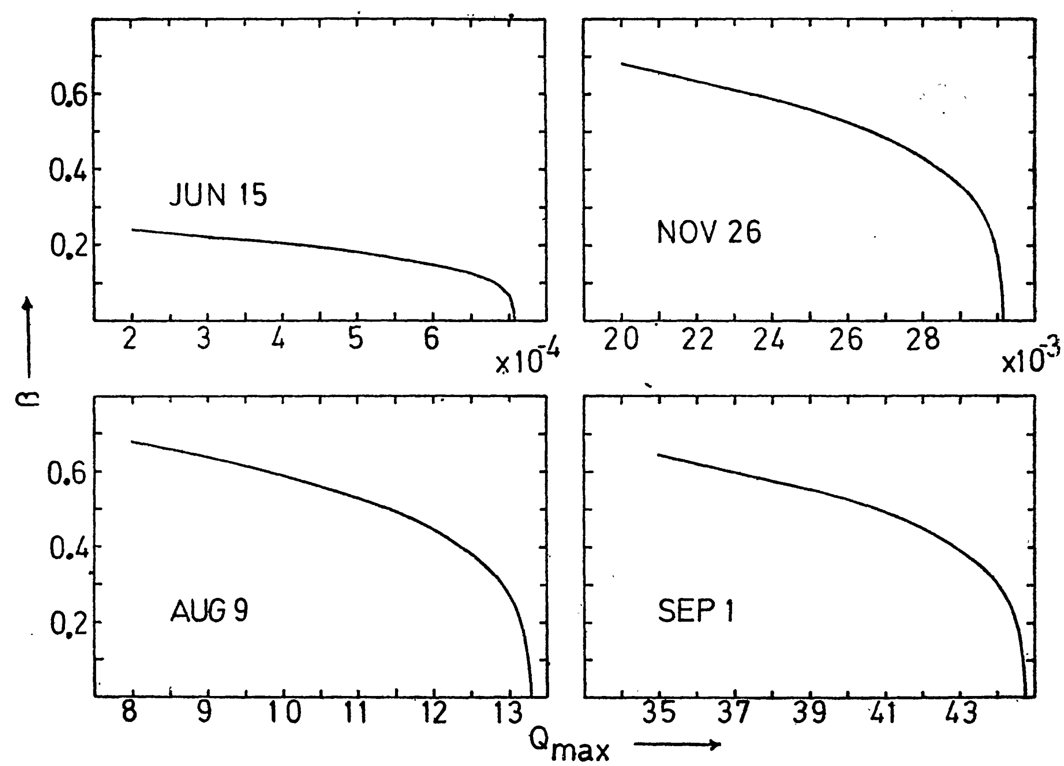

Figure 2. As figure 1, but for the point-dipole geometry.

