

## Noncosmological redshifts\*

Jayant V. Narlikar *Tata Institute of Fundamental Research,  
Homi Bhabha Road, Bombay 400 005*

### 1. Introduction

It is hardly necessary for me to explain the term 'redshift' to this assembly of astronomers. Nor is it necessary to go into details of the different types of redshifts known to the astronomers today. Nevertheless I shall spend a few minutes on these topics if only to fix my notation.

Suppose that  $\lambda$  denotes the observed wavelength of a spectral line in the spectrum of an astronomical object, and  $\lambda_0$ , the wavelength that the line has in a typical spectrum generated in the terrestrial laboratory now. Then the redshift  $z$  of the observed line is given by

$$z = \frac{\lambda - \lambda_0}{\lambda_0}. \quad \dots(1)$$

If  $z$  is positive (as is usually the case), the spectral line has shifted towards the longer wavelength end of the spectrum which, in the visual range, corresponds to the 'red' end. I shall refer to negative  $z$  as denoting blueshift.

The cause of redshift best known to the laboratory physicist is the Doppler effect described in figure 1. In the notation of the figure the redshift measured by the observer  $O$  in the spectrum of the source  $S$  is given by

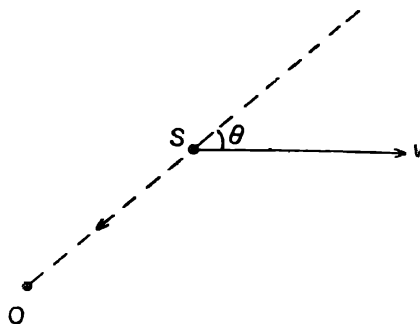


Figure 1. The source  $S$  is moving with a velocity  $v$  whose direction makes an angle  $\theta$  with the line of sight from  $O$  to  $S$ .

\*Presidential address delivered at the ninth meeting of the Astronomical Society of India 1983 November at Hyderabad.

$$z = \left(1 + \frac{v}{c} \cos \theta\right) \left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1, \quad \dots(2)$$

where  $c$  = speed of light. Notice that the Doppler effect can also lead to blueshifts for

$$\frac{v}{c} \cos \theta < \left(1 - \frac{v^2}{c^2}\right)^{1/2} - 1. \quad \dots(3)$$

Figure 2 describes the phenomenon of gravitational redshift which was first predicted by Einstein's general theory of relativity. It arises when light leaves the source  $S$  situated in a strong gravitational field and reaches the observer  $O$  located in a weak field. If the spacetime metric is expressible in a manifestly static form with  $S$  and  $O$  at rest in it :

$$ds^2 = g_{00}dt^2 + 2g_{0\mu}dt dx^\mu + g_{\mu\nu}dx^\mu dx^\nu \quad (\mu, \nu = 1, 2, 3), \quad \dots(4)$$

then

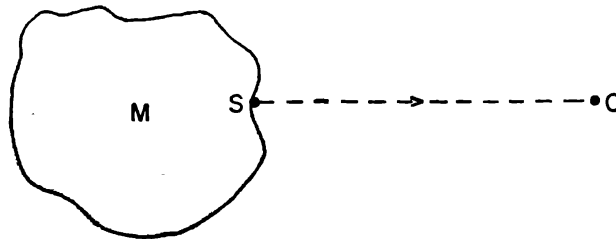
$$z = \left(\frac{g_{00}(O)}{g_{00}(S)}\right)^{1/2} - 1. \quad \dots(5)$$

The redshifts discovered by Hubble & Humason in the 1920s, following the earlier work of Slipher, are however of an altogether different class. The sources, in this case the distant galaxies, are believed to be taking part in the expansion of the universe. Figure 3 shows a galaxy  $G$  emitting light at epoch  $t_1$  which arrives at the observer (in our Galaxy) at the present epoch  $t_0$ . Denoting by  $S(t)$  the scale factor of expansion of the universe, we have  $S(t_1) < S(t_0)$  and the redshift is given by

$$z = \frac{S(t_0)}{S(t_1)} - 1. \quad \dots(6)$$

Because of its relation to cosmology, this redshift is called the cosmological redshift.

Although large Doppler shifts are commonly found in the high energy particles generated in man-made accelerators, the astronomers to date do not have cases of large red or blue shifts ( $|z| \sim 1$ ) in heavenly bodies. Stars in our Galaxy have speeds  $v \ll c$  and  $z \lesssim 0.001$ . Likewise examples of gravitational redshifts, found in white dwarf stars, do not exhibit values as high as  $10^{-3}$ . By contrast, the extension of extragalactic astronomy in the post-Hubble era has shown a large number of galaxies with  $z > 0.1$ . Galaxies with  $z \sim 1$  are also known.



**Figure 2.** A light ray emitted by source  $S$  in the vicinity of the massive object  $M$  is redshifted when received by  $O$  located far away from  $O$ .

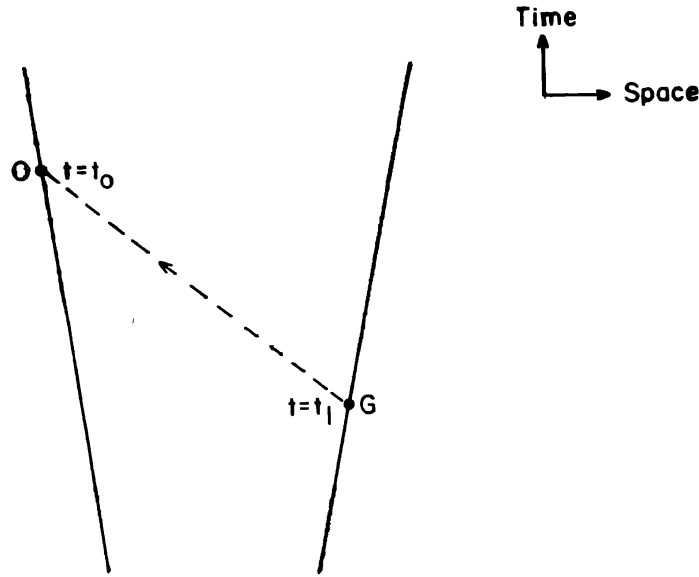


Figure 3. Spacetime diagram showing light ray leaving galaxy G at  $t_1$  and reaching observer O at  $t_0$ . The universe has expanded during the interval  $(t_1, t_0)$ .

In 1963 two objects with starlike images but identified with radio sources 3C 273 and 3C 48 were found to have  $z = 0.158$  and  $z = 0.37$  respectively. Now named quasistellar objects (QSOs) or quasars, these objects turned out to belong to a new class of astronomical bodies of which around 2000 are now known and catalogued. Starlike appearance and high redshifts (with  $z$  now going up to 3.78) are the hallmarks of quasars, although only a small fraction ( $< 10\%$ ) of the total quasar population is believed to be radio sources.

