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Irradiation effects in close binary stars

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We studied the reflection effect in 2-dimensional geometry to see how the field will change if we calculate the reflected radiation by the transfer equation in a close binary system. The reflected radiation is calculated from the extended surface of the components of a close binary system assuming 3-dimensional Cartesian coordinate geometry and circular orbits. The specific intensity of the radiation field is estimated along the line of sight for an observer at infinity. It appears that radiation field changes depending upon the position of the secondary component.

1. Introduction

Most stars are found in groups that are gravitationally bound to each other. The majority of these stars are found in binary systems which are systems of two stars in orbit around a common center of mass. Binaries are useful systems for astronomers because two stars in orbit obey well understood laws of motion. From their orbital velocities and periods, it's possible to calculate the combined and individual masses of the stars in a system. When the stars are plotted by their brightness and their spectral type on Hertzsprung-Russell diagram, most stars falls on to a narrow band called main sequence. But the stars that compose close binary systems are not the same luminosity as star of similar type, so they fall slightly off the main sequence-the primary stars tend to appear below it, while the secondary above it.

Kopal [1] first classified all close binaries into three groups - detached, semi-detached, and contact systems. In detached systems both star remain well within their respective Roche lobes (β -Aurigae). In semi-detached systems however, one component fills its Roche lobes (Algol, β Persi). Contact binaries are still exotic -both components fill their Roche lobes and continually interact (44-Bootis B). One can think of a quite a number of binary star configurations in which relatively simple numerical treatment of reflection will be adequate. Most obvious is that of slowly rotating stars, which are well detached from their limiting lobes and are therefore, not far from spherical. Systems of main sequence stars with radii of the order of 10%-15% of their separation fall into this category and or reasonably common (for stars which are smaller than this there is, in most cases, hardly any reflection effect).

In a close binary system each component will receive the radiation from its companion. If a star is to remain in radiative equilibrium, all the energy received from outside must be re-emitted, without altering the rate of escape of radiation from the deep interior. This phenomenon, known as the " reflection effect " is inevitable in close binaries. To explain this effect properly further study of the problem is necessary to answer - or at least to give some clues to - several fundamental parameters such as mass and radii. In radiative transfer theory, we solve the equation of radiative transfer by assuming certain geometrical configuration such as plane parallel or spherical symmetric stratification of the media. These geometrical configurations assume symmetric boundary conditions and whenever we have asymmetric incident radiation, the solutions developed in the context of symmetrical geometries as mentioned above, will have to be modified. Such problems are encountered in the evaluation of radiation from the irradiated component of the binary system. If we treat one of the component as a point source in a binary system then the problem of incident radiation from such source is equivalent to the searchlight problem. Chandrasekhar [2] has calculated the diffuse scattering function in a plane parallel medium when a pencil of beam of radiation from a point source is incident. There have been several attempts for calculating the diffuse radiation field in such simple geometries. However, the calculation of the radiation field during the eclipses in close binaries is of different complexity. There are two important aspects one should take into account: (1) the physical processes that takes place inside the atmosphere and (2) the geometrical shape of the illuminated surface which reflects the light. Generally, if the atmosphere of the component under consideration is extended or fills its Roche lobe, then the problem of determining the emergent radiation from such surfaces become very difficult. The process of estimating the radiation field from such surfaces become complicated when the various competing physical processes are taken into account. Geometrical considerations alone would complicate the calculations because of the deformed shape due to tidal effects from the neighbour and due to self radiation. The resultant shape would be an ellipsoid and the problem requires special treatment. The solution of radiative transfer equation either in plane parallel symmetry or in spherical symmetry or in cylindrical symmetry cannot accurately describe the radiation field emanating from such surfaces.

Since primary components of close binaries do not have spherically symmetric shapes, their shapes being distorted by the tidal forces due to the presence of secondary component and their self rotation. One can use the solution of radiative transfer equation developed in 3-dimensional co-ordinate axes. Peraiah [3] studied the reflection effect in 2-dimensional geometry to see how the radiation field will change if we calculate the reflected radiation by employing the transfer equation. Peraiah [4] extended this work when secondary component is an extended surface in a close binary system. Peraiah and Srinivasa Rao[5-8] studied the effect of reflection on the formation of spectral lines and also in the presence of dust. However, one has to study other methods which are available and fairly accurate. The problem of incidence from a point source or an extended source is termed as a searchlight problem. Chandrasekhar [9] and Rybicki [10] made few attempts but the problem remains unsolved in its total complexity. Buerger [11] employed plane parallel approximation in computing the continuum radiation and line radiation emitted by rotationally and tidally distorted stars which is irradiated by the light of secondary component. In this paper we have made an attempt to study the irradiation from the secondary component in 3-dimensional geometry. We have developed a method of obtaining the radiation field along the spherical surface irradiated by an external source of radiation as a preliminary step to understand the reflection effect in close binaries in 3-dimensional geometry.

