THE DUST GRAINS IN THE COMA OF COMET WEST

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Abstract. The observed variation of reddening as function of the heliocentric distance and the spatial variation of reddening within the coma of Comet West in the visual wavelength range have been considered to infer the properties of the cometary dust grains. The relevant model incorporates the variation in the size distribution function as well as the composition of the spherical grains. The real part of the complex index of refraction ($m = m' - im''$) is chosen such that $m' = 1.6$. The imaginary part is required to vary from $m'' = 0.2$ to 0.05 over the wavelength range 0.4 to 0.7 μm. This choice of refractive index corresponds to dirty silicate grains. As a by-product, the model also satisfies the observed polarization and albedo for the Comet West.

1. Introduction

The dust grains present in a cometary environment play a crucial role in many of the observed phenomena in comets. Unfortunately the nature and composition of the dust grains is still not very clear at the present time although there have been many investigations on this subject (see, for example, Wilkening, 1982; Hanner, 1984; Krishna Swamy, 1986; Anon, 1986). There are various kinds of observations which can be used to infer the nature of the dust grains (see, for example, Ney, 1982; Hartmann and Cruikshank, 1984). Comet West which was quite bright and rich in dust gave an opportunity for making a variety of observations (Ney and Merrill, 1976; Oishi *et al.*, 1978a, b; Michalsky, 1981; Sivaraman *et al.*, 1979; Bappu *et al.*, 1980). The observations and analysis in the infrared region have already been presented by Ney and Merrill (1976) and Oishi *et al.* (1978a, b). The present study in the visual wavelength region thus complements their work. The particular observations of interest here are mainly those of Sivaraman *et al.* (1979) and Bappu *et al.* (1980) who have measured for the first time the reddening curve as a function of the heliocentric distance. They have also measured reddening as a function of radial distance from the center of the nucleus of Comet West. Our objective here is to consider these observations among others and to construct plausible grain models which can explain them.

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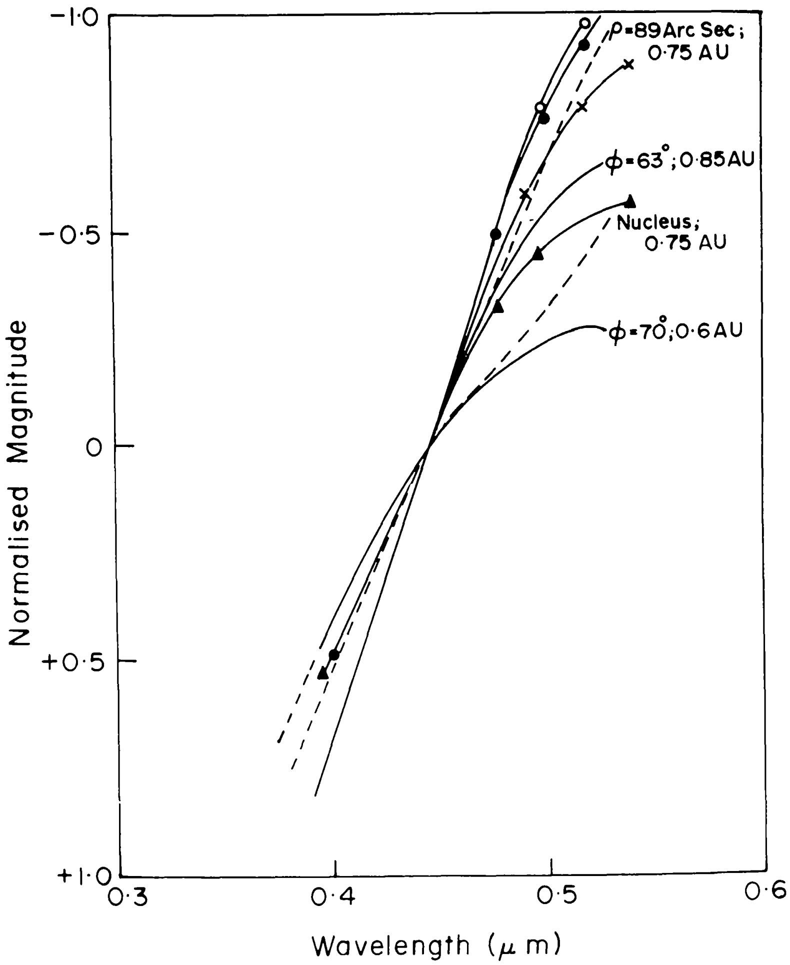

