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On the temperature distribution along a solar flare loop

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Abstract. We have obtained expressions for temperature distribution along a solar flare loop in the presence of a source of continued heating in constant cross section and line dipole geometries under nonstatic conditions. The quantitative results are presented for the flare kernel of 1973 September 1 for sources of small and large extensions.

It is found that the fall of temperature in constant cross section geometry is faster than that in line dipole geometry. The rate of fall is faster under static conditions than under nonstatic ones. Further, the fall of temperature is faster if the source is stronger and confined to a smaller region at the top of the loop than when the whole loop is heated by a weaker source.

Key words : temperature distribution—flare loop

1. Introduction

Rocket and satellite observations indicate that the flaring regions consist of a number of loops of different sizes. The magnetic fields in these regions are such that the cross section of the loops varies from top to bottom, For some loops the conditions are such that conduction dominates over other cooling mechanisms (Krieger 1978). In such cases loop geometry plays an important role (Antiochos & Sturrock 1976). In many cases the observed and the calculated conductive cooling times of flare loops do not agree with each other and the inclusion of geometrical inhibition of conduction does not in general help (Krieger 1978). This discrepancy can always be removed by postulating the existence of a source of heating, the indirect observational evidence of which is now becoming available (Levine & Withbroe 1977; Gerassimenko *et al.* 1978).

The evolution of temperature in a cooling flare loop is not well understood (Krall *et al.* 1978; Antiochos & Krall 1979). Therefore it is of interest to undertake such an investigation. Here we obtain expressions for the temperature distribution along a loop with and without geometrical inhibition in presence of a source of heating. Results are discussed in the context of the flare kernel of 1973 September 1.

2. Theoretical formulation

The energy equation of the problem is (Narain 1981)

$$\rho \frac{d}{dt}\left(1.5\frac{p}{\rho}\right) - \frac{p}{\rho}\frac{d\rho}{dt} = \frac{1}{A}\frac{\partial}{\partial s}\left(A\kappa\frac{\partial T}{\partial s}\right) + K_H(t)\exp\left(-s^2/\gamma^2R^2\right), \qquad \ldots(1)$$

where ρ is the plasma density, p the plasma pressure, A the area of cross section parameter, κ the coefficient of thermal conductivity, T the plasma temperature, $K_H(t)$ the time-dependent part of the source, s the arc length along the loop, γ the extension of the source and R the vertical height of the loop from bottom to top (Antiochos & Sturrock 1976). The flare loops are quite hot ($T \gg 10^6$ K). The scale height of such a hot plasma contained in the loops is $\sim 10^{10}$ cm which is larger than the height ($\sim 10^8$–10^9 cm) of the loops. Consequently gravitational effects may be ignored and the pressure taken to be independent of arclength s. For a fully ionized hydrogen plasma

$$p = 2nkT, \qquad \ldots(2)$$

where n is the electron number density and k the Boltzmann constant.

In equation (1) it is assumed that the flow of mass and energy takes place along the magnetic field lines which are current-free above the chromosphere. The coefficient of thermal conductivity along the magnetic field lines is (Antiochos & Sturrock 1976)

$$\kappa = \alpha T^{2.5} \quad (\alpha \approx 10^{-6}). \qquad \ldots(3)$$

The conduction across the field lines may be ignored. Following Krieger (1978) one can estimate the conductive and the radiative cooling times. A smaller conductive cooling time means that the conduction dominates over radiation. Equation (1) ignores the radiation loss.

In the post-flare phase the temperature and the heating both begin to decline. It is therefore expected that the physical variables may not have strong correlation in their dependence on space and time coordinates. Consequently temperature *etc.* may be written as a product of two functions one depending on arclength s and the other on time t. Thus

$$T(s, t) \equiv T_M T_s(s)\, T_t(t), \qquad \ldots(4)$$

where T_M is the temperature at $s = 0$ and $t = 0$.

