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Quantum cosmology as a cure for the three ailments of classical cosmology*

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Abstract. The standard big bang models of classical cosmology are known to possess three defects. The oldest known defect is the spacetime singularity whose existence seems inevitable within the classical framework. The second defect is the existence of a particle horizon which severely limits communications across the distant parts of the universe whose observed homogeneity therefore becomes inexplicable. Recently a third defect has been highlighted, *viz.*, the required fine tuning of the early universe close to the flat spatial model in order to account for the present range of its mean density.

We show here that the injection of quantum ideas holds out hope of a cure for all the three ailments described above. Using a simple path integral formalism for quantum cosmology we present arguments which suggest that (i) it is extremely unlikely that the universe evolved to the present state from quantum states of singularity and particle horizon; (ii) of all the possible Robertson-Walker models that could evolve out of quantum fluctuations of the empty Minkowski universe the flat model is overwhelmingly probable.

Key words : cosmology, classical—cosmology, quantum—big bang

1. Introduction

While the discovery of the microwave background, the work on primordial nucleosynthesis and the recent applications of grand unified theories to the early universe have given a boost to the standard big bang cosmology, serious difficulties with this picture are also being increasingly emphasized. Here we discuss three problems commonly associated with the early universe, *viz.* (i) spacetime singularity; (ii) the small particle horizon; and (iii) the apparent flatness of the spatial sections of the universe. Our discussion is limited to homogeneous isotropic universes which commonly form the basis of standard cosmology.

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In the comoving spherical polar coordinates (r, θ, ϕ) and cosmic time t, a homogeneous isotropic spacetime is described by the Robertson-Walker line element

$$ds^2 = dt^2 - S^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta \, d\phi^2) \right]. \qquad \ldots(1)$$

Here $S(t)$ is the scale factor for the universe and $k(= 0, \pm 1)$ denotes the sign of the curvature of the spatial sections.

We first note that with one trivial exception this line element describes a singular spacetime provided $S \to 0$ at some epoch. The exception is that of the flat spacetime given by $\dot{S} = 1$, $k = -1$.

The actual behaviour of $S(t)$ is of course determined by Einstein's equations. However, it is well known that for a wide variety of equations of state for matter (which are physically reasonable) $S(t)$ goes to zero at some finite moment in the past (Hawking & Ellis 1973). It is virtually impossible to avoid this geometrical singularity within the domain of classical physics.

This existence of singularity leads to a *beginning* for the universe, which is usually taken to be at $t = 0$. At any cosmic time t, a typical observer at $r = 0$ can receive signals only from within $r = r(t)$ where,

$$\int_0^{r(t)} \frac{dr}{\sqrt{1 - kr^2}} = \int_0^t \frac{du}{S(u)} \equiv Q(t, 0). \qquad \ldots(2)$$

In physically reasonable models for the universe $Q(t, 0)$ is a finite quantity, implying a finite—and, in fact, small—horizon size at the earliest epochs. For example at the redshift of a thousand when the radiation background decoupled from matter, the universe consisted of nearly 90 causally disconnected bits all of which were in the same physical condition. The observed high degree of isotropy of the background radiation therefore implies highly special initial conditions—a problem that is commonly known as the *horizon problem*. It is worth noting that the smallness of $Q(t, 0)$ arises because we were prevented from extending the time coordinate to values less than zero, the epoch of singularity. This fact is partly responsible for the existence of the horizon problem.

The *flatness problem* (Dicke & Peebles 1979; Guth 1981) also concerns itself with the special choice of the initial conditions. It turns out that the expansion rate and the density of our universe would have to be extremely fine tuned to account for their relationship observed at present. For example, the fine tuning needed is to one part in 10^{80} if the conditions were fixed at the epoch of baryosynthesis.

Recently, various types of inflationary scenarios were suggested as a possible explanation to two of the three above mentioned problems. However, all the models currently available in literature seem to either (i) run into serious difficulties of their own or (ii) invoke fine tuning at some other level (Barrow & Turner 1982). Besides, none of these models address themselves to the all important question of cosmological singularity, which is taken for granted.

We describe here a scenario based on quantum gravity which seems to be capable of tackling all the three ailments of classical cosmology. The formalism that we shall use has been discussed by us in earlier papers (*cf.* Narlikar 1979, 1981; Padmanabhan & Narlikar 1982; Padmanabhan 1982) and is first outlined below.

2. Quantum cosmology via path integrals

Consider a spacetime sandwich bounded by spacelike hypersurfaces Σ_1 and Σ_2. Let J denote the total Einstein-Hilbert action and suppose that boundary conditions are specified in suitable way on Σ_1 and Σ_2 (for details see Misner *et at.* 1973; Isenberg & Wheeler 1979). The classical solutions of Einstein's equations may be looked upon as a time development of 3-geometries starting from Σ_1 and ending on Σ_2 with the prescribed conditions. This time development we denote by a path $\bar{\Gamma}$. Any other time development with the same prescribed conditions, we denote by Γ. $\bar{\Gamma}$ is obtained by the stationary principle $\delta J = 0$.

