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Stellar velocity perturbations in colliding galaxies I

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Abstract. Expressions for the stellar velocity perturbations in colliding galaxies, valid for any distance of closest approach, are derived analytically under the impulsive approximation for spherical galaxies using a Newtonian potential with a softening parameter. The results are compared with those of earlier workers.

Key words : galaxies : collisions—stellar dynamics

1. Introduction

Using the analysis of spherically symmetric matter by Limber (1961), Alladin (1965) developed a theory for determining the interaction potential energy of two galaxies treated as superposition of polytropes, taking into account the departure from the inverse square law force due to overlap of galaxies. He tabulated the functions needed to calculate the potential energy and used them to make numerical estimates for the fractional change in the internal energy U (*i.e.* the energy due to the distribution and motion of stars in the galaxies). An alternative method of taking forces due to overlap into account and of suppressing two-body relaxation effects that arise due to a small number of particles chosen to represent a galaxy in numerical simulations of galactic collisions is to soften the potential by introducing a 'softening parameter' ϵ (White 1978, 1980; Roos & Norman 1979; Aarseth & Fall 1980; Aarseth 1980; Dekel *et al.* 1980; Dekel 1980). Thc aim of this paper is to derive an analytic expression for the velocity perturbations $\Delta\vec{V}$ of the representative stars in a galaxy due to tidal accelerations, using a Newtonian potential with a softening parameter ϵ and to highlight the ease with which the results of Alladin (1965), Sastry & Alladin (1970), Sastry (1972) and Alladin *et al.* (1975) can be obtained.

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2. Velocity perturbations

If the mass distributions in galaxies of masses M_1 and M_2 are represented by polytropes of common radius R and indices n_1 and n_2 respectively, the interaction potential energy of the pair is given by (Alladin 1965)

$$W(r) = -GM_1M_2 \left\{ \psi \left(n_1, n_2, \frac{r}{R} \right) r^{-1} \right\}, \quad \ldots(1)$$

r being the distance between the centres of the two galaxies. The function $\psi(n_1, n_2, r/R)$ which takes into account the departure from the inverse square force due to overlap has been tabulated by Alladin (1965) for two spherically symmetric galaxies of equal sizes and by Potdar & Ballabh (1974) in the case of unequal sizes.

We introduce a softening parameter ϵ in the Newtonian form of potential energy and write W in the form

$$W(r) = - GM_1M_2(r^2 + \epsilon^2)^{-1/2}. \quad \ldots(2)$$

Alladin & Narasimhan (1982) have pointed out that $W(r)$ given by equations (1) and (2) agree closely if

$$\epsilon = \left[\left(\frac{\psi}{r} \right)_{r=0} \right]^{-1}. \quad \ldots(3)$$

Limber's (1961) analysis leads to the potential at a point r' due to a galaxy of mass M_2 and radius R given by

$$\vartheta_2(r') = -\frac{GM_2}{R} \Phi_2 \left(n_2, \frac{r'}{R} \right) \quad \ldots(4)$$

The function $\Phi \left(n_2, \frac{r'}{R} \right)$ has been tabulated by Limber (1961). We shall represent this potential in the form

$$\vartheta_2(r') = -GM_2(r'^2 + \epsilon_2^2)^{-1/2}. \quad \ldots(5)$$

This is also the potential energy of a Plummer model galaxy of scale length ϵ_2. The function $\vartheta_2(r')$ given by equations (4) and (5) agree closely if

$$\epsilon_2 = \left[\left\{ \Phi_2 \left(n_2, \frac{r'}{R} \right) \right\}_{r'=0} \right]^{-1}. \quad \ldots(6)$$

While the earlier workers had used equations (1) and (4) to obtain numerically the change in the velocity of representative stars in the test galaxy, we shall use equations (2) and (5) to obtain this change analytically.

As in Sastry & Alladin (1970), we take M_1 and M_2 as the masses of the two galaxies and define a fixed coordinate system X, Y and Z with origin at the centre of mass of M_2, the X-axis in the direction of M_1 at minimum seperation and the Y-axis parallel to the velocity of M_1 at closest approach and the Z-axis perpendicular to the orbital plane. Let the position of the centre of M_1 be $\vec{r}\,(x, y, 0)$ and that of the representative star S in M_2 be $\vec{r'}\,(x', y', z')$. Let $\vec{r''}\,[(x' - x), (y' - y)\ (z' - z)]$ be the radius vector drawn from the star to the centre of M_1.

