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Stellar velocity perturbations in colliding galaxies II

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Abstract. An analytical relationship is obtained between the stellar velocity perturbations and the eccentricity of the galactic orbit, under the impulsive approximation, for a hyperbolic encounter between two spherical galaxies.

Key words : galaxies : collisions—stellar dynamics

1. Introduction

Alladin & Narasimhan (1982) have given the stellar velocity perturbations in a galactic encounter under the impulsive approximation. Their results given in equation (3.4.8) are valid only for $e \leqslant 1$. In this paper we give the velocity perturbations for $e > 1$ and show that in the limiting case of large e, they reproduce the well-known results of Spitzer (1958).

2. Velocity perturbations

We obtain analytical expressions for the velocity perturbations of a star in a galaxy in a distant encounter with another galaxy under the assumptions that the galaxies remain spherically symmetric and that the motions of stars may be neglected (impulsive approximation).

Let M and M_1 be the masses of two spherically symmetric stellar systems. We shall consider the tidal effects of M_1 on M. Using the coordinate system as defined in Alladin & Parthasarathy (1978) and Alladin & Narasimhan (1982) and the analyses therein, we get the following expressions for the three components of the tidal acceleration

$$\left.\begin{aligned} f_x &= \frac{dv_x}{dt} = \frac{GM_1}{r^3}[x'(2 - 3\sin^2\theta) + 3y'\sin\theta\cos\theta] \\ &\qquad + \text{higher order terms} \\ f_y &= \frac{dv_x}{dt} = \frac{GM_1}{r^3}[3x'\sin\theta\cos\theta + y'(2 - 3\cos^2\theta] \\ &\qquad + \text{higher order terms} \\ f_z &= \frac{dv_z}{dt} = \frac{-GM_1}{r^3}z' + \text{higher order terms} \end{aligned}\right\} \quad \ldots(1)$$

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where (x', y', z') are the coordinates of the test star in M and (r, θ) are the polar coordinates of the centre of M_1. We take the relative orbit to be the conic

$$\frac{l}{r} = 1 + e\cos\theta, \quad e > 1. \qquad \text{...(2)}$$

Assuming that the representative star is fixed in M and using the relations

$$r^2\dot{\theta} = [\mu l]^{1/2}, \quad \mu \equiv G(M + M_1), \quad l = a(e^2 - 1) \qquad \text{...(3)}$$

we get from the first term of the tidal acceleration the following velocity increments

$$\left.\begin{aligned} \Delta V_x &= \left(\frac{GM_1^2}{M + M_1}\right)^{\frac{1}{2}} \frac{x'}{p^{3/2}} e_x, \\ \Delta V_y &= \left(\frac{GM_1^2}{M + M_1}\right)^{\frac{1}{2}} \frac{y'}{p^{3/2}} e_y, \\ \Delta V_z &= \left(\frac{GM_1^2}{M + M_1}\right)^{\frac{1}{2}} \frac{z'}{p^{3/2}} e_z, \end{aligned}\right\} \qquad \text{...(4)}$$

where p is the distance of closest approach of the galaxies and

$$\left.\begin{aligned} e_x &\equiv \frac{1}{\sqrt{e+1}}\left[\frac{1}{e+1}\cos^{-1}(-e^{-1}) + \frac{2e^2 - 1}{e^2}\sqrt{\frac{e-1}{e+1}}\right] \\ e_y &\equiv \frac{1}{\sqrt{e+1}}\left[\frac{1}{e+1}\cos^{-1}(-e^{-1}) + \frac{1}{e^2}\sqrt{\frac{e-1}{e+1}}\right] \\ -e_z &\equiv \frac{2}{\sqrt{e+1}}\left[\frac{1}{e+1}\cos^{-1}(-e^{-1}) + \sqrt{\frac{e-1}{e+1}}\right]. \end{aligned}\right\} \qquad \text{...(5)}$$

The functions e_x, e_y and e_z are given in table 1 for a few values of $e > 1$, for a given p. The angle of deflection $\psi(= 2\tan^{-1}(e^2 - 1)^{-1/2})$ of the galactic orbit for various eccentricities ($e \geqslant 1$) is also indicated therein.

