Bull. Astr. Soc. India (1983) 11, 63–69

Phase retardation of light due to scattering by interstellar nonspherical grains*

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Received 1982 November 19; accepted 1982 December 28

Abstract. The phase retardation by cylindrical grains composed of pure (index of refraction, $m = 1.33 - i0.0$) and dirty ($m = 1.33 - i0.05$) ice materials has been investigated. It has been found that the resulting interstellar circular polarization is likely to be more pronounced in the near- and far-infrared wavelength regions. The orientation of the grains, even in the case of dielectric materials, plays an important role in producing interstellar circular polarization.

Key words : interstellar grains—interstellar circular polarization—phase retardation-scattering of light

1. Introduction

Although the existence of interstellar dust grains, submicron sized solid particles, was established about 50 years ago, the nature and physical properties of the grains are not still well understood. The observations which give some clues to the nature of the grains include, for example, extinction and linear polarization of starlight, diffuse galactic light and infrared radiation from certain celestial sources. However, there is no unique model of grains which can simultaneously explain all the relevant observations. Therefore, further observational and theoretical studies are necessary. The possibility that the interstellar grains can be birefringent was hinted upon by van de Hulst (1957). The consequent interstellar circular polarization measurements have since been reported by several workers (see *e.g.* Martin *et al.* 1972; Martin 1974: Stokes *et al.* 1975; Michalsky *et al.* 1974; Lonsdale *et al.* 1980). The related theoretical studies on circular polarization can certainly provide additional clues for testing various options on the models of the interstellar grains proposed so far.

The interstellar linear polarization gives evidence that the grains are nonspherical and/or anisotropic particles oriented by some alignment mechanism (see, for references, Greenberg 1978). The linear polarization is thought to be caused by linear dichroic property of the grains characterized by the difference in the extinction

*Preliminary results of this paper were presented at the second meeting of the Astronomical Society of India 1975 March 12–14 at Kodaikanal.

cross sections for two orthogonal states of polarization with electric vector along and perpendicular to the reference plane defined by the direction of propagation and the outstanding axis of symmetry of the grain. In favourable situation, relative phase retardation and consequent circular polarization would also result. This can happen if the medium containing grains possess the property of linear birefringence. This means that the ray propagation velocities and therefore the indices of refraction are different for two orthogonal states of polarization. Here one may recall the phenomenon of double refraction described in conventional optics text books. A useful theoretical formulation for interstellar phase retardation and circular polarization has been given by Serkowski (1962). Subsequently Martin (1972) presented some calculations based on infinite homogeneous circular cylinder composed mainly of quartz- or glass-like materials. In what follows, we present the results on more realistic pure and dirty ice cylindrical grains. The preliminary comparison with the observations of the interstellar circular polarization of the Crab nebula lends support to the existence of dirty ice as an important constituent of the interstellar grains.

2. Theoretical considerations

Let us replace the interstellar medium between the star and the observer by an equivalent homogeneous slab. Its formal index of refraction as per the prescription of van de Hulst (1957) can be specified by

$$\tilde{m}_j = m_j' - i m''_j = 1 - i\, 2\pi\, N k^{-3}\, T_j\,(0^\circ), \qquad \ldots(1)$$

where N is the number density of the grains assumed to be of uniform size in the region of interest; $k = 2\pi/\lambda$ is the propagation constant; λ the wavelength of the incident light; and $T_j(0^\circ)$ the forward scattering amplitude function of the grains. The suffixes $j = 1, 2$ denote the polarization states of the light, viz. $E \parallel$ and $H \parallel$, respectively. Note that $E \parallel$ ($H \parallel$) means the electric (magnetic) vector of the light beam parallel to the reference plane containing the long axis of the cylindrical grains and the direction of incidence.

The real part of $\tilde{m}_j$ determines the phase retardation and the imaginary part connects to the extinction. Thus we can write the phase retardation, ϕ_j and extinction in magnitude, A_j, along the line of sight as follows :

$$d\phi_j = \frac{N\lambda^2}{2\pi}\, \mathrm{Im}\,\{T_j(0^\circ)\}\, ds, \qquad \ldots(2)$$

$$dA_j = \frac{1.086 N\lambda^2}{\pi}\, \mathrm{Re}\,\{T_j(0^\circ)\}\, ds, \qquad \ldots(3)$$

where s = the distance travelled through the medium. For a given j and constant s one obtains

$$\left(\frac{\phi}{A}\right)_j = 0.4604\, \frac{\mathrm{Im}\,\{T_j(0^\circ)\}}{\mathrm{Re}\,\{T_j(0^\circ)\}}\,. \qquad \ldots(4)$$

So far no shape and composition of the grains have been assumed in equations (1) to (4). We will choose cylindrical shape of the grains and use the theory of scattering for obliquely oriented infinite cylinder (Lind & Greenberg 1966). The corresponding computer program developed by Shah (1967) has been adopted throughout with suitable modification.