Fig. 1. The observed and calculated reddening curves for Comet West are plotted as functions of wavelength. The curves are normalized to $\lambda = 4465$ Å. Observations: The solid line curves show the variation due to heliocentric distance (r) and correspond to the two extreme phase angles $\phi = 63°$ ($r = 0.85$ AU) and $\phi = 70°$ ($r = 0.6$ AU). The dashed curves show the observed spatial variation within the coma when the telescope pointed towards the center of the nucleus, i.e. $\rho = 0$ and towards the offset angle $\rho = 89$ arc sec, the heliocentric distance in either case being $r = 0.75$ AU. Calculations: The solid line curves running through the points represent the present model and correspond to the parameter, α, as follows: (○ ○ ○ ○ ○) $\alpha = +2$; (● ● ● ● ●) $\alpha = 0$; (× × × × ×) $\alpha = -2$; (▲ ▲ ▲ ▲ ▲) $\alpha = -4$. Note that $a_{min} = 0.1$ μm, $a_{max} = 0.5$ μm; index of refraction $m(\lambda) = 1.6 - im''(\lambda)$ with $m''(\lambda)$ varying smoothly between 0.2 to 0.05 in the range 0.40 μm $\leq \lambda \leq$ 0.70 μm.

2. Model Calculations and Discussion

2.1. Continuum Scattering

The analysis of the continuum observations of comets has been one of the main sources of information for deducing the properties of dust grains. The observations of several comets show that there appears a reddening in the scattered radiation relative to the solar spectrum as one goes from visual to near infrared wavelength region (Liller, 1960; Stokes, 1972; A'Hearn *et al.*, 1980). The observations of good quality exist for Comet West which was one of the bright dusty comets. Extensive

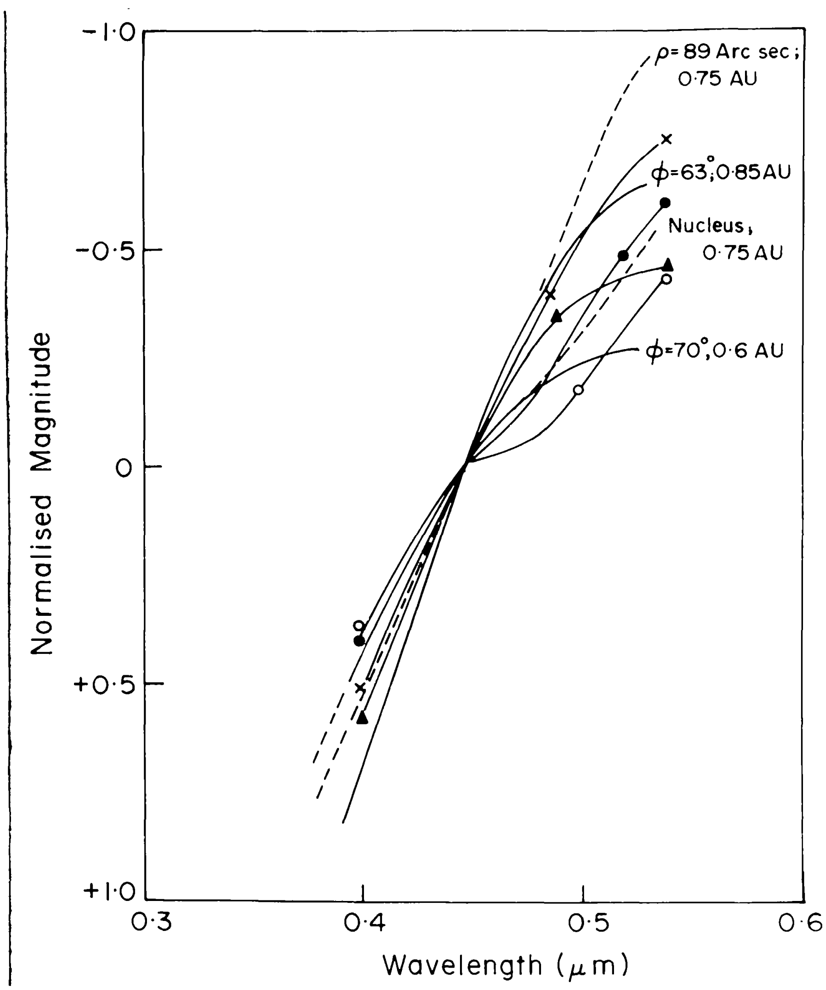

Fig. 2. The observed and calculated reddening curves for Comet West are plotted as functions of wavelength. The curves are normalized to λ = 4465 Å. Observations: Same as Figure 1. Calculations: Same as Figure 1, but for the size range a_{min} = 0.1 μm and a_{max} = 1.0 μm.