For the purpose of comparison with earlier results of Spitzer (1958) and Alladin & Narasimhan (1982) we also write the above equations in the following alternative form

$$\Delta V_x = \frac{GM_1 x'}{p^2 V_p} e'_x, \quad \Delta V_y = \frac{GM_1 y'}{p^2 V_p} e'_y, \quad \Delta V_z = \frac{GM_1 z'}{p^2 V_p} e'_z, \qquad \text{...(6)}$$

where

$$e'_x \equiv \sqrt{e+1}\, e_x, \quad e'_y \equiv \sqrt{e+1}\, e_y, \quad e'_z = \sqrt{e+1}\, e_z \qquad \text{...(7)}$$

and

$$V_p^2 = \frac{\mu}{p}(e + 1). \qquad \text{...(8)}$$

From the velocity perturbations, the fractional change in the energy of the galaxy $\Delta U / |U|$ can be obtained as in Paper I (Zafarullah *et al.* 1983). In the impulsive approximation, the change in the internal energy, ΔU, of the galaxy is the same as the change in its kinetic energy. From equations (4), we get

$$\langle \Delta V^2(r') \rangle = \frac{1}{4\pi} \int_0^{\pi} \int_0^{2\pi} [\Delta V_x^2(r') + \Delta V_y^2(r') + \Delta V_z^2(r')] \sin\theta' \, d\theta' \, d\varphi' \qquad \ldots(9)$$

$$= \frac{GM_1^2}{p^3(M+M_1)} \langle e_1^2 \rangle \, r'^2 , \qquad \ldots(10)$$

where (r', θ', φ') are the spherical polar coordinates of the test star in M and

$$\langle e_1^2 \rangle \equiv \tfrac{1}{3}(e_x^2 + e_y^2 + e_z^2). \qquad \ldots(11)$$

If $r' = R_h$ the median radius, we get for the change in the binding energy per unit mass at R_h

$$(\Delta U)_{sh} \equiv \tfrac{1}{2}\Delta V^2(R_h) \qquad \ldots(12)$$

$$\equiv \frac{1}{2} \frac{GM_1^2}{p^3(M+M_1)} \langle e_1^2 \rangle \cdot R_h^2 \cdot \qquad \ldots(13)$$

The binding energy, U, per unit mass of the galaxy is given by

$$U = -\beta_n \frac{GM}{2R_h}, \qquad \ldots(14)$$

where β_n depends slightly on the model.

Using equations (13) and (14), we get

$$\frac{(\Delta U)_{sh}}{|U|} = \frac{\langle e_1^2 \rangle}{\beta_n} \frac{M_1^2}{M(M+M_1)} \left(\frac{R_h}{p}\right)^3. \qquad \ldots(15)$$

If $M_1 \gg M$, equation (15) reduces to

$$\frac{(\Delta U)_{sh}}{|U|} = \frac{\langle e_1^2 \rangle}{2(3+e)\beta_n} \cdot \frac{\rho_R}{\rho} \qquad \ldots(16)$$

where $\rho_R \equiv M_1 (3+e)/\frac{4}{3}\pi p^3$ is the modified Roche density as derived from King (1962) and $\rho = \frac{1}{2} M/\frac{4}{3}\pi R_h^3$ is the mean density within the sphere of radius R_h. Equations (15) and (16) may be compared with the corresponding equations (3.4.14) and (3.4.15) for an elliptic orbit in Alladin & Narasimhan (1982).

3. Discussion

The stellar velocity perturbations given in equations (3.4.8) in Alladin & Narasimhan (1982) are valid only for $e \leqslant 1$, as the limits of integration for θ have to be from $-\cos^{-1}(-e^{-1})$ to $\cos^{-1}(-e^{-1})$ for hyperbolic orbits and not from 0 to 2π as taken in that paper. Those equations should read

$$\Delta V_x = \frac{\pi G M_1 x'}{(1+e)p^2 V_p}, \quad \Delta V_y = \frac{\pi G M_1 y'}{(1+e)p^2 V_p}, \quad \Delta V_z = \frac{-2\pi G M_1 z'}{(1+e)p^2 V_p}, \quad e \leqslant 1. \qquad \ldots(17)$$

To obtain the velocity increments for a relative hyperbolic orbit, the values as given by equations (17) have to be multiplied by $e_x^{\bullet}$, $e_y^{\bullet}$ and $e_z^{\bullet}$ respectively, where

$$e_x^{\bullet} = (e+1)^{3/2}\frac{e_x}{\pi}, \quad e_y^{\bullet} = (e+1)^{3/2}\frac{e_y}{\pi}, \quad e_z^{\bullet} = -(e+1)^{3/2}\frac{e_z}{2\pi}. \qquad \ldots(18)$$

From equations (5), (6) and (7), letting $e \to \infty$, we get the approximate (as V_p also is dependent on e) expressions

$$\Delta V_x = \frac{2GM_1 x'}{p^2 V_p}, \quad \Delta V_y = 0, \quad \Delta V_z = \frac{-2GM_1 z'}{p^2 V_p}. \qquad \ldots(19)$$

Equations (19) are identical with equations (8) of Spitzer (1958) for a perturber moving with a uniform large velocity in a rectilinear orbit.