In quantum gravity we replace the classical prescription by the propagator

$$K[2; 1] = \int \exp [iJ(\Gamma)/\hbar] \, \mathcal{D}\Gamma. \qquad \ldots(3)$$

The functional integral in equation (3) poses many conceptual and technical difficulties. However, if we make the reasonable assumption that in our quantization scheme we shall restrict Γ to all geometries which preserve the lightcone structure and causal connections existing in the classical Einstein geometry $\bar{\Gamma}$, we are able to make progress with the evaluation of the integral (3). For, if the classical metric tensor for $\bar{\Gamma}$ is $\bar{g}_{ik}$, a typical non-classical Γ will have the metric given by

$$g_{ik} = \Omega^2 \bar{g}_{ik} \qquad \ldots(4)$$

where Ω is a function of the spacetime coordinates. It may (and will) happen that a conformal transformation like equation (4) can allow us to extend an existing manifold. If so, we will include such extensions also.

If we are interested in homogeneous isotropic models only, then our task is further simplified. We denote by Γ the classical standard model $\bar{\mathcal{M}}$ given by

$$d\bar{s}^2 = dt^2 - \bar{S}^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta \, d\phi^2) \right] \qquad \ldots(5)$$

and consider Γ to be a general, nonclassical model $\mathcal{M}$ with

$$ds = \Omega(t) \, d\bar{s}, \qquad S(t) = \Omega(t) \, \bar{S}(t). \qquad \ldots(6)$$

If we redefine a new time coordinate τ by

$$d\tau = \Omega(t) \, dt, \qquad \ldots(7)$$

our line element is again of the form (1) with τ replacing t.

We can label the surfaces Σ_1, Σ_2 by $t = t_1, t_2$ respectively. Since we are interested in speculating about the initial states of the universe from which it came to the final present state we will phrase our quantum cosmological problem in the following way. Define $\phi(t) = \Omega(t) - 1$ as the quantum conformal fluctuation (QCF) from the classical solution. The present value of $\phi = \phi_2$ at $t = t_2$ is small whereas the past values of ϕ at $t = t_1 < t_2$ may have been considerably larger (than unity). Let us denote the state of the universe at $t_1(t_2)$ by a wavefunction $\psi_1(\phi_1)$ $[\psi_2(\phi_2)]$ and write

$$\psi_1(\phi_1) = \int K[\phi_2, t_2; \phi_1, t_1] \, \psi_2(\phi_2) \, d\phi_2. \quad \ldots(8)$$

Given ψ_2, our problem is to evaluate ψ_1.

In equation (8), the propagator K is to be determined by evaluating the path integral (3) and the integral (8) tells us how to calculate the initial state ψ_1 from the prescribed present state ψ_2. Detailed calculation shows that (*cf.* Narlikar & Padmanabhan 1983)

$$K[\phi_2, t_2; \phi_1, t_1] = \alpha(t_1, t_2) \exp \sum_{A,B=1}^{2} i\beta_{AB}\phi_A\phi_B \quad \ldots(9)$$

where α, β_{AB} are *known* functions determined by the classical solution $\bar{S}(t)$.

3. Did the universe have a singular origin?

Let us denote by C_S the class of all such Robertson-Walker manifolds $\mathcal{M}$, given by equations (6) for which singularity at $t = 0$ is *not* removed by the conformal transformation, *i.e.*, for which $\Omega S \to 0$ as $t \to 0$. Since the present state of the universe is almost classical we can approximate ψ_2 by a wavepacket centred on the classical value $\phi_2 = 1$:

$$\psi_2(\phi_2) = (2\pi\Delta_2^2)^{-1/4} \exp\left[-\frac{\phi_2^2}{4\Delta_2^2}\right]. \quad \ldots(10)$$

We use equations (8) and (9) to evaluate $\psi_1(\phi_1)$. The quadratic form in the exponential makes the calculation easy. The answer is that the initial state at t_1 was also describable by a wavepacket centred on the classical value $\phi_1 = 1$ but with a dispersion which behaves as

$$\Delta_1 \sim a/\bar{S}(t_1) \quad \text{as} \quad t_1 \to 0. \quad \ldots(11)$$

Thus we find that the QCFs diverge at the classical singularity thereby rendering the classical average solution of doubtful validity. In fact we now show that amongst the full range of nonclassical solution C, those belonging to the singular set C_S occur with vanishing probability

To see this result construct the unit Gaussian variable $x = \phi_1/\Delta_1$, for the probability function $|\psi_1|^2$, and note that for the singular class, the value of x is given by

$$x = \frac{S(t_1) - \bar{S}(t_1)}{a} \to 0 \quad \text{as} \quad t_1 \to 0 \quad \ldots(12)$$

since both $S(t_1)$ and $\bar{S}(t_1)$ tend to zero as $t_1 \to 0$. Thus, given the full range of nonclassical solutions available at $t = t_1 \approx 0$, the probability that our present state evolved out of a singular state becomes vanishingly small.