2. Method of calculation

All the calculations are performed in the 3-dimensional X-Y-Z Cartesian geometry as shown in figure 1. The binary model can be used for a system B type primary (of all different spectral types) and a hot white dwarf secondary. In this method we assume a spherical shell of the primary star with inner and outer radii R_{in} and R_{out} respectively. The center of the star is at the origin of coordinates. We assume that radiation is incident from a point source at B moving on a circle of radius R in the X-Z plane. We calculate the radiation field reflected from the spherical shell. We divide the shell into several circular slices such MNPQ parallel to Z-Y plane, with their centers lying on X-axis. We consider the transfer of radiation along the lines such as QS_2RO. Therefore in the X-Y-Z Cartesian coordinate system we should be able to determine the coordinates of any point.

We assume that the secondary component is situated at the point B and a ray from B intersects the outer surface at S_1 and passes through the point S_2 on the line QS_2RO. Let the coordinates of the points S_2 and B (in figure 1) be (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively. The equation of this line is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}. \tag{1}$$

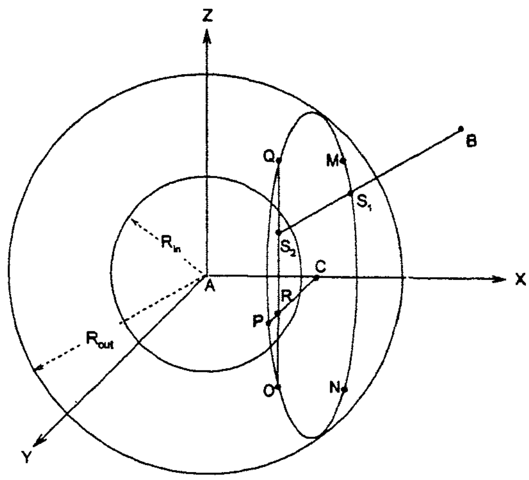

Figure 1: Shows the Schematic diagram of the Cartesian co-ordinate system in X-Y-Z geometry

By knowing the coordinates of the points S_2 and B in advance, the coordinates of the points S_1 are obtained by solving equation (1) and the equation of the sphere, whose center is at A. The equation of the sphere is given by

$$x^2 + y^2 + z^2 = R_{\text{out}}^2. \tag{2}$$

The length of the segments are calculated using distance formula. We need to avoid all points where the incident radiation does not reach such as the shadow cone cast by the central star. We calculate the equation of the cone from the enveloping sphere

$$x^2 + y^2 + z^2 = R_{\text{in}}^2. \tag{3}$$

This is given by

$$(x^2 + y^2 + z^2 - R_{\text{in}}^2)(x_2^2 + y_2^2 + z_2^2 - R_{\text{in}}^2) - (xx_2 + yy_2 + zz_2 - R_{\text{in}}^2)^2 = 0. \tag{4}$$

The points that lie in the shadow of this cone should satisfy the relation

$$(x^2 + y^2 + z^2 - R_{\text{in}}^2)\ (x_2^2 + y_2^2 + z_2^2 - R_{\text{in}}^2) - (xx_2 + yy_2 + zz_2 - R_{\text{in}}^2)^2 \le 0 \tag{5}$$

all those points which satisfies the relation are eliminated from calculation.

We estimate the radiation field that is transfered along the line segments such as S_1S_2 to obtain the source function along the lines QS_2RO. We employed the procedure described in [3].