2.1 *Constant cross section geometry*

In this case the area of cross section of the loop is throughout the same and the area of cross section parameter is given by (Antiochos & Sturrock 1976)

$$A(s) = 1, \qquad \ldots(5)$$

On combining equations (1) through (5) with the equation of continuity and the gas equation we get

$$\left(-\frac{dp}{dt} + \frac{2.5\,p}{T_t}\frac{dT_t}{dt}\right)\left(\frac{1}{\alpha\, T_M^{3.5}\, T_t^{3.5}}\right) + T_s^{1.5}\left(\frac{dT_s}{ds}\right)^2 +$$

(*equation continued on p. 58*)

$$+ \frac{K_H(t)\, I_1(\gamma s)}{\alpha T_M^{3\cdot 5}\, T_t^{3\cdot 5}\, T_s} \frac{dT_s}{ds} - \frac{1.5\, s}{\alpha T_M^{3\cdot 5}\, T_t^{3\cdot 5}\, T_s} \frac{dp}{dt} \frac{dT_s}{ds}$$

$$= \frac{d}{ds}\left(T_s^{2\cdot 5} \frac{dT_s}{ds}\right) + \frac{K_H(t) \exp\,(-\, s^2/\gamma^2 R^2)}{\alpha T_M^{3\cdot 5} \quad T_t^{3\cdot 5}}, \qquad \ldots(6)$$

where

$$I_1(\gamma s) \equiv \int_0^s \exp\,(-s^2/\gamma^2 R^2)\; ds. \qquad \ldots(7)$$

Equation (6) when solved leads to the temperature distribution along the loop.

The temperature falls and the electron density rises with time in the loops (Krall *et al.* 1978; Nagai 1980). Their variations are such that the pressure p falls slowly with time (Antiochos & Krall 1979). Consequently we may set $dp/dt \approx 0$ or $p = p_0 =$ constant till radiation becomes comparable to conduction. The time-dependence of the source of heating is not known. Either it can be fixed through observational results for a particular event (Strauss & Papagiannis 1971) or by assuming a particular time dependence and then comparing it with the observational results for different events. We assume the following time dependence for the source :

$$K_H(t) = Q_{\max}\, T_t^{3\cdot 5}. \qquad \ldots(8)$$

This renders equation (6) separable.

Under the boundary condition $T_t = 1$ at $t = 0$ the time dependent part integrates to :

$$T_t = (1 + (t/\tau_c))^{-2/7}, \qquad \ldots(9)$$

with

$$\tau_c = 2.5 p_0/\alpha T_M^{3\cdot 5} k_c^2\,, \qquad \ldots(10)$$

in which k_c^2 is the constant of separation. The part depending on arclength s has the form

$$\frac{d^2\psi}{ds^2} - \frac{2}{7\psi}\left(\frac{d\psi}{ds}\right)^2 - \frac{Q_{\max}\, I_1(\gamma s)}{\alpha T_M^{3\cdot 5}\, \psi} \frac{d\psi}{ds}$$

$$+ \frac{3.5\, Q_{\max} \exp\,(-\, s^2/\gamma^2 R^2)}{\alpha T_M^{3\cdot 5}} = -\, k_c^2\,, \qquad \ldots(11)$$

where

$$\psi \equiv T_s^{3\cdot 5}. \qquad \ldots(12)$$

One of the boundary conditions for solving equation (11) numerically is

$$\psi = 1,\; \frac{d\psi}{ds} = 0 \text{ at } s = 0. \qquad \ldots(13)$$

The other condition, following Antiochos & Sturrock (1978), is

$$\psi = 0 \text{ at } s = s_b. \qquad \ldots(14)$$

2.2 Line dipole geometry

Here the loop has a larger area of cross section at its top than that at its base. Following Antiochos & Sturrock (1976), we have

$$s = R\theta \qquad \ldots(15)$$

and

$$A = \cos^2 \theta. \qquad \ldots(16)$$

Proceeding in the same way as in the constant cross section case we get an equation which separates into θ- and time-dependent parts. The time-dependent part, under the boundary condition $T_t = 1$ at $\theta = 0$, integrates to

$$T_t = (1 + (t/\tau_D))^{-2/7}, \qquad \ldots(17)$$

with

$$\tau_D = 2.5 p_0/\alpha T_M^{3\cdot5}\, k_D^2, \qquad \ldots(18)$$

where k_D^2 is the constant of separation for the line dipole geometry. The θ-dependent part is

$$\frac{d^2\psi}{d\theta^2} - \frac{2}{7\psi}\left(\frac{d\psi}{d\theta}\right)^2 - 2 \tan\theta \frac{d\psi}{d\theta} - \frac{Q_{max}\, R^2 I_2(\gamma\theta)}{\alpha T_M^{3\cdot5}\, \psi \cos^2\theta}\frac{d\psi}{d\theta} + \frac{3.5\, Q_{max}\, R^2 \exp(-\theta^2/\gamma^2)}{\alpha T_M^{3\cdot5}} = -k_D^2 R^2, \qquad \ldots(19)$$

where

$$I_2(\gamma\theta) = \int_0^\theta \exp(-\theta^2/\gamma^2) \cos^2\theta \, d\theta. \qquad \ldots(20)$$