The redshifts of quasars could in principle be due to any of the three causes, but the fact that in 1963 only the cosmological redshift hypothesis had known examples of large  $z$  led to the general belief that quasar redshifts are also cosmological in origin. One consequence of this belief is that the quasars are necessarily very distant objects. For, in a typical Friedmann cosmology we can express the luminosity distance  $D$  as a function of redshift in the following way (Narlikar 1983) :

$$D = \frac{c}{H_0} f(z). \tag{7}$$

Here  $H_0$  is the present value of Hubble's constant and for small  $z$ ,  $f(z) \sim z$ . For  $H_0 = 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , 3C 273 is at a distance of  $\sim 1600$  million light years from us. The most remote known quasar would similarly be as far away as  $\sim 40,000$  million light years.

If the above cosmological hypothesis (CH in brief) is valid, the quasars as a class are the most remote objects in the universe and hence also the probes of the remote past of the universe. Because of their potential value as probes of how the universe has evolved over the last  $\sim 10,000$  million years, it is necessary to subject the cosmological hypothesis to a critical examination. Should it turn out that this hypothesis is not adequate to account for the observed features of quasar redshifts it may be necessary to examine other alternatives. We shall refer to all such

alternatives, including the Doppler and gravitational options as 'noncosmological' redshifts.

## 2. The evidence for cosmological redshifts

Most cosmologists believe that Hubble's law holds for galaxies. Figure 4 shows an example where the logarithm of redshift is plotted against the apparent magnitude of a sample of elliptical galaxies. Although there is some scatter, the linear trend is evident and conforms to relation (7). The scatter increases if we consider galaxies of all types. Nevertheless a definite trend can still be spotted wherein the magnitude increases with redshift.

A similar result for quasars would naturally confirm the cosmological hypothesis. However, attempts to establish the Hubble relation for quasars have so far proved inconclusive. The  $(m, z)$  plot for all quasars (see figure 5) is a complete scatter diagram. This could either mean that a wide dispersion in absolute luminosities of quasars leads to the scatter around the Hubble line or that the redshift and distance are totally uncorrelated. Some supporters of the cosmological hypothesis have tried to show that suitably chosen samples of quasars do yield a Hubble line but

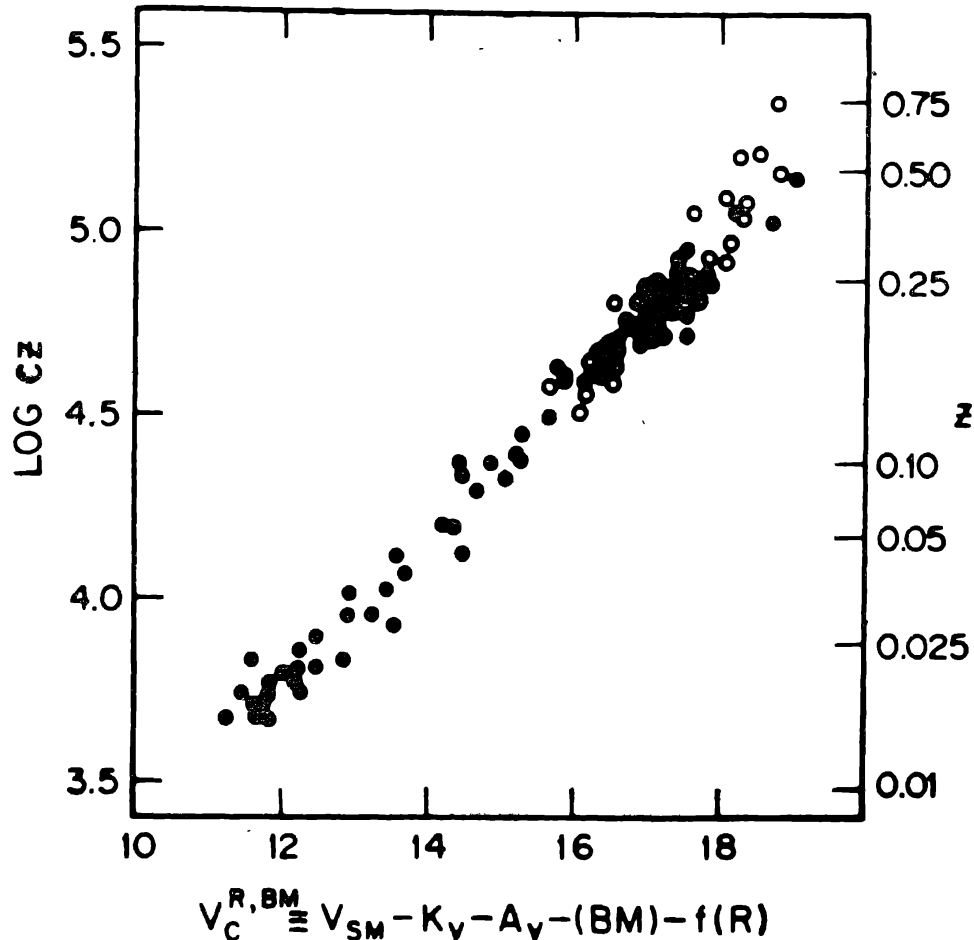


Figure 4. The redshift magnitude relation for galaxies which are the brightest members of their clusters, based on figure 3 of Kristian *et al.* (1978 : *Ap. J.* 221, 383.)