2.1 Calculation of radiation filed due to irradiation

We shall briefly describe the calculation of the source functions derived in the 1-dimensional rod model. The total source function including the diffuse radiation field given by

$$S_d^+(\tau) = S^+(\tau) + \omega(\tau)\Big[p(\tau)I_1 e^{-\tau} + (1 - p(\tau))I_2 e^{-(T-\tau)}\Big], \tag{6}$$

$$S_d^-(\tau) = S^-(\tau) + \omega(\tau)\Big[(1 - p(\tau))I_1 e^{-\tau} + p(\tau)I_2 e^{-(T-\tau)}\Big], \tag{7}$$

where $S^+(\tau)$ and $S^-(\tau)$ are the source functions at the optical depth τ (for details see [3]). $\omega(\tau)$ is the albedo for single scattering and p is the phase function equal to $\frac{1}{2}$ in this case.

We set $\omega(\tau) = 1$ which corresponds to pure scattering in the medium. I_1 and I_2 are the incident specific intensities at the boundaries at $\tau = 0$ and $\tau = T$ respectively. For isotropic scattering $S_d^+ = S_d^-$ and equation (6) and (7) will reduce to

$$S_r = \frac{1}{2}\left[I^+ + I^-\right] + \frac{1}{2}\left[I_1 e^{-\tau} + I_2 e^{-(T-\tau)}\right], \tag{8}$$

where T is the total optical depth.

We set $\tau = 0$ at point S_1 (see figure 1) where the incident ray enters the medium, and we set $\tau = T$ at the point S_2 where the source function is calculated.

$$I^+(\tau) = I_1 \frac{1 + \left[T - \tau\right]\left[1 - p\right]}{1 + T\left[1 - p\right]}, \tag{9}$$

and

$$I^-(\tau) = I_1 \frac{\left[T - \tau\right]\left[1 - p\right]}{1 + T\left[1 - p\right]}, \tag{10}$$

$$I^-(\tau = T) = 0 = I_2, \tag{11}$$

$$I^+(\tau = 0) = I_1, \tag{12}$$

therefore

$$I^+(\tau = T) = I_1 \frac{1}{1 + \frac{1}{2}T}, \tag{13}$$

$$I^-(T) = 0. \tag{14}$$

At $\tau = T$, the source function becomes,

$$S_r = \frac{1}{2}\left[I^+ + I^-\right] + \frac{1}{2}\left[I_1 e^{-T}\right] \tag{15}$$

Introducing equation (9) and (10) into the above equation with $p = \frac{1}{2}$, we obtain

$$S_r = \frac{1}{2} I_1 \left[\frac{2}{2 + T} + e^{-T}\right] \tag{16}$$

Using the above analysis we can calculate the source functions according to one-dimensional rod model.

In addition to the irradiation from the secondary component we have the radiation from the primary star itself. In the next section we shall describe method of calculation of self radiation of the primary component.

2.2 Calculation of self radiation of the primary component

The radiative transfer equation in a spherically symmetric approximation is

$$\mu \frac{\partial I(r,\mu)}{\partial r} + \frac{1}{r}\frac{\partial}{\partial \mu}\left[(1 - \mu^2) I(r,\mu)\right] + \sigma(r) I(r,\mu) = \sigma(r)\left[S_s(r) - I(r,\mu)\right], \tag{17}$$

