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2004 Phys. Scr. 2004 149

(http://iopscience.iop.org/1402-4896/2004/T113/037)

Physica Scripta. Vol. T113, 149–150, 2004

Dust-Alfvénic Shocklets in Dusty Plasmas

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Received April 27, 2004

PACS number (s): 52.27.Lw, 52.35.Bj, 52.35.Tc

Abstract

We show that the Hall dust magnetohydrodynamic equations admit a non-stationary dust-Alfvénic shock solution in dusty plasmas. The dust-Alfvénic shocklet (DAS) occurs on a long temporal (dust gyroperiod) and spatial (dust skin depth) scales. The present result can have relevance to acceleration of charged grains by the DAS in space dusty plasmas.

Space dusty plasmas are usually embedded in an external magnetic field. Dusty magnetoplasmas support a great variety of low-frequency electrostatic and electromagnetic waves [1–6]. The latter include shear and compressional dust Alfvén waves whose linear properties have been discussed in detail [5, 6].

Our objective here is to discuss the nonlinear properties of perpendicularly propagating (with respect to the magnetic field direction) compressional dust Alfvén waves in complex (dusty) plasmas. We consider an electron-ion-dust (EID) plasma in an external magnetic field $B_0\hat{z}$, where B_0 is the magnitude of the external magnetic field and $\hat{z}$ is the unit vector along the z axis. At equilibrium, we have $en_e - q_d n_{d0} = en_{i0}$, where e is the magnitude of the electron charge, n_{j0} is the unperturbed particle number density of the particle species j (j equals e for electrons, i for ions, and d for dust grains), and q_d is the dust charge. For negatively (positively) charged dust grains, we have $q_d = -eZ_d$ (eZ_d), where Z_d is the number of charges residing on the dust grain surface. The electric and magnetic fields, $\boldsymbol{E}$ and $\boldsymbol{B}$, in an EID plasma associated with the dust Alfvén waves are governed by Faraday's law

$$\frac{\partial \boldsymbol{B}}{\partial t} = -c\nabla \times \boldsymbol{E}, \tag{1}$$

and Ampère's law

$$\nabla \times \boldsymbol{B} = \frac{4\pi e}{c}\boldsymbol{J}, \tag{2}$$

where $\boldsymbol{J}_p = en_i\boldsymbol{v}_i - en_e\boldsymbol{v}_e + q_d n_d \boldsymbol{v}_d$ is the plasma current density, n_j is the total density, $\boldsymbol{v}_j$ is the fluid velocity, and c is the speed of light in vacuum. We have neglected the displacement current in (2), since the phase velocity of the dust Alfvén waves is much smaller than the speed of light.

The electric field in our cold dusty plasma is obtained by adding the inertialess electron and ion momentum equations, and using the equilibrium quasi-neutrality condition. The result is

$$\boldsymbol{E} = -\frac{\boldsymbol{v}_d \times \boldsymbol{B}}{c} + \frac{\boldsymbol{J}_p \times \boldsymbol{B}}{n_d q_d}, \tag{3}$$

which can be inserted into (1) to obtain

$$\partial_t \boldsymbol{B} = \nabla \times \left[\left(\boldsymbol{v}_d - \frac{c\nabla \times \boldsymbol{B}}{4\pi n_d q_d}\right) \times \boldsymbol{B}\right]. \tag{4}$$

We note that the second term in the parenthesis on the right-hand side of (4) represents the dust Hall effect in complex plasmas.

We now need an equation relating the dust fluid velocity, the dust number density, and the magnetic field. For this purpose, we substitute (3) into the dust momentum equation and use (2) to obtain

$$\partial_t \boldsymbol{v}_d + \boldsymbol{v}_d \cdot \nabla \boldsymbol{v}_d = \frac{(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}}{4\pi\rho_d}, \tag{5}$$

where $\rho_d = n_d m_d$ is the dust mass density, which is determined from the dust continuity equation

$$\partial_t \rho_d + \nabla \cdot (\rho_d \boldsymbol{v}_d) = 0, \tag{6}$$

where m_d is the dust mass.