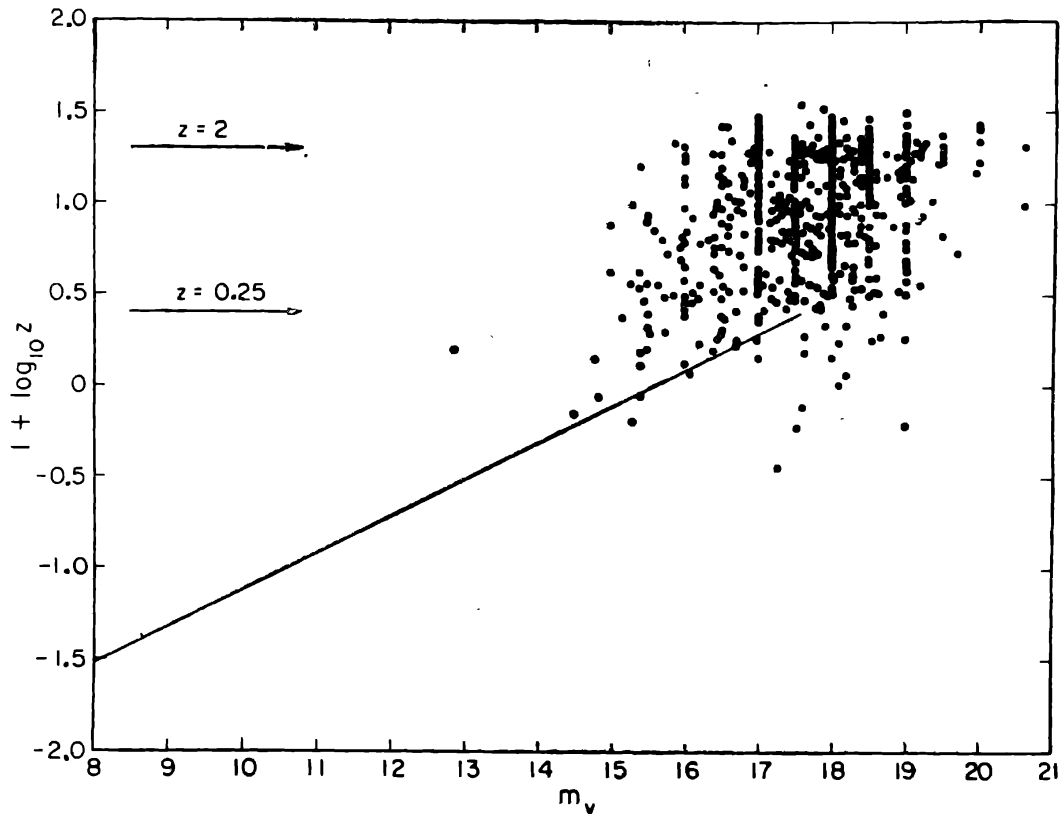


Figure 5. The  $m$ - $z$  plot of about 1500 quasars. The bunching round integral magnitudes is because the magnitude estimates are approximate.

these attempts are not very convincing (Hoyle & Burbidge 1966a; Bahcall & Hills 1973; Burbidge & O'Dell 1973).

The latter alternative implies that a substantial part of the quasar redshift is non-cosmological. However, before arriving at such a conclusion we should examine other evidence too. Table 1 lists the evidence in favour of cosmological redshifts. For reasons of brevity I cannot go into details of their discussion.

Table 1. Evidence consistent with cosmological redshifts of quasars (For details see Narlikar 1983)

No.	Evidence	Comment
1.	Historical precedent	Large redshifts known for galaxies ( $z \lesssim 1$ ).
2.	Hubble's law	Large scatter in the Hubble diagram but the believer can 'see' Hubble's law in the midst of the noise.
3.	Number counts	$\langle V/V_m \rangle$ significantly greater than 0.5 suggesting strong evolutionary effects.
4.	Angular size vs redshift	Largest angular size in any redshift bin decreases as $z^{-1}$ . Strong evolution needed to understand this effect.
5.	Absorption line systems	Many quasar spectra show absorption line systems which can be attributed to absorption by intervening galaxies. However, the probability of galaxies along line of sight is low unless their absorbing cross section is considerably higher than their observed sizes.
6.	Gravitational lens	Two or three cases of close pairs (one triplet) of nearly identical quasars suggest multiple images of a single quasar by intervening galaxy.
7.	Morphology	Quasars and nuclei of active galaxies like Seyferts show a continuity of physical properties suggesting a possible evolutionary sequence.
8.	Quasar-galaxy associations	Low redshift quasars found very close to galaxies of nearly the same redshifts.

It should be mentioned that most of the items in table 1 relate to evidence which is *consistent* with the cosmological hypothesis, but this interpretation cannot be considered unique or compelling, with probably one exception. The exception relates to the work of Stockton (1978). In a careful survey of a small area of the sky Stockton finds quasars very close to galaxies with almost identical redshifts. Since, as I have said earlier, the galaxy redshift is cosmological, so must be the quasar redshift.

This is not, however, the end of the argument and I shall return to this point later.

### 3. Evidence for noncosmological redshifts

I have summarized in table 2 the arguments casting doubts on the cosmological interpretation. For details see Narlikar (1983). Again, I shall not go through each argument here but single out the work on groupings of quasars and galaxies.

**Table 2.** Evidence posing difficulties for the cosmological hypothesis (For details see Narlikar 1983)

No.	Evidence	Comment
1.	Hubble's law	The $m-z$ diagram poses difficulties of reconciling the Hubble's law with quasar data.
2.	Number counts	$\langle V/V_m \rangle$ does not differ significantly from 0.5 if quasars are considered local.
3.	Absorption line systems	If these are produced by intervening galaxies why are there a few cases of absorption redshift > emission redshift ?
4.	Variability	Short term variability poses problems of energy generating mechanisms in quasars. These problems are alleviated if quasars are local.
5.	Superluminal separation	VLBI observations show quasar components separating from each other at speeds several times the speed of light. Contrived scenarios are needed to explain these cases unless quasars are local.
6.	Peculiarities of redshift-distributions	Some analyses have revealed periodicities in redshift distributions of quasars. These are hard to explain on the cosmological hypothesis. Supporters of the hypothesis doubt the validity of the above result.
7.	Anomalous redshifts	Quasars of high redshifts found very close to galaxies of low redshifts. This effect is discussed in the text.

The astronomer cannot directly measure distances of extragalactic objects. He can accurately measure their directions. Thus two objects A and B (see figure 6) are seen by O as projected on the celestial sphere at A' and B'. If he finds A' and B' very close to each other, can he conclude that A and B are also very close to each other ?

Recall that Stockton's argument in favour of cosmological redshifts depends on the answer to above question being 'yes'. Although the astronomer has no cast-iron proof for this conclusion the answer is made plausible by statistical arguments. If, as is the case, the quasar population on the sky appears to be fairly sparse, the probability of finding the projection A' of quasar A close enough to the projection B' of galaxy B is very small, unless A and B are physically close to each other. This was the basis of Stockton's conclusion.

