SPECKLE INTERFEROMETRY AT IIA AND ELSEWHERE: AN OVERVIEW

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ABSTRACT. The technique of speckle interferometry, used for compensating the blurring effects of the earth's atmosphere, is briefly outlined. The achievements of this technique by different groups in the world are listed. We then review our own experiments in speckle interferometry using the 1-m telescope at the Vainu Bappu Observatory at Kavalur. Finally we describe some computer simulations of speckles.

1 IMAGING THROUGH TURBULENCE

Conventional mode of imaging in optical astronomy cannot achieve resolution better than 1 arcsecond or so. This resolution is comparable to the diffraction limit of a telescope with an aperture of 10cm diameter. Atmospheric turbulence causes this degradation and prevents the complete utilisation of the resolving capabilities of large optical telescopes (Fried, 1966).

The power spectral density of refractive index fluctuations caused by atmospheric turbulence follows a power law with large eddies having greater power (Tatarski, 1961). When a wavefront passes down through such inhomogeneities, it suffers phase fluctuations and reaches the entrance pupil of a telescope with patches of random excursions in phase.

The variance of phase difference fluctuations between any two points in the wavefront increases as the 5/3 power of their separation. When this variance exceeds Π^2 rad for some separation r_o, then all details in the image smaller than λ/r_o will be obliterated in long exposure images. If the exposure time is shorter than the evolution time of the phase

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inhomogeneities, then each patch of the wavefront with diameter r_c will act independently of the rest of the wavefront resulting in multiple images of the source.

These sub-images or 'speckles' as these are called, can occur randomly along any direction within an angular patch of diameter λ/r_c. A snap shot taken later will show a different pattern but with similar probability of the angular distribution. A sum of several such short exposures is the conventional image. It is easy to visualise that the sum of several statistically uncorrelated speckle patterns from a point source can result in an uniform patch of light a few arcseconds wide.

We have attempted some computer simulations of this process. A power spectral density of the form

$$\rho(k) \propto L_0^{11/3}/(1+k^2L_0^2)^{11/6}$$

was multiplied with a random phase factor $\exp(i\phi)$, one for each value of (k_x,k_y), with Φ uniformly distributed between $-\pi$ and π. The resulting 2-D pattern in k_x, k_y space was Fourier transformed to obtain one realisation of the wavefront $W(x,y)$. The Fraunhofer diffraction pattern of a piece of this wavefront with the diameter of the entrance pupil gives angular distribution of amplitudes, while the squared modulus of this field gives the intensity distribution in the focal plane of the telescope. Fig.1 shows one such distribution for r_c = 10cm, L_o = 200cm for an entrance pupil of 200cm diameter. The smallest contours have the size of the airy disc of the telescope. Fig.2 shows the result of summing 100 such distributions. The contours of equal intensity are almost concentric circles.

The above example demonstrates how atmospheric turbulence can destroy finer details of an image. The aim of speckle interferometry and speckle imaging is to recover information about the image right up to the diffraction limit of the telescope. This will be illustrated in the next two sections.

2 TECHNIQUE OF SPECKLE INTERFEROMETRY

Let us write the image of a source $O(\alpha,\beta)$ as $I(\alpha,\beta)$. For incoherent imaging, we have

$$I(\alpha,\beta) = O(\alpha,\beta) * B(\alpha,\beta) \quad (1)$$

where $B(\alpha,\beta)$ is the image of a point source produced by the telescope - atmosphere combination in the 'snap-shot' mode and * represents convolution. Fourier transforming equation (1) we have