The corresponding boundary conditions for this case are

$$\psi = 1, \quad \frac{d\psi}{d\theta} = 0 \quad \text{at } \theta = 0; \quad \psi = 0 \quad \text{at } \theta = \theta_b. \qquad \ldots(21)$$

3. Method and results

The temperature distribution along the loop is given by equations (2), (4), (9), (10) through (14) and (17) through (21). An examination of these equations shows that one needs n, T_M, τ_c, p_0, k_c^2 Q_{max}, τ_D and k_D^2 for obtaining temperature distribution in constant cross section and line dipole geometries.

For the flare kernel of 1973 September 1 the necessary data are (Krieger 1978)

$$\left.\begin{array}{ll} n_0 = 2.0 \times 10^{11}\ \text{cm}^{-3} & \tau_{obs} = 4.5 \times 10^2\ \text{s} \\ T_M = 8.0 \times 10^6\ K & 2s_b = l = 1.5 \times 10^8\ \text{cm} \end{array}\right\} \qquad \ldots(22)$$

Here l is the total length of the loop and τ_{obs} the observed cooling time of the event. The quantity $p_0 = 2n_0kT_M$ can be evaluated from the data in a straight-forward manner. $s_b = R\theta_b$ determines θ_b when $R = 5.6 \times 10^7$ cm (Elwert & Narain 1980) is used.

We set $\tau_C = \tau_D = \tau_{obs}$ and determine k_C^2 or k_D^2 through equations (10) or (18). Q_{max} for constant cross section geometry is obtained by integrating equation (11) using Runge-Kutta method under boundary conditions (13) and (14). In line dipole geometry case Q_{max} is obtained from equations (19) through (21). The calculated data are exhibited in table 1.

Table 1. Computed parameters

Q_{max} (erg cm^{-3}s^{-1})				$k_C^2 = k_D^2$	p_0 (dyne cm^{-2})	θ_b (radian)
$\gamma = 0.1$		$\gamma = 1.0$				
$A = 1$	$A = \cos^2\theta$	$A = 1$	$A = \cos^2\theta$			
1150	350	185	55	1.69×10^{-18}	441.6	1.34

In figure 1 temperature has been plotted against arclength s for $\gamma = 1.0$ and $t = 100$ s for constant cross section and line dipole geometries. Figure 2 exhibits time-independent temperature against arclength along the loop for $\gamma = 1.0$ for static and nonstatic conditions. Figure 3 compares temperature distribution for $\gamma = 0.1$ and 1.0 as a function of θ.

4. Discussion and conclusions

An examination of figure 1 shows that the fall of temperature in constant cross section geometry is faster than in the line dipole geometry, as expected. The flow of heat is restricted in line dipole geometry where half of the loop is like a funnel but it is unhindered in the constant cross section geometry.

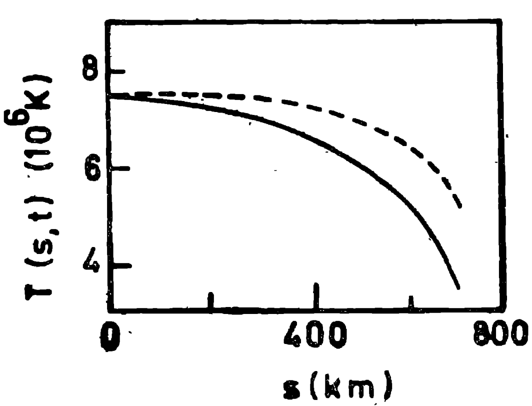

Figure 1. Plot of temperature $T(s, t)$ vs arclength s along the loop: dotted line refers to line dipole geometry and solid line to constant cross section geometry ($\gamma = 1.0$ and $t = 100$ s).

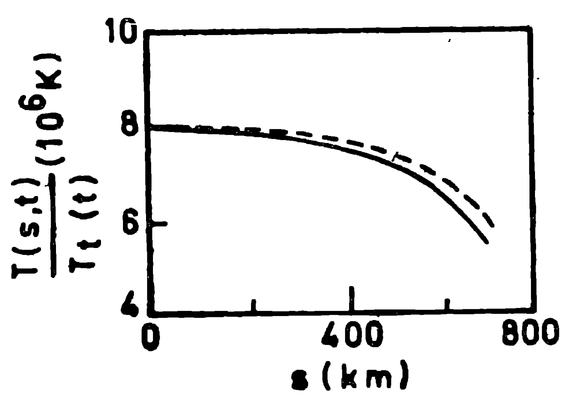

Figure 2. Plot of $T(s, t)/T_t(t)$ vs arclength s along the loop: dotted line refers to nonstatic case and solid line to static case ($\gamma = 1.0$).