However, this argument has proved to be a double-edged sword. Suppose you find quasar A projected close enough to the projection of galaxy B but discover that the redshifts  $z_A, z_B$  of the two differ substantially. By the law that what is sauce for the goose is sauce for the gander we have to conclude that A and B are

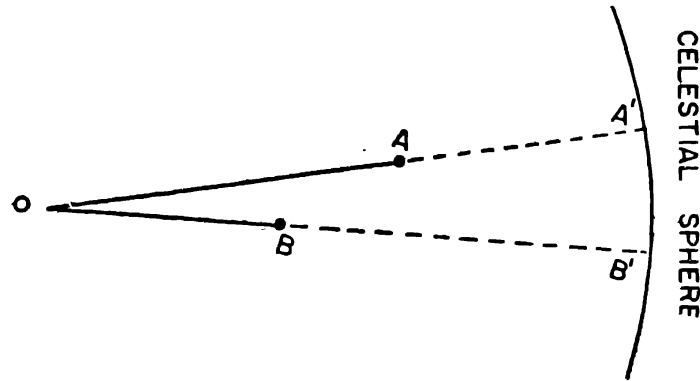


Figure 6. If the directions OA and OB are very close to each other, A, B are projected on the celestial sphere at A', B', very close to each other.

near each other despite their different redshifts. Thus Hubble's law must break down for one of the two (if not for both). Since galaxy redshifts are believed to be cosmological we have to conclude that not all of  $z_A$  is cosmological. The non-cosmological part  $z'_A$  of A's redshift is given by

$$(1 + z'_A) = \frac{1 + z_A}{1 + z_B} \quad \dots(8)$$

Arguments like these have been given by several observers but the most persistent advocate for noncosmological redshifts has been Chip Arp. Arp has demonstrated (either alone or in collaboration with others) several examples of the following types (see figures 7-12) :

(i) abnormal concentration of high redshift quasars near low-redshift (NGC) galaxies (Arp 1981); (ii) alignments of quasars across galaxies with discrepant redshifts (Arp *et al.* 1979); (iii) exact linearly aligned triplets of quasars of different redshifts (Arp & Hazard 1980); (iv) quasar concentration in jet-like distribution emanating from low redshift galaxies (Arp 1983a); and (v) low-redshift galaxies connected to companion galaxies of higher redshifts by filamentary structures (Arp 1983b).

The last type of evidence hits at the very base of Hubble's law since it calls into question the validity of the law for companion galaxies.

When confronted with such data the proponents of cosmological redshifts always seek a way out through statistics. They question Arp's selection criteria and argue that what he calls as grouping or association is simply chance projection effects. In other words, in terms of figure 6, although A' and B' are close together A and B are not. How far such a defence can be pushed depends on individual prejudices but I personally feel that the 'chance projection' hypothesis is becoming hard pressed under Arp's evidence.

In a recent interview to *Sky and Telescope* (see Smith 1983) Martin Rees likened doubting 'cosmological redshift' to doubting 'special relativity'. I think the comparison is unfair. Special relativity has been amply confirmed in laboratory experiments and there are no outstanding observational/experimental anomalies confronting the theory. The few who still doubt its validity do so on philosophical/conceptual/aesthetic grounds. By contrast there is plenty of observational evidence

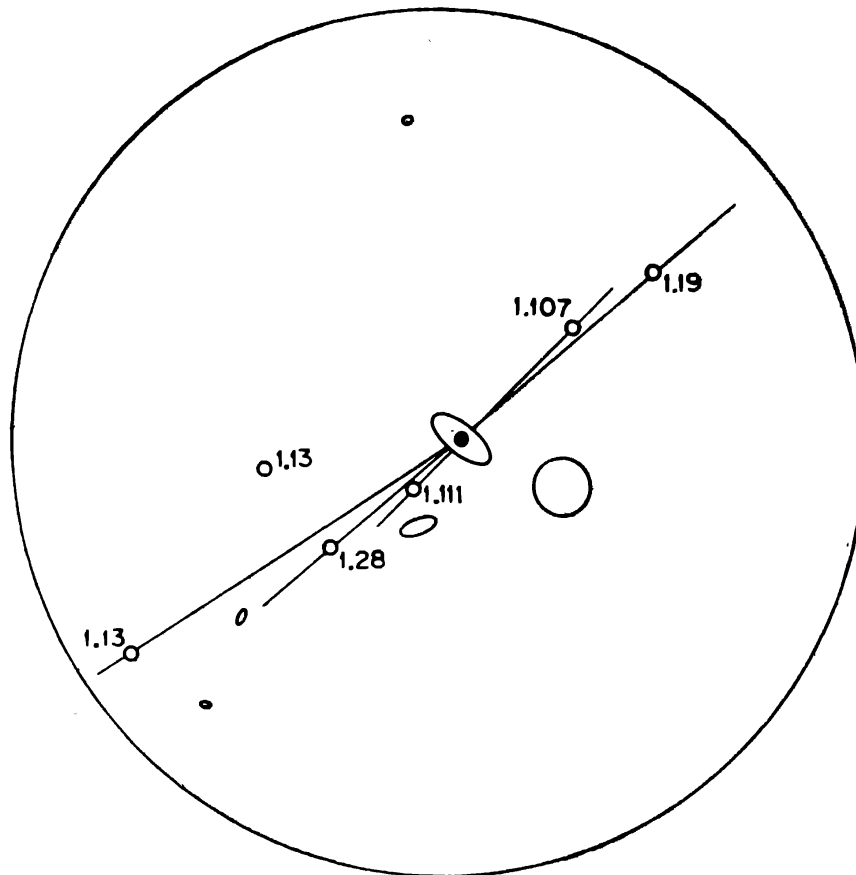


Figure 7. The quasars found by Arp (1979 *Ap. J.* 229, 489) in the neighbourhood of the galaxy NGC 3384. The quasar redshifts are given at their location. Are the alignments and the concentration of quasars pure chance effects ?

questioning Hubble's law for quasars and the awkward cases have not been explained away except through the steadily shrinking loophole of 'chance projection'.

In the remaining part of this talk I shall briefly review three alternative of non-cosmological nature for understanding the redshifts of quasars.

#### 4. The Doppler option

Terrell (1964) and Hoyle & Burbidge (1966b) proposed scenarios in which bulk of the quasar redshift was explained as Doppler effect. In Terrell's model quasars were proposed as stars ejected from our Galaxy.

Although the galactic centre has a strong mass concentration it does not show evidence of the kind of high energy activity that would eject quasars at relativistic speeds. In any case activity of that order would have disrupted stellar orbits in the Galaxy.

In the Hoyle-Burbidge theory quasar ejection was associated with active galactic nuclei. This got round the objection to Terrell's hypothesis but led to another difficulty which I shall refer to as the blueshift catastrophe. As shown by Strittmatter (1967) the number of blueshifted quasars was expected to be predominantly higher than the number of redshifted quasars in any flux-limited sample. Since no quasars have so far been seen with blueshifts this theory did not gain credibility.