$$i(u,v) = O(u,v) \,.\, b(u,v) \quad (2)$$

Averaging over several such snap shots gives

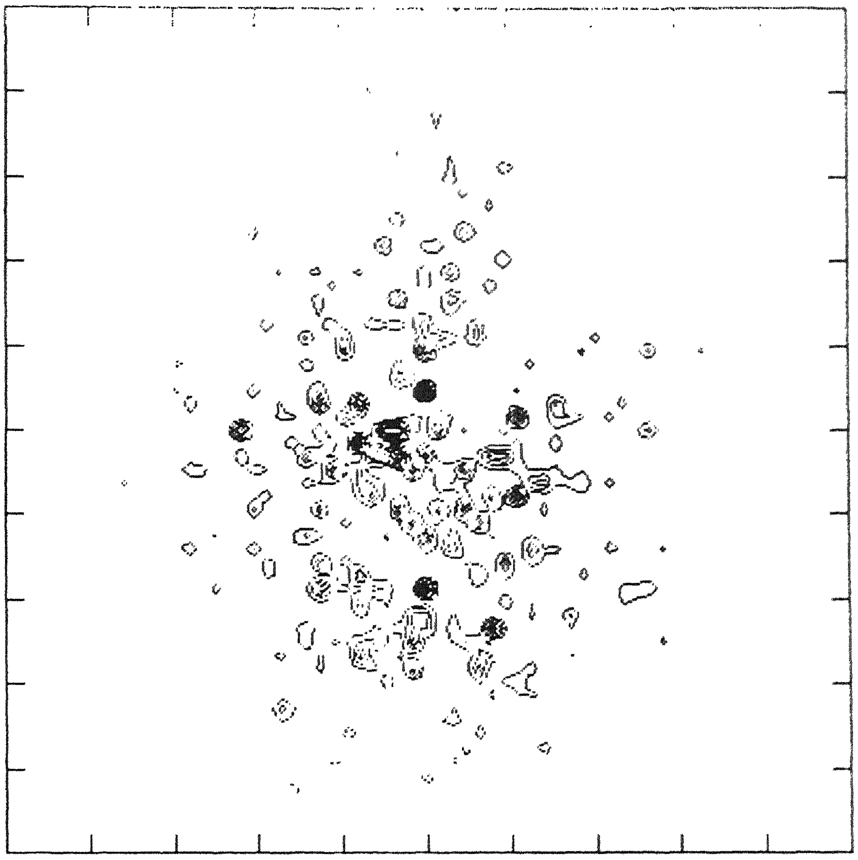

Figure 1. Intensity distribution in the focal plane of the telescope for r_c = 10cm, L_o = 200cm and for an entrance pupil of 200cm diameter.

$$\langle i(u,v)\rangle = O(u,v) \cdot \langle b(u,v)\rangle \qquad (3)$$

From the arguments presented in the previous section we know that the summed exposure is devoid of detail smaller than ≈ 1 arcsecond. Thus $\langle b(u,v)\rangle$ would be zero beyond frequencies of a few cycles/arcsecond. Hence conventional imaging fails to retain information beyond such frequencies. In speckle interferometry as invented by Labeyrie (1970), we take the squared modulus of equation (2) to obtain

$$|i(u,v)|^2 = |O(u,v)|^2 \cdot |b(u,v)|^2 \qquad (4)$$

Now $|b(u,v)|^2$ is the power spectrum of $B(\alpha,\beta)$ and thus by the Wiener theorem is the Fourier transform of the autocorrelation of $B(\alpha,\beta)$. Since $B(\alpha,\beta)$ consists of a random distribution of speckles spread over an arcsecond, its autocorrelation would consist of an 'Airy spike' riding on a broad pedestal of a couple of arcseconds. This 'Airy spike' contributes a non-zero high frequency tail to $|b(u,v)|^2$. This enables one to invert equation (4) to obtain

$$|O(u,v)|^2 = |i(u,v)|^2 / |b(u,v)|^2 \qquad (5)$$

To impart better signal over noise one takes the ensemble average of equation (5) which yields

$$|O(u,v)|^2 = \langle|i(u,v)|^2\rangle / \langle|b(u,v)|^2\rangle \qquad (6)$$

The quantity in the denominator has been modelled by Korff (1973), and is shown to consist of a low frequency spectrum similar to that of a long exposure image, while the high frequency spectrum is proportional to the telescope's transfer function scaled down by the number of speckles. In practice, $\langle|b(u,v)|^2\rangle$ is obtained from a nearby unresolved star. A danger of such a comparison is a possible change in the statistics of atmospheric turbulence along different directions. A smaller (larger) "seeing" for the comparison star would result in the suppression (enhancement) of intermediate spatial frequencies, which unheeded could lead to the "discovery" of rings or discs around some poor unassuming star! One must be very cautious in interpreting high resolution data, all the more while on virgin land.