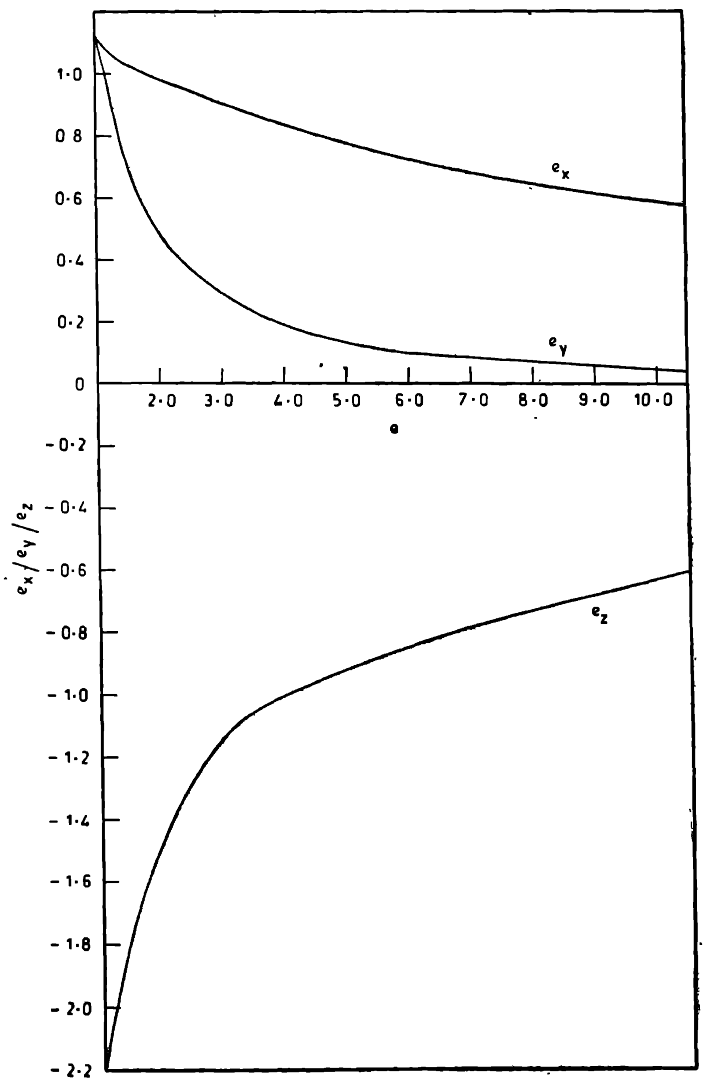

Figure 1. Run of e_x, e_y and e_z against the orbital eccentricity e.

It may be noted that under the assumption that the motion of stars may be neglected in comparison with the orbital motion of the galaxies, the velocity perturbations are symmetric with respect to the x and y axes for elliptic and parabolic orbits and become asymmetric for hyperbolic orbits, the asymmetry increasing with e.

In figure 1, we give the run of e_x, e_y and e_z against e. The asymmetry in the velocity perturbations along the x and y axes for increasing e can easily be visualised from it.

In table 1, we also give the values of $\langle e_1^2 \rangle$, for a few values of e, obtained from the present analysis. The error in $(\Delta U)_{sh}/|U|$ in Alladin & Narasimhan (1982) is less than 10% for $e = 1.2$ and less than 20% for $e = 1.5$. We also find that our results agree very well with those of Spitzer (1958) for eccentricities larger than 3.

Table 1. The Values of ψ e_x, e_y and e_x for various e.

e	1.0	1.2	1.5	2.0	2.5	3.0	5.0	10	50	100	∞
ψ	180°	113°	83°	60°	48°	39°	23°	12°	2°	1°	0°
e_x	1.110	1.049	1.022	0.986	0.947	0.907	0.774	0.589	0.279	0.198	0.000
e_y	1.110	0.924	0.708	0.486	0.359	0.278	0.134	0.049	0.004	0.002	0.000
$-e_z$	2.221	1.973	1.730	1.452	1.304	1.084	0.906	0.638	0.284	0.200	0.000
$\langle e_1^2 \rangle$	2.467	1.949	1.513	1.105	0.908	0.692	0.479	0.252	0.052	0.026	0.000

In this connection, it may also be pointed out that for $e = 1$, $\beta_n = 0.4$, equation (3.4.15) in Alladin & Narasimhan (1982) should read

$$\frac{(\Delta U)_{sh}}{|U|} = 0.8\frac{\rho_R}{\rho} \qquad \ldots(20)$$

and not 0.6 ρ_p/ρ as mentioned there. For $e = 2$, the numerical factor is about 0.3.

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