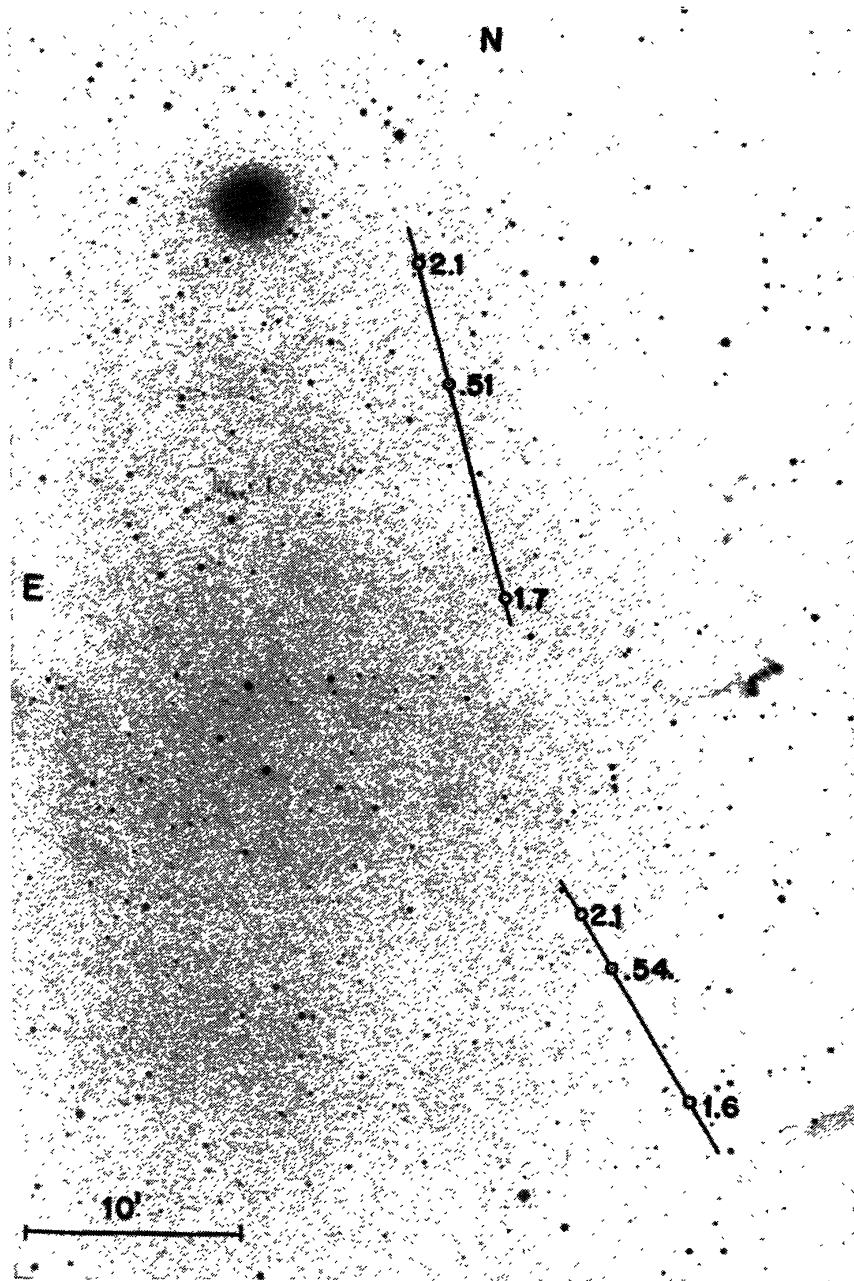
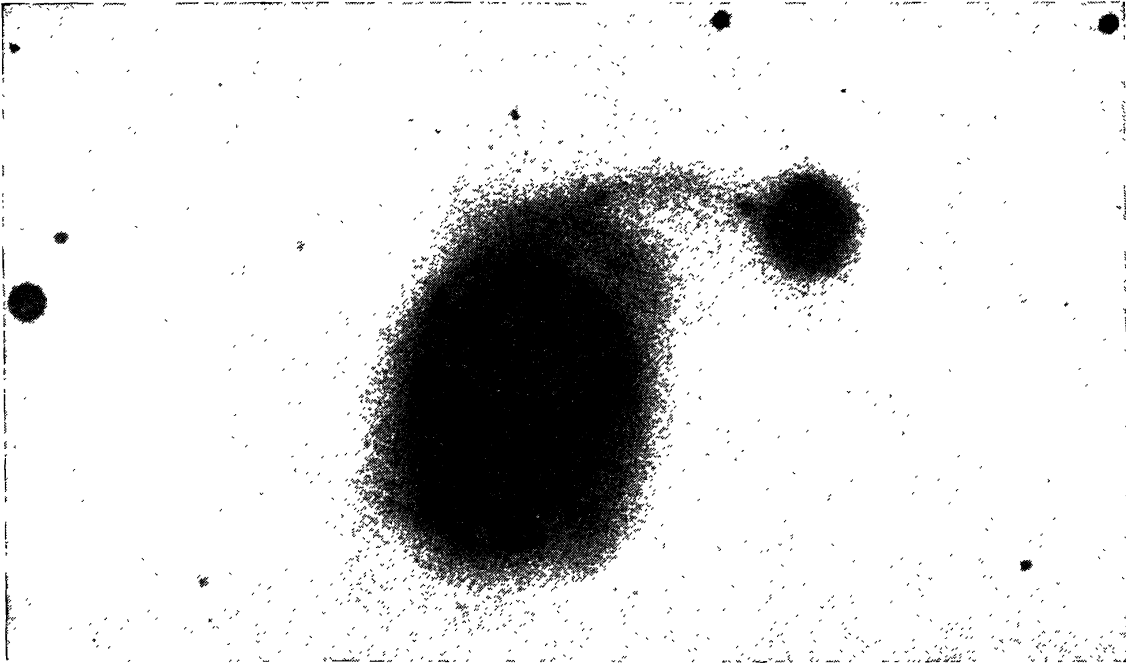


Figure 8. Two linearly aligned triplets of quasars with redshifts given at their location (Arp & Hazard 1980).