observations have been carried out on this comet as functions of the heliocentric distance, r, as well as the distance, ρ, from the nucleus in the coma (Sivaraman *et al.*, 1979; Bappu *et al.*, 1980). These results for reddening, normalized to λ = 4465 Å, are plotted in Figures 1 to 3. The continuum light scattered by the dust grains within the coma suffers reddening with reference to the original solar spectrum. One may notice a gradual increase in reddening with heliocentric distance or with decrease in the phase angle ϕ. This feature is illustrated by two solid line curves specified by the extreme values of phase angle $\phi = 63°$ and 70° in each of the Figures 1 to 3. The corresponding heliocentric distances are 0.85 and 0.6 AU, respectively. Figures 1 to 3 also include the reddening curves for the radial distance ρ from the nucleus of the coma when the Comet West was at $r = 0.75$ AU. They are shown by two dashed line curves for the extreme values of $\rho = 0$ (i.e. nucleus) and $\rho = 89$ arc sec. The most interesting result is the progressive increase in reddening with increasing radial distance, ρ, from the center of the coma. The variation in

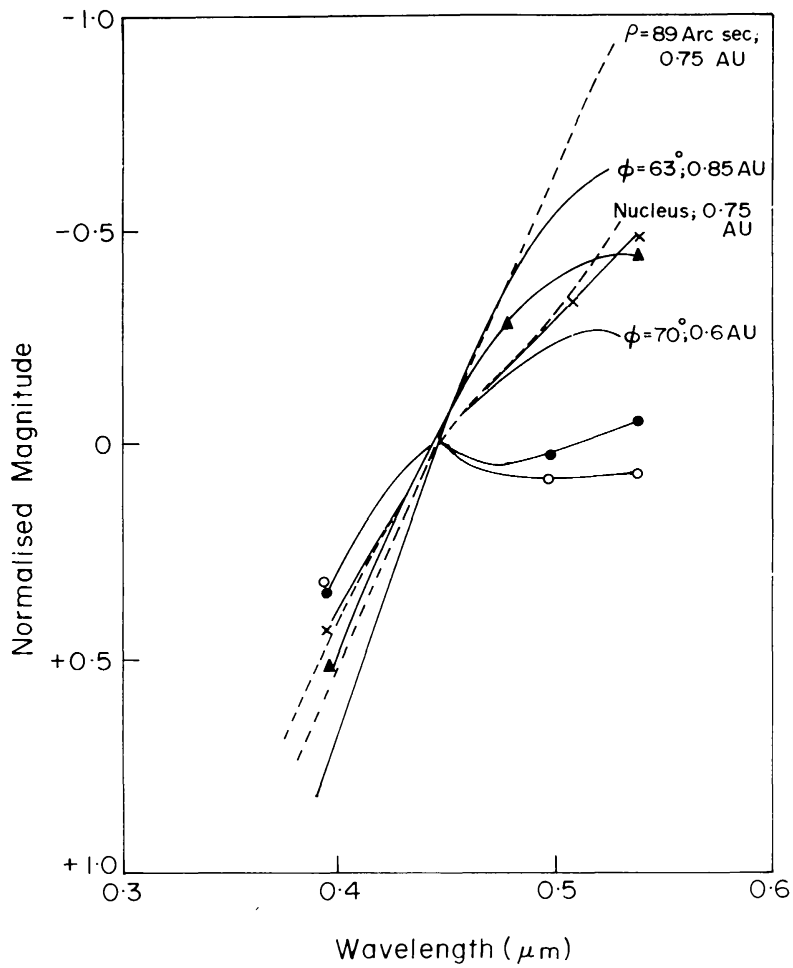

Fig. 3. The observed and calculated reddening curves for Comet West are plotted as functions of wavelength. The curves are normalized to $\lambda = 4465$ Å. Observations: Same as Figure 1. Calculations: Same as Figure 1, but for the size range $a_{min} = 0.1$ μm and $a_{max} = 2.0$ μm.

reddening, as one goes from $\rho = 0$ to $\rho \simeq 90$ arc sec, amounts approximately to 0.5 mag at $\lambda = 5265$ Å (with reference to $\lambda = 4465$ Å).