The tidal force, $\vec{f_T}$, per unit mass on $S\,(x', y', z')$ due to the gravitational attraction of M_1 is

$$\vec{f_T} = \vec{f_*} - \vec{f_G}. \qquad \ldots(7)$$

Here $\vec{f_*} = -\nabla''\vartheta$ and $\vec{f_G} = -\nabla W$ the accelerations towards M_1, of the star and the centre of mass of the galaxy M_2 respectively are obtained from equations (2) and (5). In the impulsive approximation, the change $\Delta\vec{V}$ in the velocity of S during the collision follows from

$$\Delta\vec{V} = \int_{-\infty}^{+\infty} f_T dt. \qquad \ldots(8)$$

On integrating, we obtain in the scalar form, the velocity increments

$$\left.\begin{aligned} \Delta V_x &= \frac{2GM_2}{V}\left[\frac{(p - x')}{(p - x')^2 + z'^2 + \epsilon_1^2} - \frac{p}{p^2 + \epsilon^2}\right], \\ \Delta V_y &= 0, \\ \Delta V_z &= -\frac{2GM_1}{V}\left[\frac{z'}{(p - x')^2 + z'^2 + \epsilon_1^2}\right], \end{aligned}\right\} \qquad \ldots(9)$$

where ϵ_1 is scalelength of the galaxy M_1 and V is the uniform relative velocity of the galaxies and p, their minimum seperation. In equations (9) $\Delta V_y = 0$ as the relative motion of the galaxies takes place in the XY plane parallel to the axis of X. All the values of $\Delta\vec{V}$ obtained numerically by Sastry (1972) can be obtained readily, using equations (9).

3. Fractional change in binding energy

From the velocity perturbations, the fractional change in internal energy of galaxy of mass M_2, *i.e.* $\frac{\Delta U}{|U|}$ can be readily obtained.

The change in the kinetic energy of each shell of the stars of the test galaxy characterised by a common distance r', from the centre of M_2 ($= \Delta U$ in the impulsive approximation) is given by

$$\langle \Delta U(r') \rangle = \frac{1}{2N}\sum_{i=1}^{N}\left[(\Delta V_x)_i^2 + (\Delta V_y)_i^2 + (\Delta V_z)_i^2\right]. \qquad \ldots(10)$$

The change in the binding energy of the galaxy is

$$\Delta U = \int_0^R \langle \Delta U(r') \rangle \frac{d}{dr'}[M_2(r')]\, dr', \qquad \ldots(11)$$

where the mass interior to r' is given by

$$M_2(r') = M_2\left(1 + \frac{\epsilon_2^2}{r'^2}\right)^{-3/2}. \quad \ldots(12)$$

The self-gravitation potentital energy of M_2 is given by

$$\Omega = -\frac{3\pi}{32}\frac{GM_2^2}{\epsilon_2}, \quad \ldots(13)$$

which is $2U$ according to the virial theorem.

For $p = 0$, equation (11) can be integrated analytically and we obtain for $\epsilon_1 = \epsilon_2$ in conformity with Toomre (1977)

$$\frac{\Delta U}{|U|} = \frac{2.33}{\epsilon_2}\frac{GM_1^2}{V^2 M_2} \text{ for } \epsilon_1 = \epsilon_2 \quad \ldots(14)$$

4. Discussion

For a spherical galaxy of polytropic index $n = 4$, $\epsilon_2 = 0.107R$ (Limber 1961) and for a pair of identical spherically. Symmetric galaxies of polytropic indices $n_1 = n_2 = 4$, $\epsilon = 0.167\ R$ (Alladin 1965). In figure 1, we compare the values of the function ψ/s where $s = r/R$ as obtained from equations (1) and (2) and find the agreement to be good. Equally good agreement we notice in the comparision, in figure 2, of the respective values of $\frac{d}{ds}\left(\frac{\psi}{s}\right)$ giving the gravitational force between the pair of galaxies as a function of s. In figure 3 we