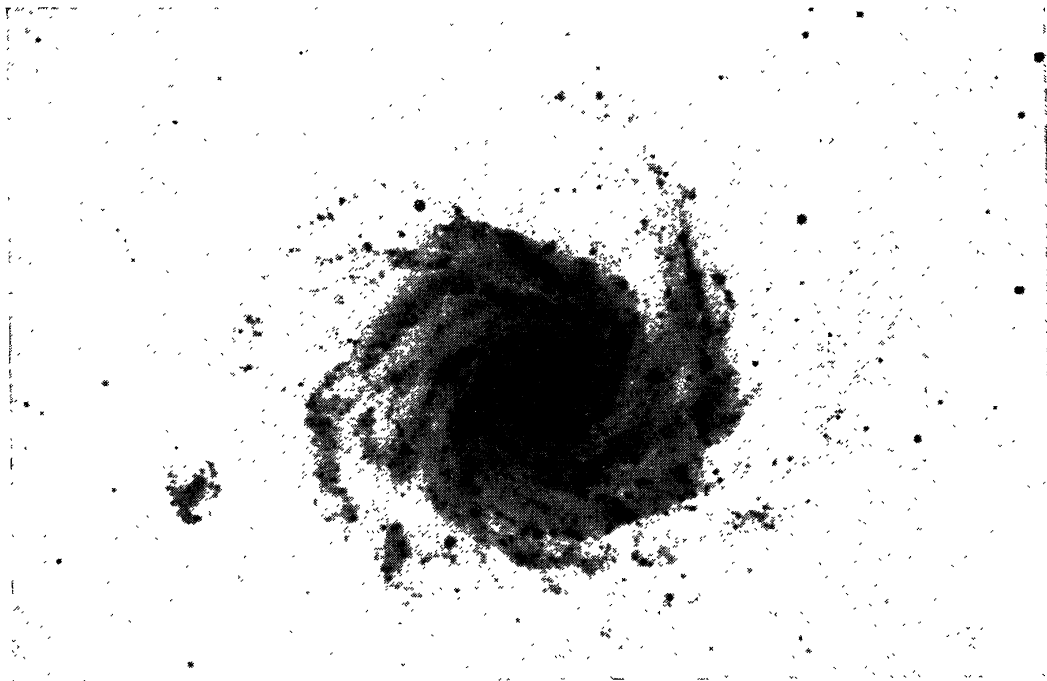
It was felt by Strittmatter (1967) and emphasized more recently by Hoyle (1980) that the blueshift catastrophe can be averted if Doppler quasars selectively emit all their radiation in backward direction. As shown by Hoyle, if a quasar moving with speed  $v$  relative to the cosmological substratum emits all its radiation in a cone of semivertical angle

$$\theta = \cos^{-1} \frac{c}{v} \left\{ 1 - \left( 1 - \frac{v^2}{c^2} \right)^{\frac{1}{2}} \right\} \quad \dots(9)$$

as measured in its rest frame, *no* observer at rest in the cosmological substratum will see it blueshifted.



**Figure 9.** The large galaxy NGC 7603 appears to be connected by a filament to a small companion. The redshifts of the two are 0.0290 and 0.0563 (Arp 1983).



**Figure 10.** The spiral galaxy NGC 1232 and its small companion NGC 1232A (bottom left) appear to be related. But the latter has an excess redshift of 0.016 (Arp 1983).

Why should a high speed quasar selectively emit its radiation in a backward cone? Strittmatter and Hoyle have not given any *astrophysical* reason for such emission. This problem was discussed recently by Narlikar & Subramanian (1983).

We found that the explanation could lie in an adaptation of the twin exhaust model of Blandford & Rees (1974) to fast moving quasars.

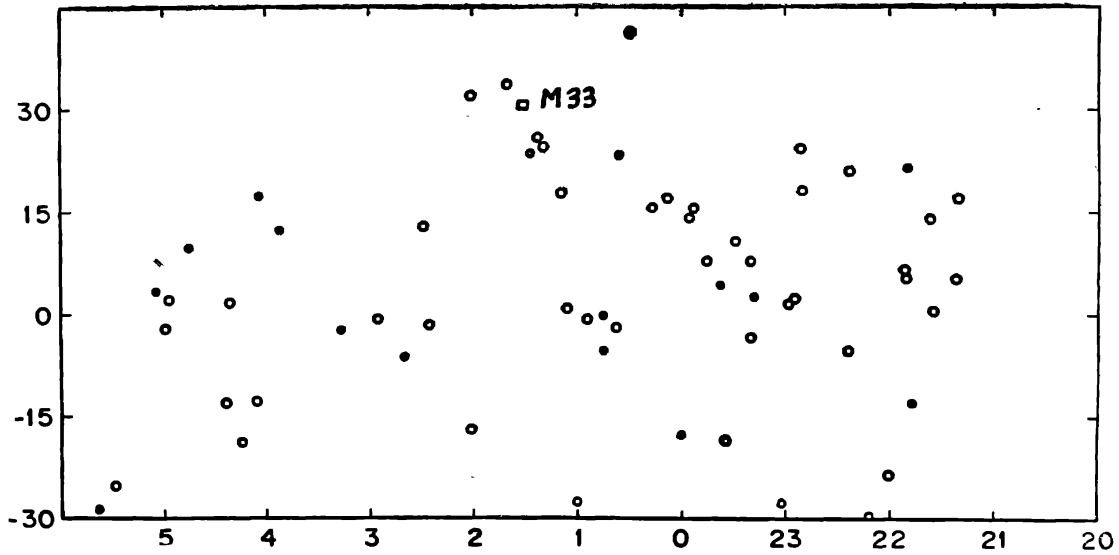


Figure 11. A plot of all Parkes and 3 CR radio quasars from the Hewitt-Burbidge catalogue, which have  $1.4 < z < 2.8$  and  $V < 20^m$ . Small filled circles represent quasars with  $V < 19^m$ . There appears to be a jet-like concentration of quasars from the square box where M33 is located. (From Arp's preprint).