It is assumed that the grains are homogeneous smooth spheres. Therefore, the calculations are based on Mie theory of scattering of electromagnetic waves (see, for example, van de Hulst, 1957). It is further assumed that single scattering holds.

The expected intensity of the scattered light at wavelength λ and phase angle $\pi - \theta$ has been calculated according to the usual relation

$$I(\lambda, \theta) = \frac{I_0(\lambda)\, \lambda^2}{8\pi^2 r^2\, \Delta^2} [\bar{i}_1(\lambda, \theta) + \bar{i}_2(\lambda, \theta)], \quad (1)$$

where

$I_0(\lambda)$ = the solar intensity at 1 AU,
r = the Sun-Comet distance in AU,
Δ = the Comet-Earth distance,

θ = the scattering angle, i.e. the supplement of the phase angle, ϕ, and
$i_{1,2}$ = the orthogonally polarized intensity scattering functions. The suffices 1 and 2 are for components perpendicular and parallel, respectively, to the scattering plane defined by the Sun-Comet-Earth plane.

A bar over a quantity denotes the average with respect to the size distribution function. Thus the intensity functions i_1 and i_2, polarized with electric vectors of the incident light beam perpendicular and parallel, respectively, to the scattering plane, have been averaged over the size distribution function as

$$\bar{i}_{1,2}(\lambda, \theta) = \frac{\int_0^\infty f(a)\, i_{1,2}(\lambda, \theta)\, \mathrm{d}a}{\int_0^\infty f(a)\, \mathrm{d}a}, \tag{2}$$

where $f(a)$ denotes the size distribution function and $f(a)$ da represents the relative number of dust grains in the size interval a to $a + da$. The question of the size distribution function in comets is still open. The power law distribution function for the grain sizes has been generally accepted in many investigations (see, for example, Remy-Battiau, 1964; 1966; Donn *et al.*, 1967; Finson and Probstein, 1968a, b; Oishi *et al.*, 1978b; A'Hearn *et al.*, 1984). Therefore, we have chosen here a representative power law for size distribution as defined by

$$f(a)\, \mathrm{d}a = \text{const.} \times a^\alpha\, \mathrm{d}a, \tag{3}$$

where α is a parameter and the grain radius a is covered in a range between minimum ($a_{\min}$) and maximum ($a_{\max}$) values. The calculations of the scattering intensity functions i_1 (λ, θ) and i_2 (λ, θ) involve a knowledge of the shape and composition of the grains. The laboratory measurements on individual interplanetary dust particles collected from the stratosphere (see, for example, Fahey *et al.*, 1985) indicate the presence of certain silicate materials in comets. Some features also show the presence of carbonate minerals similar to those found in the spectra of some protostars. Cometary dust models based on microwave analog experiments (Weiss–Wrana *et al.*, 1985a) indicate the role of silicate materials in explaining the observed intensity of Comet West reasonably well but simultaneous fit to the degree of polarization is lacking. Oishi *et al.* (1978b) have included a mixture of metallic dust particles (graphite or iron) and dielectric dust (silicate) particles to reproduce some of the observed features of the energy spectra and polarization. However, the form of the phase function as well as the characteristic 10 μm feature in Comet West are indicative of the dielectric nature of the scattering particles (see, for example, Ney and Merrill, 1976; Michalsky, 1981; Ney, 1982).

Therefore, we have adopted the models based on dirty silicate spherical grains with representative refractive index $m = 1.6 - im''$. The imaginary part m'' is varied until the calculated results agree with the observations. Furthermore, we have considered the variation of grain size distribution by varying the parameter α in the size distribution function as well as the upper ($a_{\max}$) and the lower ($a_{\min}$) bounds on the radii of the grains.