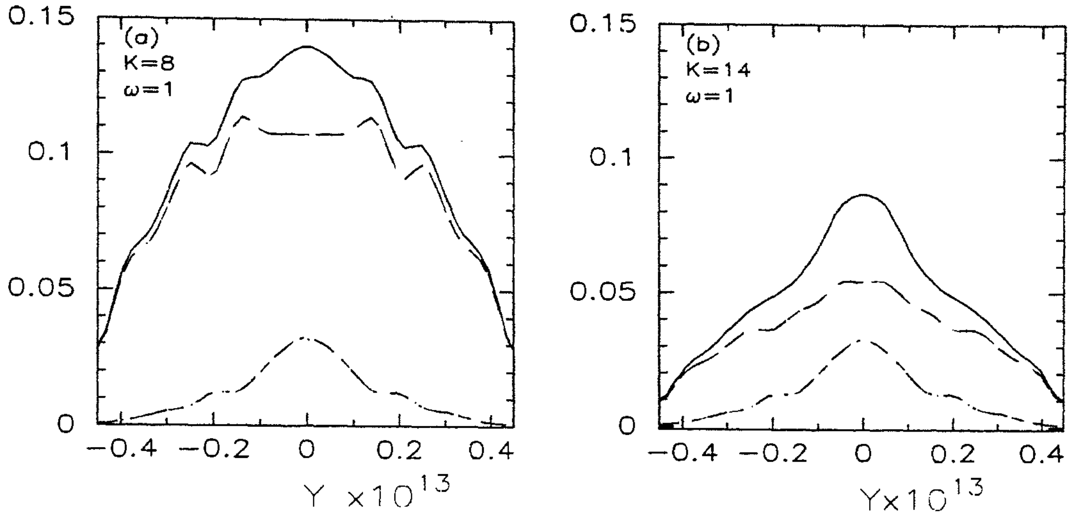

Figure 2: Shows the comparison of self radiation represented by line dash dot, reflected radiation represented by the line dash dash and the total radiation is represented by continuous line along the Y-axis in case 1

where

$$S_s(r) = \frac{1}{2}\int_{-1}^{+1} p(r,\mu,\mu')I(r,\mu')d\mu'. \tag{18}$$

Here $I(r,\mu)$ is the specific intensity of the ray making an angle $\cos^{-1}\mu$ with the radius vector.

The quantities $\sigma(r)$ and $S_s(r)$ are the absorption coefficient and the source function respectively and $P(r,\mu,\mu')$ is the phase function for isotropic scattering and function is normalized such that

$$\frac{1}{2}\int_{-1}^{+1} p(r,\mu,\mu')I(r,\mu')d\mu' = 1 \tag{19}$$

and $\quad p(r,\mu,\mu') \geq 0 \quad and \quad -1 \leq \mu,\ \mu' \leq 1$

2.3 Brief description of the numerical method for solving the radiative transfer equation in spherical symmetry

The solution of radiative transfer equation (17) in spherical symmetry is developed by using discrete space theory of radiative transfer [12]. In general the following steps are followed for obtaining the solution.

(i) We divide the medium into a number of "cells" whose thickness is less than or equal to the critical (τ_{crit}). The critical thickness is determined on the basis of physical characteristics of the medium. τ_{crit} ensures the stability and uniqueness of the solution.

(ii) Now the integration of the transfer equation is performed on the "cell" which is two-dimensional radius - angle grid bounded by $[r_n, r_{n+1}] \times [\mu_{j-\frac{1}{2}}, \mu_{j+\frac{1}{2}}]$ where $\mu_{j+\frac{1}{2}} = \sum_{k=1}^{j} C_k, j = 1, 2 \ldots, J$, where C_k are the weights of Gauss Legendre formula.

(iii) By using the interaction principle described in [12], we obtain the reflection and transmission operators over the "cell"

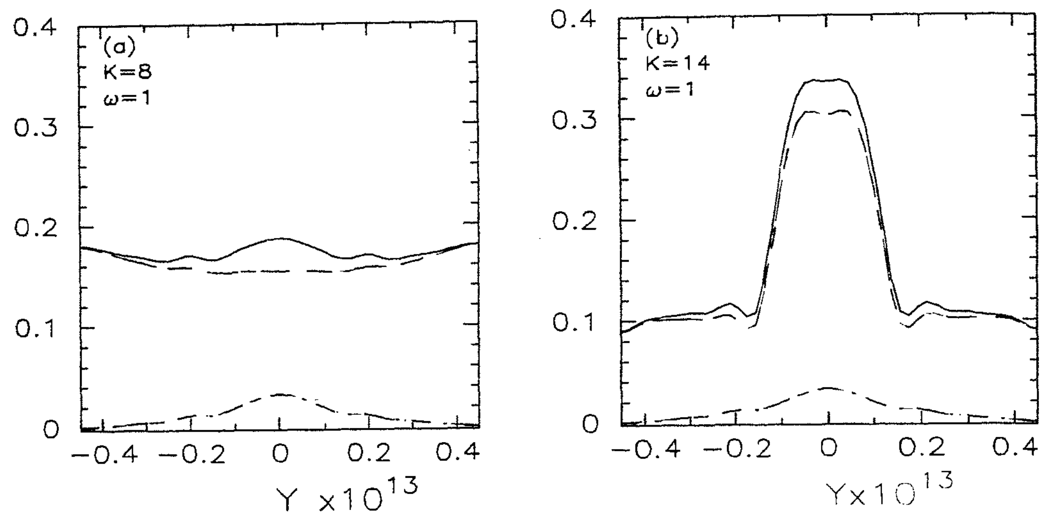

Figure 3: Shows the comparison for case2. The notation is same as case 1

(iv) Finally we combine all the cells by the star algorithm described in [12] and obtain the radiation field.