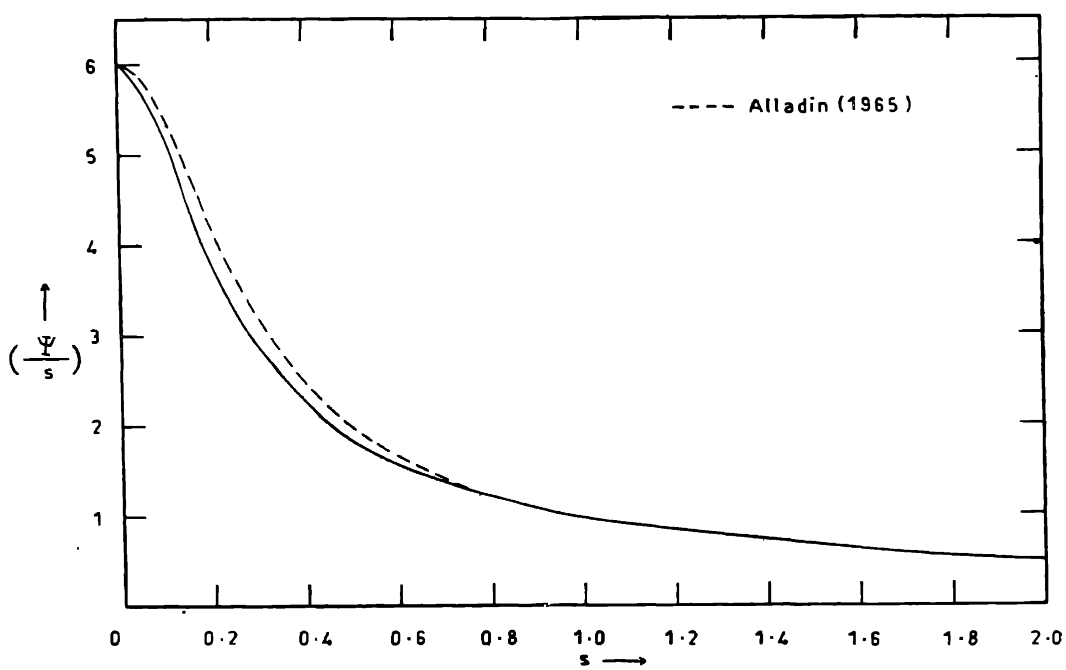

Figure 1. Comparison of interaction potential energies of a pair of galaxies $n_1 = 4$; $n_2 = 4$.