In real situation, the scattering particles in comets or interstellar medium could be regular and irregular in shape, fluffy or nested conglomerate of small particles. The theoretical treatments of irregular particles are mainly based on the use of Fresnel reflection coefficients for particles large in size or size-to-wavelength ratio. This means that one has to restrict to the geometrical optics regions of the scattering domain. This necessarily implies a scalar wave theory which is limited in application. However, in the case of small irregular and/or inhomogeneous particles in ultraviolet, visual and ifnrared wavelength regions, one definitively needs a vector electromagnetic wave theory of scattering equivalent to Mie-like theories for spheres (van de Hulst, 1957; Güttler, 1952), cylinders (Lind and Greenberg, 1966; Shah, 1970) and spheroids (Asano and Yamamoto, 1975; Schaefer, 1980). Other theoretical methods for irregular shapes and the laboratory microwave analog methods pioneered by Greenberg *et al.* (1961) and Giese and Siedentopf (1962) in this connection have advantages as well as limitations discussed by Greenberg *et al.* (1971), Shah and Vardya (1972), Shah (1976), and Zerull (1985). At present there is a great handicap in constructing the models of scattering involving odd shapes of scatterers. The use of irregular particles as model of dust grain has begun to be in vogue in the recent past (see, for example, Schuerman, 1980; Giese and Lamy, 1985). However, we wish to point out the following remark made by Lamy (1985) in summarizing the recent work on the properties and interactions of interplanetary dust: 'Looking in retrospect to past conferences in our field, has led me to refrain from drawing any definite conclusions'. The observational, laboratory and theoretical studies need to be pursued further to arrive at clear cut picture about the nature of the dust grains. But at the present time the situation is very confusing. Because of these considerations, the present calculations have been based on the classic Mie theory for spheres (see, for example, van de Hulst, 1957).

We have performed the calculations for various values of refractive indices and the parameter α in Equation (3). The real part of the refractive index is kept at 1.6 with the corresponding imaginary part to be $m'' = 0.02$, 0.2 and 0.5. The parameter α in Equation (3) has been varied from $+4$ to -4 with several combinations of $a_{\min}$ and $a_{\max}$ as the cut off radii. The calculated results on the reddening agreed with the observations in the near ultraviolet region but not in the near infrared region. However, we could infer from these calculations that a proper variation of the refractive index of the model grains is required. Therefore, we tried as an experiment a variation of the refractive index in the form $m = 1.6 - 0.2i$ in the wavelength range $0.40 \leq \lambda < 0.58$ μm and $m = 1.6 - 0.05i$ for $0.58 \leq \lambda \leq 0.70$ μm. This produced the variation of the reddening in the correct direction. Since a step function for the refractive index does not seem to be physically realistic one in the wavelength region under consideration, we have chosen a smooth variation of m'' from 0.20 to 0.05 between $\lambda = 0.40$ to 0.70 μm. The present choice of the index of refraction conforms to the ranges, $1.3 < m' < 2$ and $0.01 < m'' < 0.1$, specified by Michalsky (1981) except that out imaginary part extends beyond the upper limit. These particular ranges of m' and m'' are characteristic of a variety of impure silicates such as