4. Horizon-free cosmologies

We now show that particle horizons can be eliminated for all those Robertson-Walker cosmologies which are non-singular at $t = 0$. First note that the past lightcone from any $t > 0$ back to $t = 0$ is invariant under the conformal transformation. The value of the τ-coordinate which corresponds to $t = 0$ is given by equation (7) as

$$\tau_0 = \tau_1 - \int_0^{t_1} \Omega(t)\, dt \qquad \ldots(13)$$

where $\tau = \tau_1$ at $t = t_1$. For nonsingular models $\Omega(t)\ \bar{S}(t) \rightarrow b > 0$ so that close to $t = 0$,

$$\int_0^{t_1} \Omega(t)\, dt \sim b \int_0^{t_1} \frac{dt}{\bar{S}(t)} \sim bQ(t_1, 0), \qquad \ldots(14)$$

and τ_0 as defined above is finite. Hence unlike for the classical solution the light-cone can be continued to the past of $\tau = \tau_0$ and thus the particle horizon eliminated (Such extension is not possible for singular models.)

5. Resolution of the flatness problem

We begin with the assumption that the empty Minkowski spacetime is unstable to quantum fluctuations. (For explicit demonstrations of such instabilities *see* Padmanabhan 1983; Lindley 1981; Atkatz & Pagels 1982; Brout *et al.* 1980). The path integral technique described above can be used to compute the probability of transition from the Minkowski spacetime to Robertson-Walker spacetimes. Our results will show that the probability is the highest for $k = 0$ model and makes it overwhelmingly probable.

Robertson-Walker spacetimes are conformally flat. Infeld & Schild (1945) have given an explicit transformation of equation (1) to the form

$$ds^2 = \Omega_0^2\,[dt^2 - dx^2 - dy^2 - dz^2]. \qquad \ldots(15)$$

The function Ω_0 depends on t and $\mathbf{r} \equiv (x, y, z)$ except in the special case of $k = 0$ when it depends on t only. We consider the propagator which describes transition from $\Omega_0 = \Omega_1$ at $t = t_1$ to $\Omega = \Omega_2$ at $t = t_2$, for the Hilbert action for empty spacetime. In this case the classical Einstein metric is $\bar{g}_{ik} \equiv \eta_{ik}$, the Minkowski metric. The path integral (3) over Ω becomes

$$K[\Omega_2, t_2; \Omega_1, t_1] = \int \exp\left\{-\frac{3i}{8\pi\hbar}\int_0 \Omega_i\Omega^i d^4x\right\} \mathcal{D}\Omega, \qquad \ldots(16)$$

where $\Omega_i \equiv \Omega_{,i}$.

Suppose our initial state wavefunctional $\psi_1(\Omega_1)$ is strongly peaked at $\Omega_1 = 1$. Let the final state wavefunctional be strongly peaked at $\Omega_2 = \Omega_0$. The transition probability is then found to be

$$|\langle \psi_2 | \psi_1 \rangle|^2 = N \exp\left(-\frac{3W}{8\pi\hbar}\right) \quad ...(17)$$

where N is a normalizing constant and

$$W = \int\int \frac{\nabla_1\Omega_0(\mathbf{r}_1)\cdot\nabla_2\Omega_0(\mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|^2} d^3\mathbf{r}_1 d^3\mathbf{r}_2. \quad ...(18)$$

It is clear from equations (17) and (18) that the probability is the highest and W the least when Ω_0 does not depend on space coordinates, *i.e.*, when $k = 0$. Since for $W \neq 0$ (for $k \neq 0$), the exponent $W/8\pi\hbar \gg 1$, the probability for $k = \pm 1$ comes out to be vanishingly small.

Hence if the universe evolved out of the flat Minkowski spacetime through QCF, then it is most likely to have made a transition to the $k = 0$ Robertson-Walker model. Note that in the above analysis our assumption of the initial state to be the Minkowski spacetime is dictated by the requirement of simplicity. Since all Robertson-Walker models are conformally flat, *any* such model could have arisen from the initial state by QCF. It is the dynamics which determines that the models with $k = \pm 1$ occur with very low probabilities.

6. Conclusion

The three outstanding ailments of the standard classical cosmology—singularity, horizon and flatness—therefore receive natural cures from quantum cosmology. Although our work is limited to homogeneous isotropic universes and our approach confined to conformal fluctuations, the results achieved above hold out hope for further generalizations.

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