It is convenient to use a set of four Stokes parameters I, Q, U and V to describe the propagation of partially polarized light through a dichroic and/or birefringent medium. These quantities can be defined from operational point of view as follows (cf. Shurcliff 1966):

$I =$ total intensity of the beam of incident light including polarized components;

$Q =$ the component of the linear polarization with preference of the electric vector in the horizontal direction;

$U =$ the component of the linear polarization with preference of the electric vector at an angle of 45° with respect to Q ;

$V =$ the component with the circular polarization.

The differential equations which govern the state of polarization of light as it marches through the medium have been derived by Serkowski (1962). They need correction for two sign errors (Martin 1974). The complete set of equations is as follows :

$$I^{-1}\left(\frac{dI}{ds}\right) = -\tfrac{1}{2}N(C_{e1}+C_{e2}) - \tfrac{1}{2}N(C_{e1}-C_{e2})\left(\frac{Q}{I}\right), \qquad \ldots(5)$$

$$\frac{d(Q/I)}{ds} = \tfrac{1}{2}N(C_{e1}-C_{e2}) - \tfrac{1}{2}N(C_{e1}-C_{e2})\left(\frac{Q}{I}\right)^2, \qquad \ldots(6)$$

$$\frac{d(U/I)}{ds} = -\tfrac{1}{2}N(C_{p1}-C_{p2})\left(\frac{V}{I}\right) - \tfrac{1}{2}N(C_{e1}-C_{e2})\left(\frac{Q}{I}\right)\left(\frac{U}{I}\right), \qquad \ldots(7)$$

$$\frac{d(V/I)}{ds} = \tfrac{1}{2}N(C_{p1}-C_{p2})\left(\frac{U}{I}\right) - \tfrac{1}{2}N(C_{e1}-C_{e2})\left(\frac{Q}{I}\right)\left(\frac{V}{I}\right), \qquad \ldots(8)$$

where

C_{e1}, C_{e2} = the extinction cross sections per grain for $E \parallel$ and $H \parallel$, respectively,

$$C_{ej} = \sigma_{ej} l, \quad j = 1, 2,$$

l = length of the cylindrical grain,

σ_{ej} = the extinction cross section per unit length of the cylinder,

$$= \frac{4}{k}\,\mathrm{Re}\,\{T_j(0°)\}, \quad j=1, 2. \qquad \ldots(9)$$

Similarly, we can define

$$C_{pj} = \frac{2\lambda l}{\pi}\,\mathrm{Im}\,\{T_j(0°)\}, \quad j = 1, 2. \qquad \ldots(10)$$

It is implied that the Stokes parameters are functions of wavelength as well as distance s. In astrophysical cases of interest here, the quantities (Q/I), (U/I), and (V/I) are small. Therefore the product of any two or square of any one of them can be ignored. The two equations relating U and V then reduce to simple forms:

$$\frac{d}{ds}\left(\frac{U}{I}\right) = -\Delta C_p\left(\frac{V}{I}\right), \qquad \ldots(11)$$

$$\frac{d}{ds}\left(\frac{V}{I}\right) = \Delta C_p \left(\frac{U}{I}\right), \qquad ...(12)$$

where $\Delta C_p = \frac{1}{2} N (C_{p1} - C_{p2})$.

Direct integration of each of these equations is not permissible because U and V both are functions of s. An intuitive method is as follows :

$$\frac{d}{ds}\left(\frac{V}{U}\right) = \frac{V'(U) - V(U')}{U^2}, \qquad ...(13)$$

where prime denotes the derivative with respect to s. Using equations (11) and (12), we obtain.

$$\frac{d}{ds}\left(\frac{V}{U}\right) = \frac{\Delta C_p U^2 - \Delta C_p V^2}{U^2}. \qquad ...(14)$$

$$\therefore \quad \tanh^{-1}\left(\frac{V}{U}\right) = \frac{1}{2}\int N(C_{p1} - C_{p2})\, ds. \qquad ...(15)$$

Since V cannot be greater than U, $(V/U)^2 < 1$. Thus we can use the approximation

$$\tanh^{-1}\left(\frac{V}{U}\right) \simeq \frac{V}{U}. \qquad ...(16)$$

The final result of integration turns out to be

$$\left\{\frac{V}{I}\right\} = 0.5Ns(C_{p1} - C_{p2})\left(\frac{U}{I}\right). \qquad ...(17)$$

Note that in order to be able to observe circular polarization, one must have the U component of linearly polarized light from the source itself or during transit of light beam through the medium. What we observe in the linear polarization of interstellar origin is combination of Q and U.