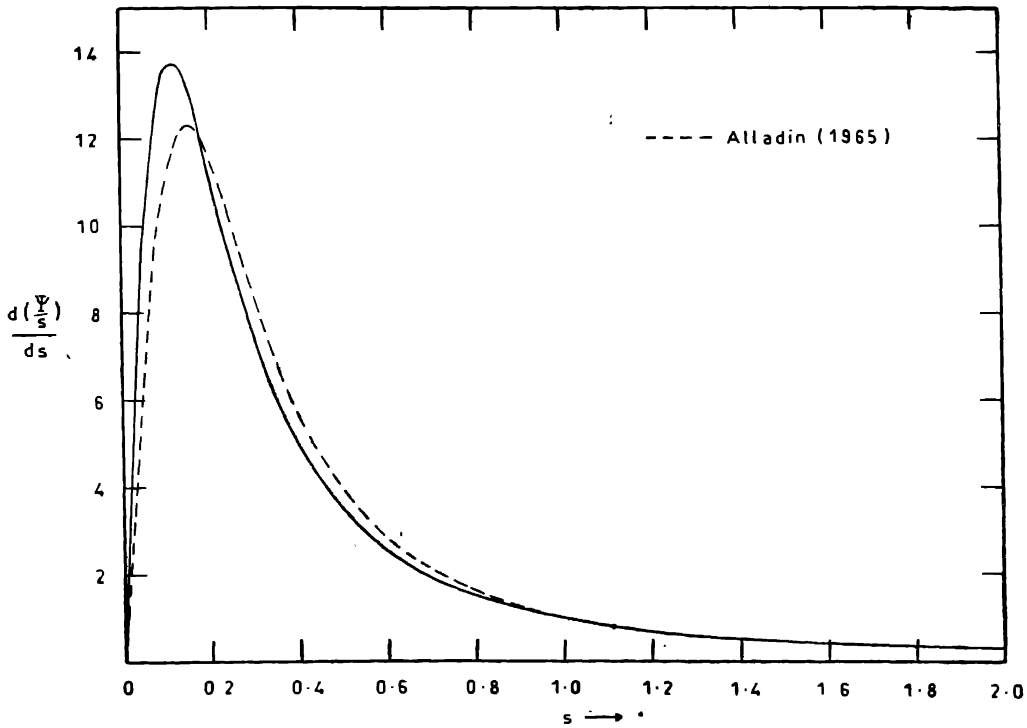

Figure 2. Comparison of gravitational force between a pair of aglaxies $n_1 = 4$; $n_2 = 4$.

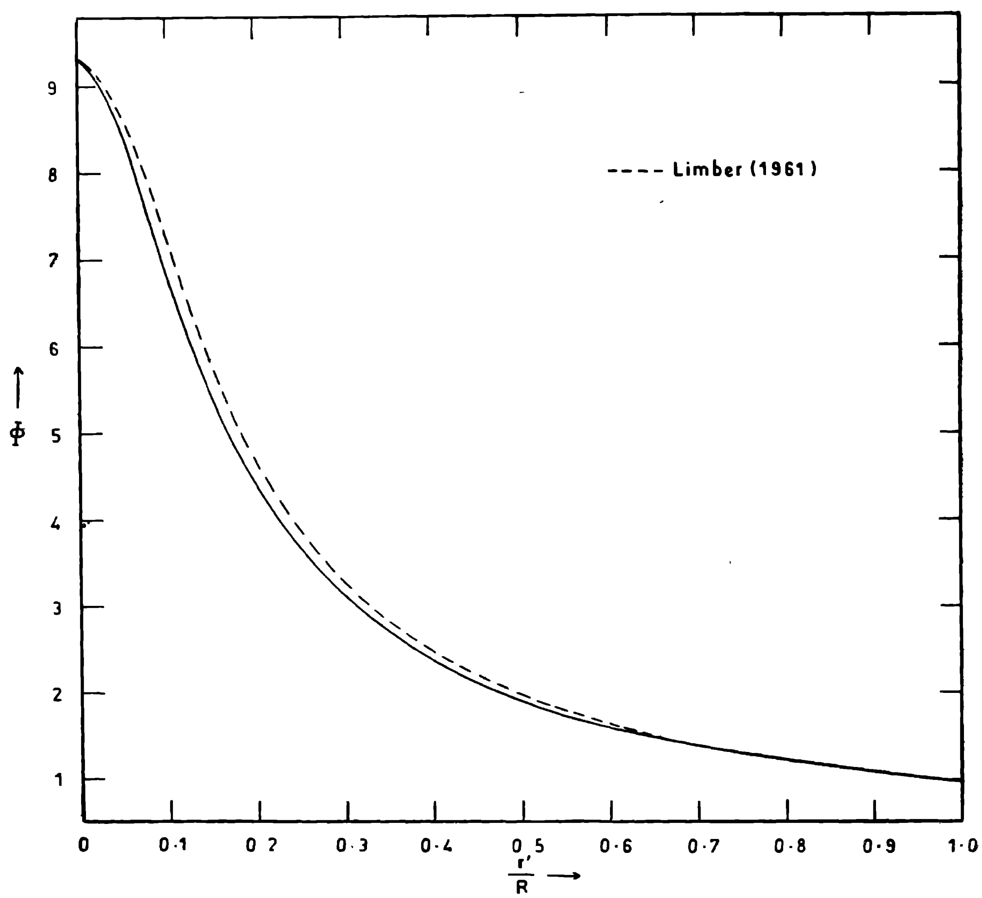

Figure 3. Comparison of potential at any point in a galaxy $n = 4$.

compare the values of Φ (n_2, r'/R) obtained from equations (4) and (5) as a function of r'/R. The agreement is good.