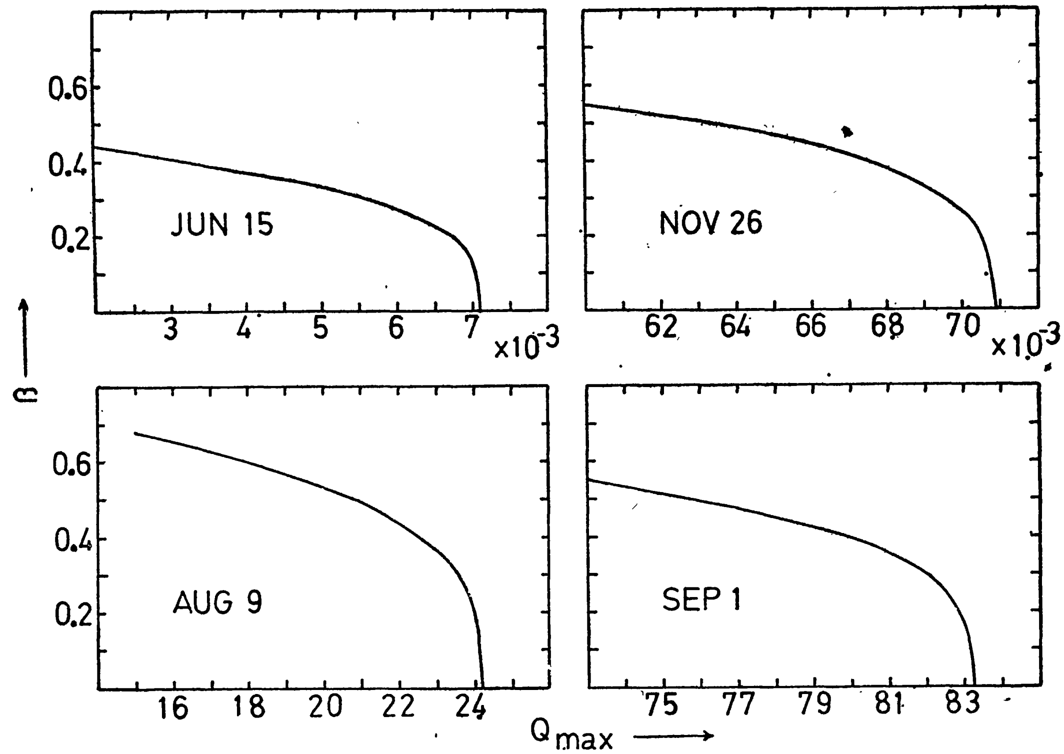

Figure 3. As figure 1, but for $\gamma = 0.5$.

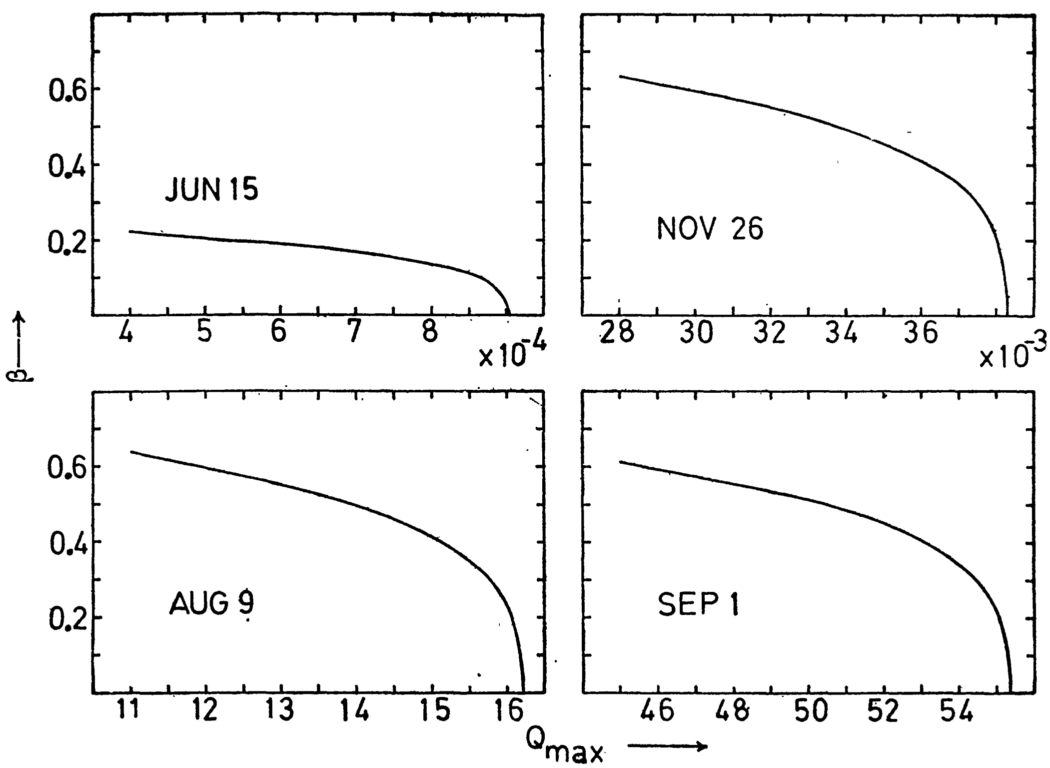

Figure 4. As figure 2, but for $\gamma = 0.5$.