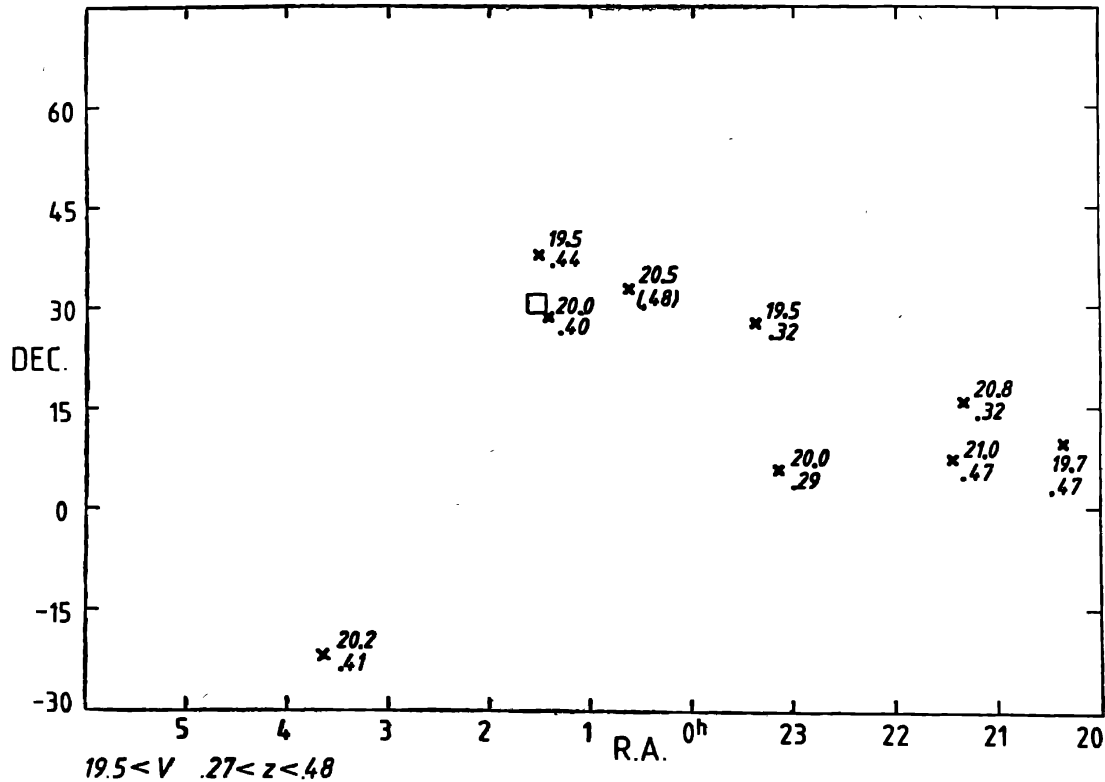


Figure 12. Plot of all bright radio quasars in the southern galactic hemisphere which have  $V < 19.8^m$  and redshifts in the range  $0.27 < z < 0.47$ . They appear to form a linear structure. (From Arp's preprint).

The idea is illustrated in figure 13. In the standard twin exhaust model a source of hot plasma surrounded by a gas cloud bound by the gravitational field of a star cluster or a massive blackhole ejects jets of plasma in a highly collimated fashion. The jets issue along lines of least resistance which, in the Blandford & Rees model, lie along the axis of rotation directed in opposite directions. In our model, a similar triple structure moving through the intergalactic medium (IGM) develops a single jet in the following way. As it moves it encounters ram pressure in the forward direction whereas there is no resistance in the backward direction. The result is that only one jet comes out preferentially in the backward direction where resistance is the least.

The Doppler model has certain attractive features since it makes definite predictions. The first is the obvious one that quasars should have single jets, a prediction which seems to be borne out by radio observations so far. VLBI measurements may, in near future, be able to tell whether quasars do have proper motions against the cosmological substratum, and if they do whether their jets are oppositely directed. The model is able to explain why there are comparatively few radio quasars and why it is more likely that low redshift quasars are more likely to be seen with fuzz around them. And of course quasar-galaxy associations found by Arp are easy to explain in this model.

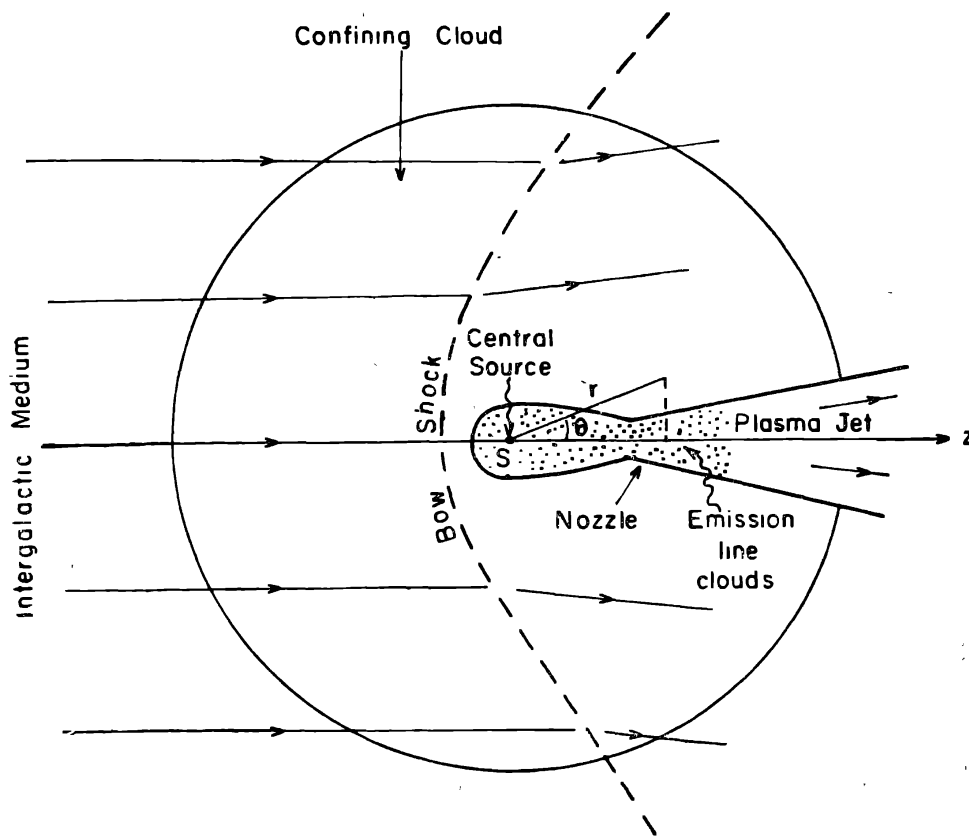


Figure 13. The single exhaust model has three basic components : a central source S ejecting fast plasma, gravitating matter (not explicitly shown here) and a confining cloud of gas and dust. The ram pressure of the intergalactic medium acts on the front portion of the object as it moves with high speed across it. The jet can issue only in the backward direction.

### 5. Gravitational redshifts

Although the compact nature of quasars naturally suggests gravitation as the explanation for their redshifts, two objections in the early days prevented their gaining much popularity. The first one was a powerful theoretical result by Bondi (1964) demonstrating that no massive object in equilibrium with equation of state having  $p \leq \frac{1}{3} \rho c^2$  for pressure and energy could have a surface gravitational redshift exceeding 0.62. The second objection was based on the spectroscopic analysis of the first two quasars 3C 48 and 3C 273 by Greenstein & Schmidt (1964) showing that in order to account for the requisite amount of radiation through the wide emission lines observed, the objects have to be as massive as galaxies and located within the Local Group. This seemed highly implausible.

To get round these problems Hoyle & Fowler (1967) suggested that quasar emission lines come not from the surface of the massive system but from hot gas trapped in its central region. For radiation to come out highly redshifted, the system had to be optically thin, yet capable of generating a deep gravitational potential well in the centre. A cloud of compact neutron stars, for example could fill the bill.

In the mid 1970s Das & Narlikar (1975, 1976) explored these ideas further. Keeping within the spectroscopic constraints mentioned above, as well as imposing requirements of stability, positive distribution functions and a distance range of 100 Mpc it is possible to have objects of masses of galactic order with central gravitational redshifts  $\sim 2-2.5$ .