In table 1, we give the values of $(\Delta V)_{rms}$ for a few values of p as obtained from the present analysis for $M_1 = M_2 = 10^{11}\ M_\odot$, $\epsilon_1 = \epsilon_2 = 1.07$ kpc, and $V = 1000$ km s^{-1}, V_{rms} for the stars in this model is 345 km s^{-1}. The corresponding values obtained by Alladin (1965) and corrected by Sastry & Alladin (1970) for $p/R = 0.2$ with $R_1 = R_2 = 10$ kpc are given in parantheses. The agreement between the values is again found to be extremely close.

Table 1. (ΔV_{rms}) in km s^{-1} for $M_1 = M_2 = 10^{11}\ M_\odot$, $\epsilon_1 = \epsilon_2 = 1.07$ kpc and $V = 1000$ km s^{-1}

r/R \ p/R	0.0	0.2	0.6
0.0	0	0	0
0.1	330 (330)	100 (105)	20 (20)
0.2	280 (300)	160 (175)	40 (40)
0.3	210 (220)	300 (320)	65 (70)
0.4	165 (170)	280 (315)	95 (100)
0.5	135 (140)	260 (285)	125 (120)
0.6	115 (120)	245 (270)	80 (88)

If we take $p \gg r'$, equations (9) lead to the stellar velocity perturbations given by equation (8) in Spitzer (1958). On the other hand, if we set $p = 0$, we get

$$\left.\begin{aligned} \Delta V_\perp &= \left[\Delta V_x^2 + \Delta V_z^2\right]^{\frac{1}{2}} = \frac{2GM_1}{V}\frac{S'}{S'^2 + \epsilon_1^2} \\ \Delta V_\parallel &= 0 \end{aligned}\right\} \qquad \ldots(15)$$

in confirmity with equation (3) in Toomre (1977) for Plummer model galaxies. For $M_1 = M_2 = 10^{11}\ M_\odot$, $R_1 = R_2 = 10$ kpc, and $V = 1000$ km s^{-1}, $p = 0$; and for polytrope $n = 4$ models for galaxies, Sastry (1972) obtained $\dfrac{\Delta U}{|U|} = 0.8$ by numerical integration. We obtain a value of 0.9 for $\dfrac{\Delta U}{|U|}$ for the above case with $\epsilon_1 = \epsilon_2 = 1.07$ kpc, which is the appropriate scalelength for $n = 4$ polytrope models for galaxies.

Thus with appropriate choice of the scale length ϵ we can obtain the results for $\Delta\vec{V}$ and $\dfrac{\Delta U}{|U|}$ for any polytropic model with considerable ease.

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References

Aarseth, S. J. & Fall, S. M. (1980) *Ap. J.* **236**, 43.

Aarseth, S. J. (1980) *Phil. Trans. R. Soc.* A **296**, 351.

Alladin, S. M. (1965) *Ap. J.* **141**, 768.

Alladin, S. M., Potdar, A. & Sastry, K. S. (1975) *IAU Symp. No. 69*, p. 167.

Alladin S. M. & Narasimhan, K. S. V. S. (1982) *Phys. Rep.* **92**, 339.

Dekel, A., Lecor, M. & Shaham, J. (1980) *Ap. J.* **241**, 946.

Dekel, A. (1980) *Nature* **286**, 135.

Limber, D. N. (1961) *Ap. J.* **134**, 537.

Potdar A. & Ballabh, G. M. (1974) *Ap. Sp. Sci.* **26**, 353.

Roos, N. & Norman, C. A. (1979), *Astr. Ap.* **76**, 75.

Sastry, K. S. & Alladin, S. M. (1970) *Ap. Sp. Sci.* **7**, 261.

Sastry, K. S. (1972) *Ap. Sp. Sci.* **16**, 284.

Spitzer, L. (1958) *Ap. J.* **127**, 17.

Toomre, A. (1977) in '*The Evolution of Galaxies and Stellar Populations* (eds : B. M. Tinsley and R. B. Larson) Yale Univ. Obs., p. 401.

White, S. D. M. (1978) *M. N. R. A. S.* **184**, 185.

White, S. D. M. (1980) *Phil. Trans. R. Soc.* **296**, 347.