Equations (4), (5) and (6) govern the dynamics of three-dimensional nonlinear dust Alfvén waves (DAWs). These equations are useful for studying the turbulence properties of fully developed dust Alfvén wave turbulence as well as coherent nonlinear structures in dusty magnetofluids. In the following, we focus on the particular case in which $\boldsymbol{v}_d = \hat{\boldsymbol{x}}u(x,t)$, $\nabla = \hat{\boldsymbol{x}}\partial_x$ and $\boldsymbol{B} = \hat{\boldsymbol{z}}B_z(x,t)$, where $\hat{\boldsymbol{x}}$ is the unit vector along the x axis. Equations (4)–(6) can then be put in the form

$$\partial_\tau H + U\partial_\xi H + H\partial_\xi U = 0, \tag{7}$$

$$\partial_\tau U + U\partial_\xi U + \frac{1}{2\rho}\partial_\xi H^2 = 0, \tag{8}$$

and

$$\partial_t \rho + U\partial_\xi \rho + \rho\partial_\xi U = 0, \tag{9}$$

where $H = B_z/B_0$, $U = u/V_{dA}$, $\rho = \rho_d/\rho_0$, $\rho_0 = m_d n_{d0}$, $\tau = \omega_{cd}t$ and $\xi = x/\lambda_d$. Here $V_{dA} = B_0/\sqrt{4\pi\rho_0}$ is the dust Alfvén speed, $\omega_{cd} = |q_d|B_0/m_d c$ is the dust gyrofrequency, $\lambda_d = c/\omega_{pd}$ is the dust skin depth, and $\omega_{pd} = (4\pi q_d^2 n_{d0}/m_d)^{1/2}$ is the dust plasma frequency, A comparison of (7) and (9) shows that $H = \rho$, which is the frozen-field relation for a magnetized dusty plasma. Hence, Eq. (8) becomes

$$\partial_\tau U + U\partial_\xi U + \partial_\xi H = 0, \tag{10}$$

Our coupled equations (7) and (10) are similar to (2b) and (2a) of Stenflo *et al.* [7] who investigated the shock wave formation in a magnetized electron-ion plasma without dust.

Following Stenflo *et al.* [7], we now discuss possible non-stationary solutions of (7) and (10). By comparing the latter equations, we have

$$H = \left(D + \frac{1}{2}U\right)^2, \tag{11}$$

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where D is an arbitrary constant. Accordingly, (10) can be written as

$$\partial_\tau U + D\partial_\xi U + 3\partial_\xi U^2 = 0, \tag{12}$$

which is an inviscid Burgers equation. A possible solution of (12) is [7]

$$U(\xi,\tau) = U_0 - U(\xi,\tau)\left[\xi - \left(\frac{3U(\xi,\tau)}{2} + 1\right)\tau\right]^2, \tag{13}$$

where U_0 is a constant. Equation (13) describes the nonlinear evolution of the initial dust velocity perturbation

$$U(\xi,\tau=0) = \frac{U_0}{1+\xi^2}. \tag{14}$$

It turns out that $U(\xi,\tau)$ develops into a shock whose profile has been depicted in Ref. [7].

In summary, we have considered the nonlinear propagation of compressive dust Alfvén waves in a uniform dusty magnetoplasma. It has been shown that the nonlinear dust Alfvén wave dynamics is governed by the dust Hall-MHD equations. The latter admit non-stationary dust Alfvénic shock structures across the external magnetic field direction. Dust Alfvénic shocklets represent a new class of discontinuities in dusty plasmas and they can serve the purpose of accelerating charges dust grains in space dusty plasmas.

Acknowledgments

The authors would like to dedicate this paper to Lennart Stenflo on the occasion of his 65th birthday.

This work was partially supported by the European Commission (Brussels, Belgium) through contract No. HPRN-CT-2000-00140 for carrying out the task of the Human Potential Research Training Networks “Complex Plasmas”, as well as by the Deutsche Forschungsgemeinschaft (Bonn, Germany) through the Sonderforschungsbereich 591 and the International Space Science Institute at Bern.

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