In a recent long preprint, Hoyle (1983) has proposed another version of the gravitational model in which he has made a strong case for the subunits of the system to be magnetic monopoles rather than neutron stars. It will be impossible for me to go into all the extensive details of Hoyle's model in the short time at my disposal. Whether you agree with the models of noncosmological redshifts or not, I strongly recommend you to read this preprint in order to appreciate refreshingly new ideas from a highly original mind. I shall mention, however, that Hoyle needs masses of the order of  $10^{15} M_{\odot}$  for companion galaxies (with higher masses for the main ones) in order to account for their excess redshifts in Arp's data. Conservative opinion may balk at such massive objects, although Hoyle anticipates the likely lines of criticism and counters them.

### 6. A new kind of redshift

Astronomy was the first to provide examples of two out of the three types of redshifts described earlier. Of these gravitational redshift depends on relatively local change of spacetime geometry whereas cosmological redshift needs a global change.

Hoyle & Narlikar (1972) had shown that the cosmological redshift in a Friedmann universe could be interpreted also as arising from variable particle masses moving in flat Minkowski spacetime. Thus, going back to equation (1), the observed value of wavelength  $\lambda$  of a spectral line in the radiation from a distant galaxy is higher than the currently measured value  $\lambda_0$  because it arose out of lighter masses. The notion of the expanding universe is therefore replaced by the notion of steadily increasing particle masses in a flat spacetime.

So far the two approaches have been mathematically equivalent although the latter finds a ready explanation in the Machian theory of gravity developed by Hoyle & Narlikar (1974). A new input was introduced (Narlikar 1977) in this picture to account for the anomalous redshifts of Arp. Qualitatively the idea is as follows.

If all matter were created at the same epoch  $t = 0$  as in the big bang universe, the masses were all zero at that epoch. They started increasing thereafter, so that all galaxies observed by us at the same epoch would have the same redshift. (This is simply a restatement of Hubble's law that all galaxies at the same distance from us have the same redshift.)

Imagine now a quasar fired out of a galaxy at some epoch  $t > 0$ . According to my new input the particles in the quasar had zero masses at epoch  $t$  and the masses started growing thereafter. Thus the masses of particles in a quasar will always be less than the masses of particles in the parent galaxy simply because they are younger. It is this effect that leads to an excess redshift in the quasar. This excess redshift decreases as the firing epoch recedes into the past.

It is possible that the companion galaxies observed by Arp were also fired out of the parent main galaxy but if the firing took place in the remote past their excess redshifts will be lower. We may in fact argue that quasars, being more recent, have not yet developed into galaxies. By the time they do so, their excess redshift will have become much smaller.

Das & Narlikar (1980) have investigated the dynamics of ejection and its observable consequences. For instance, we expect that the quasar, if fired at energies small enough for it to remain bound by the gravitational field of the parent galaxy, will move around it in a highly eccentric orbit which shrinks steadily.

This prediction and a few others seem to be borne out by Arp's examination of his own data. The grouping of quasars of the same redshift near a galaxy suggests, in our model, that they were fired out at the same epoch. The observed quasar alignments are also explicable naturally in this picture (Arp 1983c).

## 7. Conclusions

It is scientific practice to subject the most widely accepted theory to the most searching tests. As I have tried to show in my talk these tests do leave sufficient room for doubting the validity of Hubble's law for quasars. On this basis I have discussed possible alternatives.

Because they have not been taken as seriously as they deserve to be, the Doppler and gravitational models have not yet progressed to the levels of detail and sophistication that the standard cosmologically-inspired quasar models have. This in itself cannot be a criticism of these models. A *prima facie* case having been established for their study, equally searching tests can be applied to them *after* they have been developed further.

By the use of Occam's razor, we should come to new types of redshifts only if the standard three types fail to deliver the goods. In that sense it may be premature at present to investigate the variable mass idea in great detail. For this reason I shall watch with great interest the observational developments in the area of anomalous redshifts in the years to come.

## References

- Arp, H. C., Sulentic, J. W. & di Tullio, G. (1979) *Ap. J.* **229**, 489.
- Arp, H. C. & Hazard, C. (1980) *Ap. J.* **240**, 726.
- Arp, H. C. (1981) *Ap. J.* **250**, 31.
- Arp, H. C. (1983a) Talk presented at the Liege symposium.
- Arp, H. C. (1983b,c) *Sky Teles.* **65**, 307; *Ap. J.* **271**, 479.
- Bahcall, J. N. & Hills, R. E. (1973) *Ap. J.* **179**, 699.
- Blandford, R. & Rees, M. J. (1974) *M.N.R.A.S.* **164**, 395.
- Bondi, H. (1964) *Proc R. Soc. Lond.* **A282**, 303.
- Burbridge, G. R., O'Dell, S. I. & Strittmatter, P. A. (1972) *Ap. J.* **175**, 601.
- Burbridge, G. R. & O'Dell, S. I. (1973) *Ap. J.* **183**, 659.
- Das, P. K. & Narlikar, J. V. (1975) *M.N.R.A.S.* **171**, 87.
- Das, P. K. (1976) *M.N.R.A.S.* **177**, 391.
- Greenstein, J. L. & Schmidt, M. (1964) *Ap. J.* **140**, 1.
- Hoyle, F. & Burbridge, G. R. (1966a,b) *Nature* **210**, 1346; *Ap. J.* **144**, 534.
- Hoyle, F. & Fowler, W. A. (1967) *Nature* **213**, 373.
- Hoyle, F. & Narlikar, J. V. (1972) *M.N.R.A.S.* **155**, 305.
- Hoyle, F. & Narlikar, J. V. (1972) *Action at a Distance in Physics and Cosmology*, Freeman.
- Hoyle, F. (1980, 1983) *Astrophysics and Relativity*, Preprints 63 & 88.
- Narlikar, J. V. (1977) *Ann. Phys.* **107**, 325.
- Narlikar, J. V. & Das, P. K. (1980) *Ap. J.* **240**, 401.
- Narlikar, J. V. (1983) *Introduction to Cosmology*, Jones & Bartlett.
- Narlikar, J. V. & Subramanian, K. (1983). *Ap. J.* **273**, 44.
- Smith, D. H. (1983) *Sky Teles.* **66**, 107.
- Stockton, A. (1978) *Ap. J.* **223**, 747.
- Strittmatter, P. A. (1967) cited on pp. 164-174 in *Quasistellar objects* (G. R. Burbridge & E. M. Burbridge) Freeman.
- Terrell, J. (1964) *Science* **145**, 918.