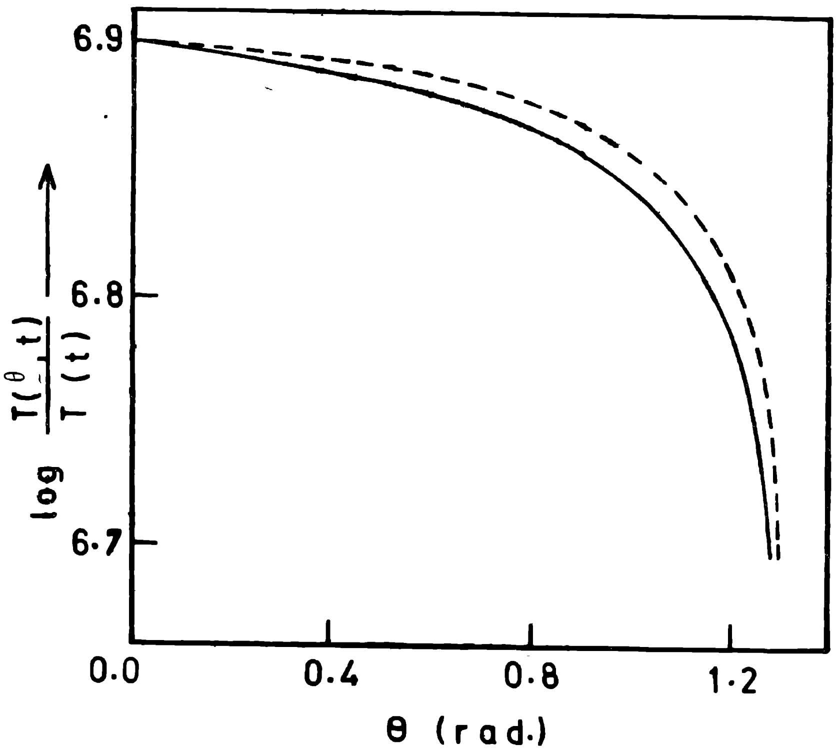

Figure. 3. Plot of log $\{T_c(\theta, t)/T_t(t)\}$ vs θ : dotted line refers to a source of larger extension (γ = 1.0) and solid line refers to a source of smaller extension (γ = 0.1).

In static case no exchange of plasma between the loop and the chromosphere takes place (Elwert & Narain 1980). Therefore no fraction of heat energy lost to chromosphere returns back to the loop. In the nonstatic (present) case the heat flux conducted to the chromosphere evaporates the chromospheric material which enters the loop through its base. Thus part of the heat energy lost to the chromosphere is gained back through the evaporated material. Therefore the fall of temperature in static case is expected to be faster than that in nonstatic case. This is what figure 2 leads one to conclude. It may be remarked that the cooling in nonstatic case is faster near the base than near the top of the loop because the cooler material from the chromosphere exchanges heat with the loop plasma near the base first and gets heated. As this material rises in the loop its temperature goes on increasing and its capacity to cool decreases.

The case $\gamma = 0.1$ corresponds to a source of smaller extension. It implies that roughly one tenth of the loop is heated. The case $\gamma = 1.0$ represents a source of larger extension. The implication of this is that the whole loop is being heated. In order that the calculated cooling time be equal to the observed cooling time the source of smaller extension ($\gamma = 0.1$) must be stronger than a source of larger extension ($\gamma = 1.0$). Table 1 clearly demonstrates this fact. A stronger source confined to a small region at the top of the loop will cool faster than a weaker one extended over the whole length of the loop. Therefore the fall of temperature for a stronger source of smaller extension will be faster than that of a weaker source of larger extension. This is what figure 3 depicts. It may be noted that the surroundings for a source with $\gamma = 0.1$ are cooler than those for a source with $\gamma = 1.0$.

The conclusions drawn in the foregoing paragraphs are not likely to be altered by a more refined theory although the assumption of keeping pressure independent of time is not very much justified and leads to overestimation of Q_{max} (Narain 1981).

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