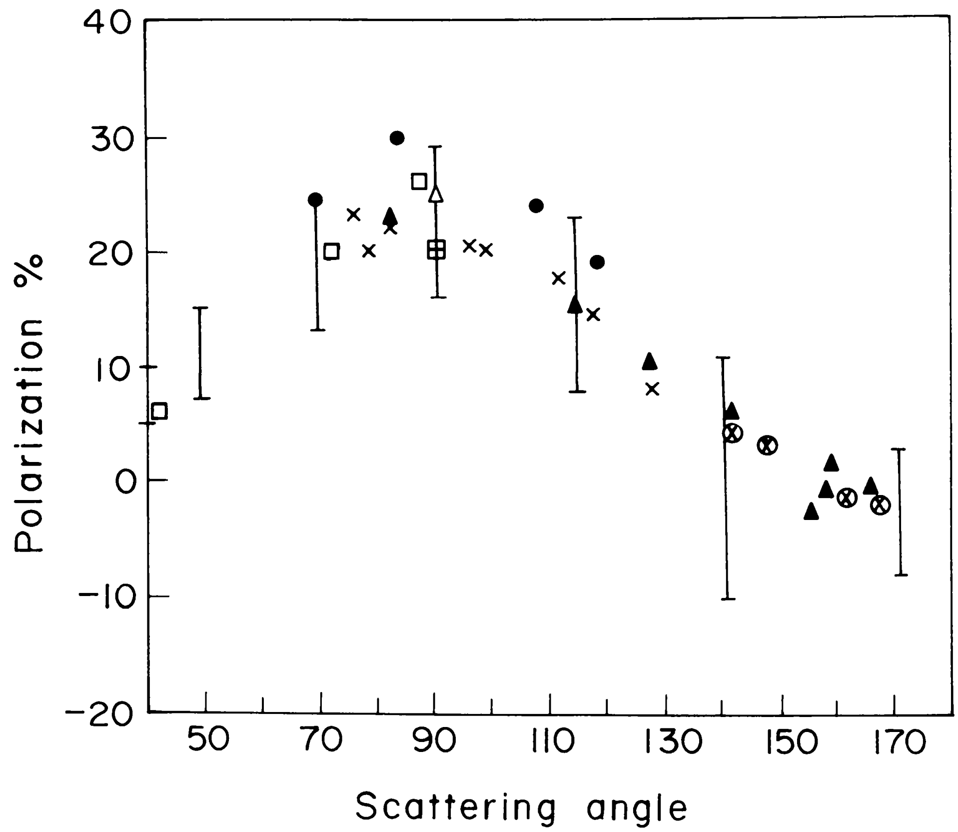

Fig. 4. A plot of the observed and calculated polarization vs scattering angle. Observations: Comet West (□ □ □ □ □) Michalsky (1981); (● ● ● ● ●) Oishi *et al.* (1978); (▲ ▲ ▲ ▲ ▲) Kiselev and Chernova (1978). Comet austin (× × × × ×). Myers and Nordieck (1984), Comet Churyumov–Gerasimenko (⊕⊕⊕⊕⊕) Myers and Nordsieck (1984), Comet Arend-Roland (⊞⊞⊞⊞⊞) Bappu and Sinvhal (1960), Comet Ikeya-Seki (ΔΔΔΔΔ) Bappu *et al.* (1967). Calculations: The vertical bars (I I I) shows the polarization variation for wavelengths between 0.4 to 0.7 μm. Note that $a_{min} = 0.1$ μm and $a_{max} = 2.0$ μm; index of refraction $m(\lambda) = 1.6 - im''(\lambda)$ with $m''(\lambda)$ varying smoothly between 0.2 to 0.05 in the range 0.4 μm $\leq \lambda \leq$ 0.70 μm and for $\alpha = -4$.

enstatite, olivine, etc. We have also been able to narrow down the range of sizes of the silicate grains as anticipated by Ney and Merrill (1976) and Michalsky (1981) on the basis of infrared observations. In Figure 1 to 3, the results of our calculations have been shown by the curves passing through open circles, dots, crosses and filled triangles which correspond to the values of the parameter $\alpha = 2, 0, -2$, and -4, respectively. As can be seen the agreement with the observed reddening is quite good provided appropriate value of α is selected with each phase angle. The observations also require the size range to be $0.1 \leq a \leq 0.5$ μm for the variation of the reddening within the coma and $0.1 \leq a \leq 1.0$ or 2.0 μm for the variation of reddening with heliocentric distance.

2.2. Polarization

The observed polarization of light scattered by dust grains in comets is another set of data which has to be explained by any dust grain model. Since circular polarization