Let us briefly look at the hardware necessary for obtaining speckle pictures. We need magnification of the focal plane image to sample at least 3 x 3 pixels within an Airy disc. We need a fast recorder to record the speckles within 10 msec or so. And because one always wants to go faint in astronomy, one requires high quantum efficiency. We in the Indian Institute of Astrophysics (IIA) have constructed a simple apparatus consisting of a Barlow lens and movie camera (Saha et al. 1987). Fig.3 shows a 20ms exposure of α Leo taken with the f/13 beam of the Kavalur 1-m telescope magnified 7 times. Of course, photography implies low quantum efficiency and some of the earlier versions of speckle cameras used image intensifiers to record the faint speckles. These have been replaced in modern versions by intensified television

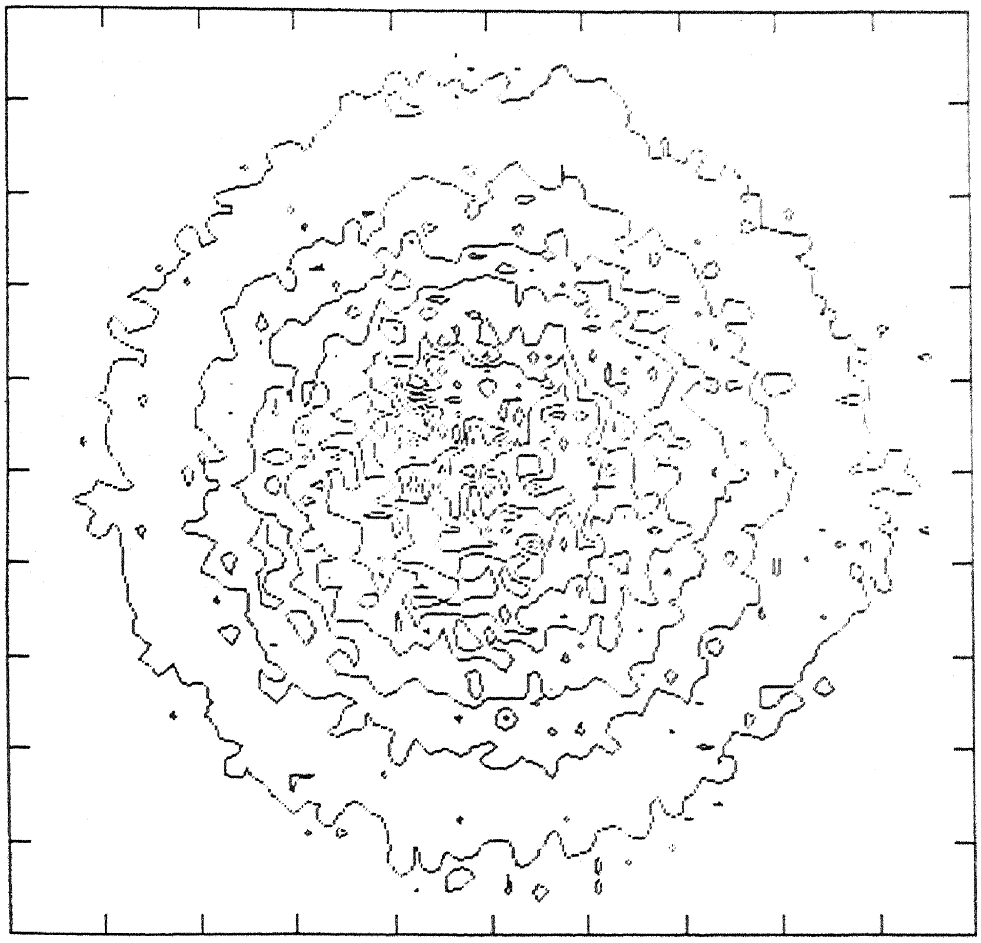

Figure 2. Result of summing 100 intensity distributions.