3. Results and discussion

We consider the grains in the form of cylinders composed of either nonabsorbing dielectric ice (index of refraction, $m_1 = 1.33 - \mathrm{i}\, 0.0$) or absorbing dirty ice ($m_2 = 1.33 - \mathrm{i}\, 0.05$). The angles of incidence of the light beam have been chosen as $\alpha = 0°$, $30°$ and $60°$. The size-to-wavelength parameter is given by $ka = (2\pi a/\lambda)$ a being the radius of the cylindrical grains and λ the wavelength, both in μm. The resulting phase retardation $(\phi/A)_1$ and $(\phi/A)_2$ for $E \parallel$ and $H \parallel$, respectively, have been listed in tables 1 and 2 for dielectric ice (m_1) and dirty ice grain (m_2), respectively. The first column gives the wavelength if one assumes the radius, a, to be uniformly 0.1 μm for all the grains. The following points may be noted in table 1. The phase retardation (ϕ/A) for a given α and $E \parallel$ or $H \parallel$ reduces monotonically as ka increases, i.e. λ decreases. For $\alpha = 0°$ and $30°$ for all ka, $(\phi/A)_2 > (\phi/A)_1$. However for $\alpha = 60°$ the trend is reversed at about $ka \simeq 2$, i.e. in the UV wavelength region. The difference in phase retardation for $E \parallel$ and $H \parallel$ is a measure of circular polarization. It is given in the last column. Notice that in the infrared, relatively large circular polarization is indicated, reducing as one goes from the IR to the UV wavelength region for $\alpha = 0°$ as well as $30°$. The case of $\alpha = 60°$ shows

Table 1. Phase retardation due to scattering by oblique cylinder

$m = 1.33 - i\,0\,0$

Wavelength λ (μm) (a = 0.1 μm)	ka	Angle of incidence α (degree)	$(\phi/A)_1$ $E\parallel$	$(\phi/A)_2$ $H\parallel$	$\Delta(\phi/A)$
3.14	0.2	0	18.86	53.36	−34.50
1.57	0.4		4.76	13.97	−9.21
0.785	0.8		1.46	4.31	−2.85
0.523	1.2		1.10	2.48	−1.38
0.314	2.0		0.64	0.93	−0.29
0.125	5.0		0.081	0.114	−0.033
3.14	0.2	30	27.18	42.23	−15.05
1.57	0.4		6.74	10.76	−4.02
0.785	0.8		1.80	3.05	−1.25
0.523	1.2		1.03	1.75	−0.72
0.314	2.0		0.59	0.79	−0.20
0.125	5.0		0.00725	0.0465	−0.039
3.14	0.2	60	36.36	29.49	6.87
1.57	0.4		8.66	7.07	1.59
0.785	0.8		1.93	1.60	0.33
0.523	1.2		0.819	0.693	0.126
0.314	2.0		0.338	0.457	−0.119
0.125	5.0		−0.272	−0.231	−0.041

Table 2. Phase retardation due to scattering by oblique cylinder

$m = 1.33 - i\,0.05$

Wavelength λ (μm) (a = 0.1 μm)	ka	Angle of incidence α (degrees)	$(\phi/A)_1$ $E\parallel$	$(\phi/A)_2$ $H\parallel$	$\Delta(\phi/A)$
3.14	0.2	0	2.26	3.41	−1.15
1.57	0.4		1.60	2.83	−1.23
0.785	0.8		0.898	1.88	−0.98
0.523	1.2		0.748	1.356	−0.61
0.314	2.0		0.484	0.682	−0.198
0.125	5.0		0.060	0.098	−0.038
3.14	0.2	30	2.48	3.34	−0.86
1.57	0.4		1.87	2.64	−0.77
0.785	0.8		1.02	1.57	−0.55
0.523	1.2		0.707	1.105	−0.398
0.314	2.0		0.444	0.596	−0.152
0.125	5.0		0.0043	0.0445	−0.0402
3.14	0.2	60	2.95	3.20	−0.25
1.57	0.4		2.24	2.27	−0.03
0.785	0.8		1.095	1.014	+0.081
0.523	1.2		0.583	0.536	+0.047
0.314	2.0		0.263	0.345	−0.082
0.125	5.0		−0.191	−0.145	−0.046

