The Motions of the Planets.

BY DR. D. N. MULLICK, B.A., F.R.S.E.

The ancient astronomers included under the name of "planets," or "wanderers" not only bodies like Mercury, Venus and the other planets known to them (excluding the Earth) but the Sun and the Moon as well. For to these astronomers, not only Mercury, Venus, Mars, Jupiter and Saturn, but also the Sun and the Moon appeared to move in somewhat erratic paths, as compared with the heavenly bodies in general. Very simple observations were, indeed, enough to convince them that, whereas the majority of the heavenly bodies—the stars—move in paths in the heavens which remained unaltered from day to day, all appearing to be chained together by an invisible bond, the paths of these wanderers baffled all attempts at bringing them under any coherent law. Yet we have now as complete a knowledge of these as we have almost of any natural phenomena. Very simple observations are enough to assure us that the stars move in fixed circles whose planes are parallel to each other and perpendicular to a certain fixed line in space, all completing a cycle in a fixed period of time, called the siderial day. If the motion observed be real, we must suppose that all the infinite number of heavenly bodies were connected together in some mysterious way. If, on the other hand, the motion is only apparent, it may be simply explained as being due to the rotation of the Earth about an axis, fixed in direction in space. The latter hypothesis giving a very simple account of a complex phenomenon, gradually gained acceptance, though it was only in the last century that positive proof was forthcoming that the Earth had actually this motion of rotation.

Proceeding now to the motion of the "planets," it is obvious that the motions they are observed to possess must be due partly to the motion of rotation of the Earth. In order to take account of this, all that is necessary to do is to observe the motion of these bodies relatively to the stars, *i.e.*, their motion among the stars.

When this is done, it is found that the Sun's path in the celestial vault (irrespective of the Earth's diurnal motion) lies on a great circle in a plane inclined to the equator (*i.e.*, plane perpendicular to the Earth's axis) and the Moon's path is also a circle in a different plane, the paths of the planets proper still presenting the most baffling complexity.

The motions of the Sun as thus observed may again be either real or apparent. Arguments, into which I do not wish to enter, lead, however, to the positive conclusion that the motion is apparent, that the Sun is really stationary or practically so and that the Earth is moving about the Sun. It follows thence that the Moon's motion cannot also be apparent but that she is really moving round the Earth.

Turning now to the planets, it is found that they appear sometimes to go forward (that is, in the same direction in which the Sun appears to move), sometimes to go backwards (in the opposite direction to that of the Sun), and sometimes to stand still. But all these complicated motions may be explained by supposing that they all move about the Sun in elliptic orbits, more or less inclined to the orbit of the Earth, that the Earth itself moves about the Sun in an elliptic orbit, and that the rates at which they severally move in their orbits are different, being connected, however, by a simple law [the third law of Kepler.]

On this simple scheme, we can not only explain completely the highly complicated motions of the planets, but the scheme itself is found to be deducible from the principle of universal gravitation.

Let us consider how this scheme can explain some of the peculiar features of planetary motion referred to above. We note in the first place that in accordance with Kepler's third law an inferior planet moves more rapidly round the Sun than the Earth. Therefore, when such a planet is between the Sun and the Earth, it will appear to move (say) from left to right (just as when two persons are going—one at 5 miles an hour and the other at 7 miles an hour, the latter will obviously go forward), while at the same time, the Sun will appear to go from right to left. On the other hand, when the Sun is between the planet and the Earth, both the planet and the Sun will *appear* to move from right to left. Therefore, also, while the planet is changing its direction, it will appear to be stationary.

The motions of the planets obey three laws, discovered by Kepler, by observation alone.

The first law may be thus stated: If a planet goes from P to T, in one day and from T to U the next day, then S being the Sun, the area S P T is equal to the area S T U. Now, if the area S P T is small, it is proportional to the product of SP^2 and the angle S P T. But S P, or the distance of the Earth from the Sun, varies inversely as the angular radius of the Sun, and the angle S P T is the change in the longitude of the Sun. Hence by observing the angular diameter and the change in longitude from day to day, the law can be verified in the *case of the Earth.* In the case of any other planet the reduction of observations will require a little more trigonometry, but the method is similar.

The second law states that these orbits are ellipses.

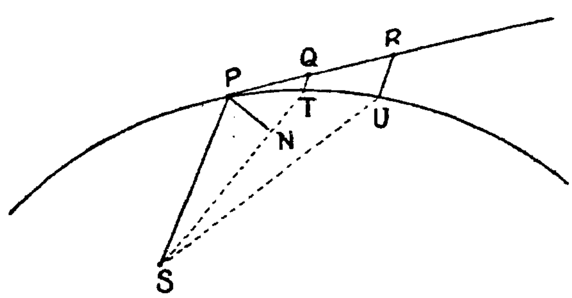
We have seen how S P varies. Representing it on any assumed scale and representing also the angles (changes in longitude) made by successive positions of S P, we can plot the curve traced out by P, if a sufficiently large number of observations shall have been made. This was what Kepler actually did and the paths were found to be ellipses.