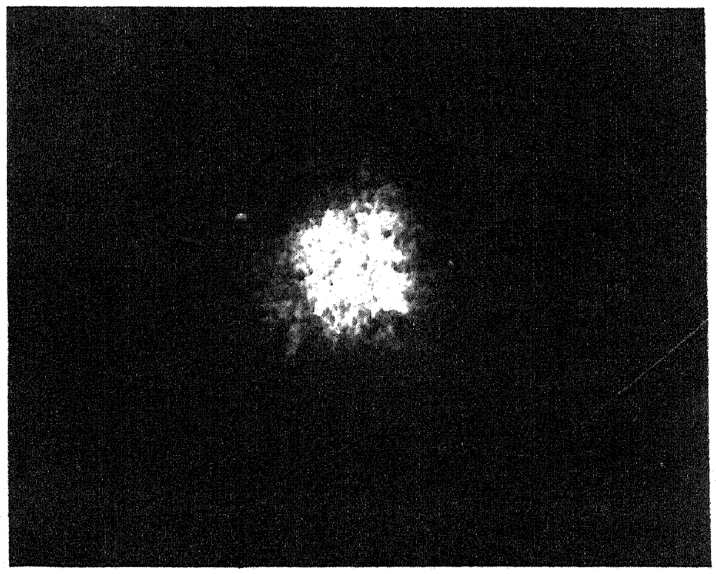

Figure 3. Single frame of α-Leo taken with F/13 beam of the Kavalur 1-m telescope.

camera (Blazit et al. 1977) or PAPA detectors (Papaliolios et al. 1985) with recording on video-tapes. We also plan to incorporate such devices in the future. Speckle interferometry provides information only on the amplitude of the FT of the object. In the next section we will briefly review the methods of phase retrieval.

3 SPECKLE IMAGING

Phase retrieval has been attempted chiefly by the following methods

a) Knox Thompson algorithm
b) Speckle Holography
c) Speckle Masking
d) Fienup algorithm

We shall very briefly outline each method.

3.1 Knox Thompson Algorithm

In the Knox Thompson procedure (Knox and Thompson, 1974), the quantities

$$F_1(u,v,\Delta u) = \langle i(u,v) \, . \, i^*(u+\Delta u,v)\rangle \quad \text{and}$$

$$F_2(u,v,\Delta v) = \langle i(u,v) \, . \, i^*(u,v+\Delta v)\rangle$$

are determined. These expressions can be further expanded as

$$F_1 = o(u,v) \, . \, o^*(u+\Delta u,v)\rangle \, . \, \langle b(u,v) \, . \, b^*(u+\Delta u,v)\rangle \quad \text{and}$$

$$F_2 = o(u,v) \, . \, o^*(u,v+\Delta v)\rangle \, . \, \langle b(u,v) \, . \, b^*(u,v+\Delta v)\rangle$$

If $\Delta u, \Delta v \ll r_c/\lambda$, then the expression within the angular brackets has a phase factor which is expected to be intrinsically small, with ensemble averaging making it yet smaller. Thus $\langle b.b^*\rangle$ will be almost real and can be estimated by observing an unresolved star. Now $o.o^*$ gives

$$|o|^2 \exp i(\Phi_1-\Phi_2) = |o|^2 \exp -i\left(\frac{\partial\Phi}{\partial u} u + \frac{\partial\Phi}{\partial v} v\right)$$

The amplitude $|o|$ is determined from speckle interferometry. Thus the arguments of F_1 and F_2 yield $\partial\Phi/\partial u(u,v)$ and $\partial\Phi/\partial v(u,v)$ respectively. The problem now remains of integrating $\partial\Phi/\partial u$ and $\partial\Phi/\partial v$ to get $\Phi(u,v)$. As is well known from the experience of numerical solution of partial differen -tial equations, such initial value problems lead to amplification of errors. These errors are sought to be minimised by averaging the phases arrived at from four neighbouring points (u,v) and using this estimate in a second round of integration; this process is continued until two consecutive iterations show small deviations from each other.

3.2 Speckle Holography

When there is a reference source within the same isoplanatic patch, then this reference can be used as a key to reconstruct the target source in the same way as a reference coherent beam is used in holographic reconstruction (Weigelt, 1978). Let O_T be the total object + reference. Then

$$O_T = O(x,y) + \delta(x-x_R,\ y-y_R)$$

Thus

$$O_T * O_T = \delta(x,y) + O(x,y) * O(x,y) + O(-x+x_R,\ -y+y_R) + O(x+x_R,\ y+y_R)$$

When x_R, y_R, > size of the object, then one easily extracts the objects from the term $O(x+x_R,\ y+y_R)$ which means a replica of the object at $(-x_R, -y_R)$. However, when this is not the case, then one must use image processing methods like subtraction of the summed long exposure image or by using the different light levels of the speckle clouds (reference and target) to extract the object.