a somewhat interesting feature. Here two prominent effects due to orientation of the grain and the wavelength dependence have been displayed. The cross over from positive to negative circular polarization occurs somewhere in the visual wavelength region. Note that the sense of polarization in the IR has changed compared to the cases of $\alpha = 0°$ and 30°. But in the far UV the sense of polarization has been retained. Such a change of sign from positive in the IR to negative in the blue wavelength region has also been reported in the observations.

In table 2, the effect of absorptivity in the index of refraction is prominently brought out. There is considerable reduction of phase retardation as well as circular polarization. The results for $\alpha = 60°$ show a maximum somewhere in the visual wavelength region. Martin *et al.* (1974) have stated that the change of sign in the middle of the optical window is known to be characteristic of dielectric but not of metallic grains. However, the present results given in tables 1 and 2 show that even for dielectric elongated grains the change of sign may not occur at all unless the alignment of the grains is favourable.

As an illustration, let us consider the actual observations of Martin *et al.* (1972) for position 3 in the Crab nebula. They have reported the linear polarization $(U/I) = 17.3\%$ and circular polarization $(V/I) = 0.235 \pm 0.092\%$ both at a mean wavelength $\lambda = 0.83\ \mu m$ in their table 1. The question is : can we use the present calculations in table 1 or 2 to reproduce this result? Let us assume cylindrical grains all of uniform radius $a = 0.1\ \mu m$ and uniform length, $l = 0.2\ \mu m$. Following Naranan & Shah (1970) we assume the number density of the grains $N = 10^{-13}\ cm^{-3}$. The distance to the Crab nebula, $s = 2$ kpc. It can be shown that the phase factors C_{p1} and C_{p2} are related to $(\phi/A)_j$ by the following relation :

$$C_{p1} - C_{p2} = 4.344 la \left[(\phi/A)_1 Q^{E}_{ext} - (\phi/A)_2 Q^{H}_{ext} \right], \quad \ldots(18)$$

where Q^{E}_{ext} and Q^{H}_{ext} are the extinction efficiencies per unit length of the cylinder for the cases of $E \parallel$ and $H \parallel$, respectively. Using equation (18), table 2 and substituting the quoted values of various quantities in equation (17), we arrive at the results given in table 3. Where necessary, appropriate linear interpolation has been done. It is now clear that the observational value of (V/I) stated above for the Crab nebula can be satisfied by aligned dirty ice grains.

Table 3. Extinction efficiencies (Q_{ext}), phase retardation (ϕ/A) and circular polarization (V/I) as function of orientation of the cylindrical grains

Radius of the grain, $a = 0.1\ \mu m$,
Length of the grain, $l = 0.2\ \mu m$,
Number density of the grains, $N = 10^{-13}\ cm^{-3}$,
Wavelength, $\lambda = 0.83\ \mu m$,
Distance to Crab nebula $s = 2$ kpcs,
The linear polarization at the source, $U/I = 17.3\%$,
Index of refraction of dirty ice, $m = 1.33 - i\ 0.05$

Angle of incidence α (degrees)	Q^{E}_{ext}	Q^{H}_{ext}	$(\phi/A)_1$	$(\phi/A)_2$	$(C_{p1} - C_{p2})$ $\times 10^{-11}$	(V/I) 10^{-3}
0	0.4255	0.1692	0.92	1.92	5.79	3.09
30	0.3776	0.2010	1.06	1.62	6.57	3.51
60	0.3290	0.3172	1.15	1.08	3.12	1.66

4. Conclusion

The present calculations substantiate the conclusion regarding the dielectric nature of the interstellar grains which can cause the circular polarization. However. the orientation of the grains also play an important role just as in the case of linear polarization. More detailed observations on the wavelength dependence of linear and circular polarization of starlight in various regions of the sky are essential to justify sophisticated models of the grains.

Acknowledgements

The calculations reported here were performed in 1968 using the CDC 6400 computer of the University of Arizona during the author's stay at the Lunar and Planetary Laboratory, Tucson, Arizona. The author thanks Dr T. Gehrels and Dr K. Serkowski for suggesting this work.

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