The third law states that the squares of the periodic times vary as the cubes of the major axes or the greatest diameters of the elliptic orbits described by the planets.

Since the orbits are known, all that has to be done is to observe the time taken by the planets to go round the Sun. For this we notice that if the Earth goes round the Sun in y days it goes $\frac{1}{y}$ of the total circuit in one day; similarly, the planet goes $\frac{1}{p}$ of the circuit in one day if its periodic time is p, then it goes ahead of the Earth by $\left(\frac{1}{p}-\frac{1}{y}\right)$ of the circuit in one day. When it has gone ahead by a whole circuit, it will be in the same relative position to the Sun and the Earth as at the beginning. The interval therefore is $\frac{1}{\frac{1}{p}-\frac{1}{y}}=\frac{py.}{y-p.}$ This interval can be observed—for it is the interval between, say, two successive conjunctions, and hence p can be determined, since y, the number of days in the year, is known.

These laws seem, at first sight, to be too complicated to be of much use. It is remarkable, however, that they are all consequences of a single law—the law of universal gravitation. And we reach this grand generalisation—that all these complicated motions of the planets are due to an attraction (directed to the Sun), varying according to the law of inverse square of the distance or the force under which a planet P moves in its orbit is μ/SP^2 where S is the position of the Sun and μ, a constant (nearly) for all the planets.

Let P be the position of planet at any time moving in the



direction P Q. If there had been no action of the Sun, the planet would have continued to move along P Q, and Q, R would have been its positions at t, 2t seconds after, *t* being small.

On account of the attraction of the Sun, however, directed along P S, it occupies positions marked T, U instead. In other words, the planet is displaced through Q T, R U, in the times t and 2t seconds. Since these displacements are due to a force acting along P S, very nearly, Q T, R U are parallel to P S, and the areas P S T, P S U are very nearly triangles. We have, accordingly, by Euclid, the triangle P Q S.

=the triangle P S T.
=Q S R.
=T S U.

That is, the areas P S T, T S U, are equal or the area described by the planet is proportional to time. Thus the first law of Kepler is seen to flow from the supposition that the motion of a planet is due to an attractive force directed to the Sun.

Again, if V is the velocity of the planet at P, P Q=Vt. and if p is the perpendicular from S on P^2Q.

We have P Q.p=Vt. p.

Which we have just proved proportional to *t*.

i.e., Vp=constant.

Calling this const. *h*, we have $V^2=\frac{h^2}{p^2}$.

But the kinetic energy acquired by a particle is equal to the work done by the force producing the motion.

Now as the planet is displaced from P to T, T being a neighbouring point, the displacement may be considered to be made up of P N (perp. to S T) and T N. The first being perpendicular to the force along S T, produces no work. The work is, therefore, measured by F. T N, where F is the force at T.

If, then, U is the velocity at T, V being the velocity at a fixed point P,

$$\tfrac{1}{2}(U^2-V^2)=-\frac{\mu}{ST^2}.\quad T\,N.=\mu\frac{SP-ST}{ST.\,SP.}\text{ very nearly,}$$

$$=\frac{\mu}{S\,T}-\frac{\mu}{SP.}\text{ very nearly;}$$

$$\text{or } U^2=\frac{2\mu}{S\,T}+V^2-\frac{2\mu}{SP.}$$

$$\text{But } U^2=\frac{h^2}{P^2}$$

$$\therefore\ \frac{h^2}{p^2}=\frac{2\mu}{D}+V^2-\frac{2\mu}{SP,}$$

where D = distance of the planet from the Sun at any point and p = perpendicular from the Sun on the direction of motion at that point.

But in a conic section,

$\frac{2}{D}$ is greater than, equal to, or less than $\frac{l}{p^2}$, where l is the semilatus rectum, according as it is an ellipse, a parabola or a hyperbola.

Hence, the path of a planet is a conic section where $h^2 = \mu l$ and it is an ellipse provided it is found to possess a suitable velocity at any particular point, that is, if V^2 is *less* than $\frac{2\mu}{SP}$, V being the velocity at a point P. As the body has always the same velocity, whenever it comes to the same position, it must have started at some unknown point of time with a certain definite velocity which was suitable for the description of the elliptic path.

The third law follows from the relations that have been obtained already, but it is unnecessary to trouble you with the analysis.

Note on a Brickwork Stage for a Siderostat.

BY W. HANLEY.

In deicding on the form of the stage for the siderostat we had to consider the ventilation of the adjacent building, and on this account it was settled to adopt the form sometimes used on railways for tank stages. This type of staging is economical in cost while providing ample base area.

In Bengal the pressure permissible on the ground may be taken as ½ to ¾ of a ton per sq. ft., but in order to avoid as far as possible any trouble from settlement, the pressure in the case of this stage was reduced to about ¼ ton per sq. ft. by floating the structure on a large base of concrete about 1 ft. thick.

The stage consists of 2 parallel rows of 3 columns of brickwork 2′—1″ × 1′—8″ each, tied at the bottom by segmental inverts so as to ensure even distribution of the weight, and at top by semicircular arches. It is hoped that this method of construction will prevent vibration. If, however, vibration