3.3 Speckle Masking

Speckle masking uses the triple correlation of the object to obtain phase information (Lohmann et al. 1983). A triple correlation is obtained by first multiplying a shifted object with the original object and then cross correlating this product with the original object. The Fourier transform of a triple correlation is the bispectrum. Consider the bispectrum of the image as

$$i_n^3(\bar{u},\bar{v}) = i_n(\bar{u}).\ i_R(\bar{v}).\ i_n(-\bar{u},-\bar{v})$$

Where n refers to the nth specklogram. Now,

$$\langle i_n^3 \rangle = o^3(\bar{u},\bar{v})\ .\ \langle b_n^3(\bar{u},\bar{v}) \rangle$$

It can be shown that $\langle b_n^3 \rangle$ is real and nonzero upto the telescope's cut off frequency. Thus phase $(\langle i_n^3 \rangle)$ = phase (o^3).

To recover the phase of o from the phase of o^3 one employs recursive image reconstruction in frequency domain. Consider the above equation in discrete form

$$o^3_{p,q} = o_p \cdot o_q \cdot o_{-p-q}$$

where $\bar{u} = p.\Delta\bar{u}$ and $\bar{v} = q.\Delta\bar{v}$. Put $p = 0$ and use the Hermitian property of o. Then $o^3_{p,q}$ = constant $|o_q|^2$. Let $r = p+q$ and $\beta_{r-q,q}$ = phase $(o^3_{r-q,q})$. Then

$$\exp\,[i\Phi_r] = \exp\left[i\,(\Phi_q + \Phi_{r-q} - \beta_{r-q,q})\right]$$

Setting $q = 1$ and $\Phi_o = 0$, we have

$$\Phi_r = r.\Phi_1 - \beta_{1,1} - \beta_{2,1} - \ldots - \beta_{r-1,1}$$

Here $r\Phi_1$, gives a linear shift in image space and can be ignored if absolute positions are not required. The same procedure could be repeated for different values of q upto q = r/2 and an average value of Φ_r could be obtained. After obtaining the phases Φ_r one can recover the object using the amplitudes determined by speckle interferometry.

3.4 The Fienup Algorithm

This is a method of reconstruction of an object using only the modulus of its Fourier transform (Fienup, 1978). It is an iterative method where, at the kth iteration, $g_k(x)$, an estimate of the object Fourier transform is compared with the measured one and made to conform with the given modulus at all Fourier frequencies. Next, this transform is inverted to yield an image $g_k'(x)$. The iteration is completed by forming a new estimate of the object that conforms to certain object-domain constraints, e.g. positivity and finite extent, such that

$$g_{k+1}(\bar{x}) = g_k'(\bar{x}), \quad \bar{x} \notin \gamma,$$
$$= 0 \quad , \bar{x} \in \gamma$$

where the region γ is the set of all points at which $g_k'(x)$ violates the constraints. The iteration is stopped when the norm of the deviation of the Fourier transform at any iteration from the measured value becomes less than some predetermined tolerance. The above procedure can be considerably accelerated if the new estimate $g_{k+1}(\bar{x})$ is formed as follows

$$g_{k+1}(\bar{x}) = g_k(\bar{x}), \qquad , \bar{x} \notin \gamma,$$
$$= g_x(\bar{x}) - \beta g_k'(\bar{x}), \quad \bar{x} \in \gamma,$$

where β is a constant. This method has not been tried on real astronomical images.

4 CONCLUDING REMARKS

Speckle imaging has been successful in recent times for solving astrophysical riddles (e.g. in the resolution of R136a by Weigelt and Baier, 1985) or in creating new puzzles (e.g. in the discovery of a close companion to α Ori by Karovska et al. 1986). There is no doubt whatsoever that this is one field where image processing of a complex nature has shown an ingenious way of counteracting the blurring effects of the earth's atmosphere and sharpened our cosmic vision.

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