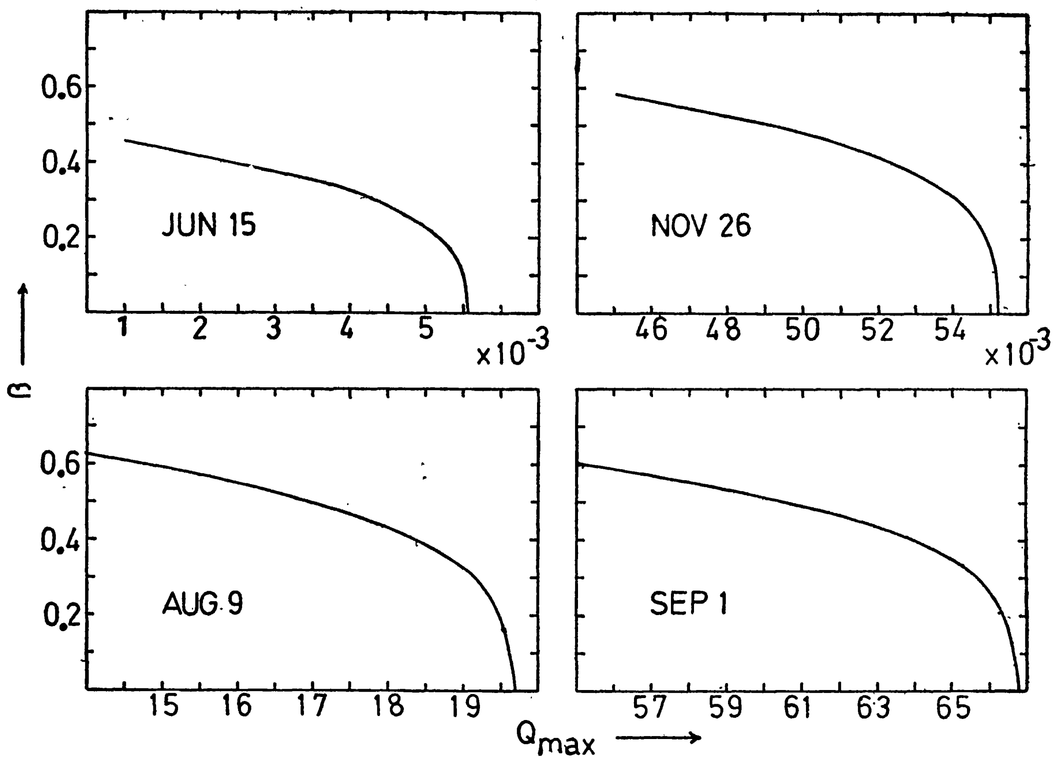

Figure 5. As figure 1, but for the second form of heating source (equation 19).

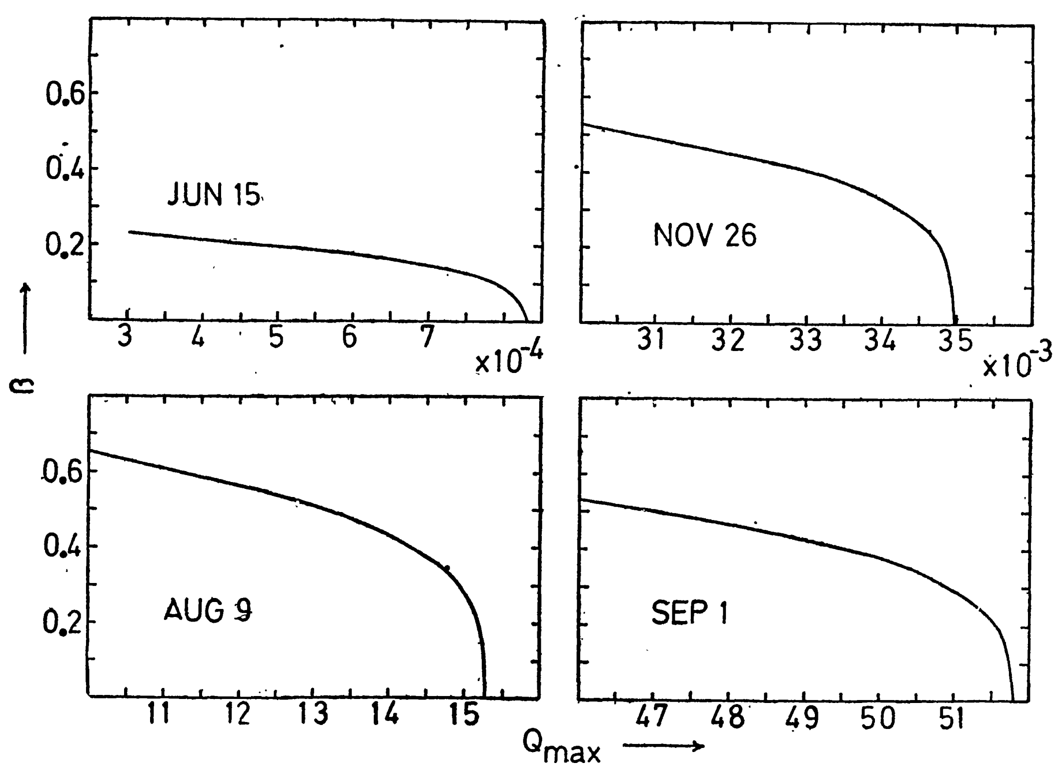

Figure 6. As figure 2, but for the second form of heating source (equation 19).