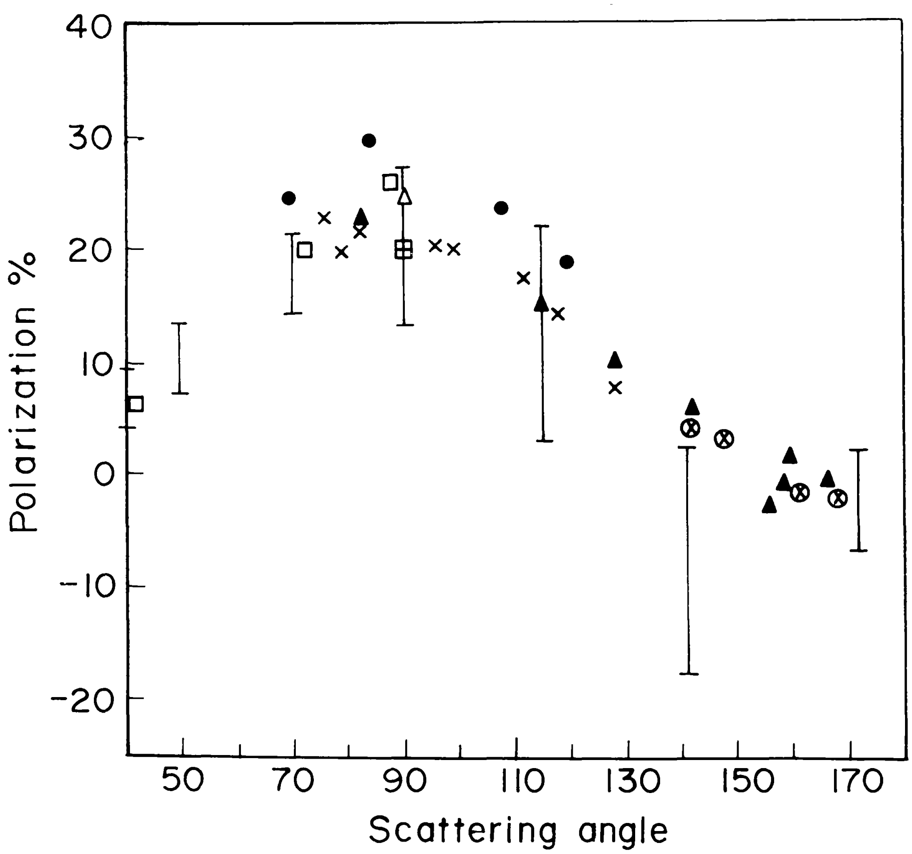

Fig. 5. A plot of the observed and calculated polarization vs scattering angle. Observations: Same as Figure 4. Calculations: The refractive index is the same as in Figure 4. The size range is $a_{min} = 0.1$ μm and $a_{max} = 0.5$ μm.

has not been observed for any comet so far, we are concerned only with the linear polarization here. A system study of the polarization is not available for a single comet and the observations are scanty at the present time (Bappu and Sinvhal, 1960; Bappu *et al.*, 1967; Myers and Nordsieck, 1984). An exception is the Comet West; there exist many more observations compared to all other comets (Michalsky, 1981; Oishi *et al.*, 1978a; Kiselev and Chernova, 1978). The available observations for various comets are shown in Figure 4. These observations seem to indicate that the variation of polarization with wavelength is quite small. In fact, the linear polarization for Comet West observed by Michalsky (1981) is stated to be wavelength independent in the visual wavelength region. Figure 4 shows that the variation of polarization with scattering angle for various comets is very similar and therefore the average curve is well defined. The model grains which satisfy the reddening in Figure 1 to 3 have been used to derive the results on polarization plotted in Figures 4 and 5. Note that the variation of the calculated polarization in the wavelength range 0.40 μm $\leq \lambda \leq$ 0.70 λm is shown by the vertical bars. Figure 4 and 5 show that the shape of the calculated polarization curve compares reasonably well with the observed variations. The shapes of the calculated curves for the size ranges $0.1 \leq a \leq 0.5$ μm and $0.1 \leq a \leq 2.0$ μm are very similar. We have seen earlier that the