Table 3. Values of the parameter $Q_{\max}$ for $\beta = 0.2$, and the parameter C

Event (1973)	$f(\theta) = \exp[-(F(\theta)/\gamma)^2]$				$f(\theta) = \exp[-F(\theta)/\gamma]$	
	$\gamma = 1$		$\gamma = 0.5$		$\gamma = 1$	
	L	P	L	P	L	P
Parameter $Q_{\max}$ for $\beta = 0.2$						
Fare loop (June 15)	4.467(−3)*	4.170(−4)	6.724(−3)	5.308(−4)	5.226(−3)	4.846(−4)
Flare loop (Nov. 26)	4.695(−2)	2.998(−2)	7.051(−2)	3.805(−2)	5.493(−2)	3.481(−2)
Flare kernel (Aug. 9)	1.664(1)	1.316(1)	2.404(1)	1.607(1)	1.951(1)	1.520(1)
Flare kernel (Sep. 1)	5.672(1)	4.457(1)	8.294(1)	5.512(1)	6.648(1)	5.159(1)
Parameter C						
Flare loop (June 15)	7.947(28)	6.313(27)	7.808(28)	6.254(27)	7.956(28)	6.311(27)
Flare loop (Nov. 26)	7.053(29)	4.030(29)	6.912(29)	3.980(29)	7.062(29)	4.025(29)
Flare kernel (Aug. 9)	1.030(29)	8.101(28)	9.732(28)	7.770(28)	1.032(29)	8.043(28)
Flare kernel (Sep. 1)	8.450(28)	6.396(28)	8.071(28)	6.182(28)	8.467(28)	6.364(28)

*The numbers in the brackets are the powers of ten; *e.g.* 4.467(−3) = 4.467×10^{-3}.

5. Total amount of heating

To calculate the total amount of heating, we assume that the radius r of the flux tube at the apex of the event is related to the length l through $r = \delta l = 2\delta s_b$, where $\delta < 1$ is an adjustable parameter. Then, the cross-section at the apex of the flux tube is

$$A(0) = 4\pi\delta^2 s_b^2, \qquad \ldots(20)$$

and the total amount of heating is given by

$$Q_T = Q_{\max} \int_0^{\tau} (1 + t/\tau)^{-1}\, dt\; 2 \int_0^{s_b} f(s)\, A(0)\, A(0)\, ds$$

$$= 4.355\, Q_{\max}\, I_1(\gamma\theta_b)\, \tau R l^2\, \delta^2 = C\delta^2 \text{ (erg)}. \qquad \ldots(21)$$

By using $Q_{\max}$ corresponding to $\beta = 0.2$, the value of the factor C is calculated and is given in table 3. Now, one can easily calculate the total amount of heating for a given value of δ. Table 3 shows that the total amount of heating for the line-dipole geometry is larger than that for the point-dipole geometry. Further, the amount of heating is larger when the source is extended one (large value of γ). The heating for the second form of the source is found larger than that for the first form in the line-dipole geometry, while in the point-dipole geometry the reverse is true.

6. Final remarks

In the present investigation, the flare loop of 1973 June 15 which was not accounted for in the final calculations of Chandra & Narain (1982) has also been considered.

The results for this event may be little less reliable because the gravitational effect, which is neglected here, may have some contribution. Present investigation differs from the work of Chandra & Narain (1982) in three aspects shown in table 4. Consequently, the difference between the results of the two investigations cannot be assigned to a single particular aspect.

Table 4. Comparison of the present investigation with that of Chandra & Narain (1982)

	Present investigation	Chandra & Narain (1982)
1. Electron density	$n = n(s, t)$	$n = n(s)$
2. Pressure	$p = p_0$ (constant)	$p = p(t)$
3. Drift velocity	$v \neq 0$	$v = 0$

Acknowledgements

I am grateful to Dr S. M. Padhye for varifying the mathematical part of this work, and for his valuable suggestions. I am thankful for the nice hospitality extended by Professor W. H. Kegel. My sincere thanks are due to Dr M. C. Pande for his encouragement.

References

Antiochos, S. K. & Sturrock, P. A. (1976, 1978) *Solar Phys.* **49**, 359; *Ap. J.* **220**, 1137.
Chandra, S. & Narain, U. (1982) *Bull. Astr. Soc. India* **10**, 18.
Elwert, G. & Narain, U. (1980) *Bull. Astr. Soc. India* **8**, 21.
Narain, U. (1983) *Bull. Astr. Soc. India* **11**, 56.