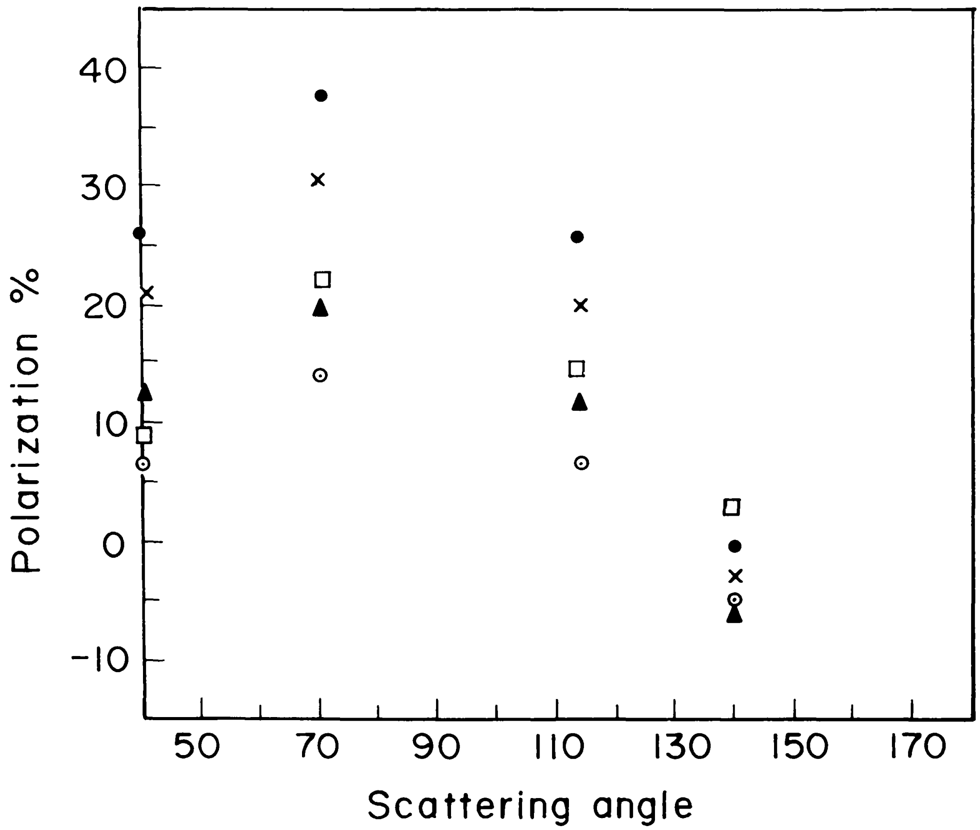

Fig. 6. A plot of the calculated polarization vs scattering angle for $\lambda = 4500$ Å and for various values of α. The refractive index and the size range are the same as in Figure 5. (● ● ● ●) $\alpha = +4$; (× × × ×) $\alpha = +2$; (▲ ▲ ▲ ▲) $\alpha = 0$; (⊙ ⊙ ⊙ ⊙) $\alpha = -2$; (□ □ □ □) $\alpha = -4$.

size range $0.1 \le a \le 0.5$ μm is consistent with the observed reddening curve. It should however, be pointed out that for polarization to be almost independent of wavelength, the scattering particles should be reasonably large such that $2\pi a/\lambda >> 1$. However, the small particles in the range $0.1 \le a \le 0.5$ μm would produce high polarization in the visual wavelengths; this goes against the observations. Such small particles can also produce some variation in polarization with wavelength. In this connection, it should be borne in mind that there is uncertainly in the measurements of polarization. More improved observations on other comets in future should clarify this point.

2.3. Albedo

The average bond albedo, A, is defined by

$$A = \frac{Q_{sca}}{Q_{ext}}, \tag{4}$$

where Q_{sca} and Q_{ext} are the scattering and extinction efficiencies, respectively. By definition, $Q_{ext} = Q_{sca} + Q_{abs}$, Q_{abs} being the absorption efficiency. Thus Equation

(4) can be reduced to the form

$$\frac{A}{1-A} = \frac{Q_{\text{sca}}}{Q_{\text{abs}}} . \tag{5}$$

Equation (5) can now be directly compared with the ratio of the scattered radiation intensities in the visual spectral region to the infrared thermal radiation flux emitted by the grains (O'Dell, 1971; Ney and Merrill, 1976). In the former case, it is the scattering efficiency of the grains which comes into the picture and in the latter case, it is the absorption efficiency of the grains which is important. Sometimes an assumption of isotropic scattering is introduced to facilitate the integration of the scattered intensities over all the scattering angles. This kind of procedure may lead to over or under-estimate of the albedo if the data on scattered intensities are inadequate. For Comet West, a conservative estimate of the albedo, A, derived from the observations lies in the range, $0.3 \leq A \leq 0.5$ (Ney and Merrill, 1976). The model grains as discussed in the present work give $A \simeq 0.5$ at $\lambda = 0.5$ μm.

3. Conclusions

It is found that, for Comet West, the observed reddening as a function of heliocentric distance as well as its spatial variation within the coma in the wavelength region 0.40 to 0.70 μm, require the model grains to have the index of refraction $m = 1.6 - im''$ and a narrow size range from 0.1 to 0.5 μm. The imaginary part m'' varies smoothly from 0.2 to 0.05 over the same wavelength region. This refractive index roughly corresponds to the dirty silicate material reminiscent of the grains envisaged in variety of environments such as circumstellar envelopes and interstellar clouds. It is hoped that comprehensive observations carried out on Comet Halley may warrant further detailed models of the dust grains.

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