2.4 Boundary conditions

The boundary conditions are assumed as follows

$$U_{N+1}^{-}(\tau = T, \mu_j) = 1 \qquad \text{for all} \quad \mu_j's \tag{20}$$

$$U_1^{+}(\tau = 0, \mu_j) = 0 \qquad \text{for all} \quad \mu_j's \tag{21}$$

Equation (20) represents the incident radiation on the atmosphere where the radius is minimum, and equation (21) represents the boundary condition at maximum radius, for $\omega = 1$.

2.5 Calculation of total source function

If $J(r)$ is the mean intensity then the total source function $S_T(x, y, z)$ is given by

$$S_T(x, y, z) = S_r(x, y, z) + J(x), \tag{22}$$

this means that total source function (S_T) is sum of the source functions due to self radiation of the primary star ($(S_s) = J(x)$)and the irradiation from the secondary component which is assumed as point source (S_r).

3. Results and Discussion

We have used the following data:

$R_{in} = 10^{12}$cm, $R_{out} = 5 \times 10^{12}$cm, R=$10^{13}$cm where R_{in}, R_{out} is the inner and outer radius of the primary star and R is the separation between two components. We assumed a constant

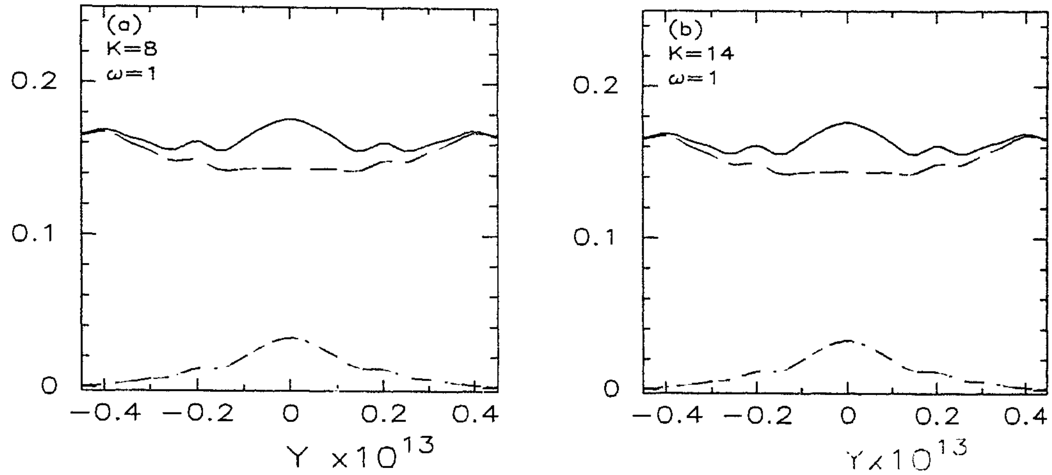

Figure 4: Shows the comparison for case3. The notation is same as case 1

electron density of $10^{12}\mathrm{cm}^{-3}$. As mentioned earlier, we calculate the intensities along the lines such as QS_2RO in a given circular slice. These slices are designated as K= 1, 2, 3,... where the slice with designation K= 1 corresponds to that at $x = R_{\mathrm{out}}$, that with K= 11 corresponds to that at $x = 0$ and that with K= 21 corresponds to that at $x = -\mathrm{R}_{\mathrm{out}}$. We give unit incident intensity at the surface $r = \mathrm{R}_{\mathrm{out}}$. We set the coordinates (x_2, y_2, z_2) of the secondary component as the point source in the following cases. We can consider many number of cases keeping secondary component in different positions in a circular orbit. We can obtain many possible cases but we consider following cases.

Case 1: $x_2 = \mathrm{R}$, $y_2 = 0$, $z_2 = 0$;
Case 2: $x_2 = \mathrm{R}\sin\frac{\pi}{4}$, $y_2 = 0$, $z_2 = \mathrm{R}\cos\frac{\pi}{4}$; and
Case 3: $x_2 = 0$, $y_2 = 0$, $z_2 = \mathrm{R}$.

In the above cases we have placed the secondary component on the X-axis at a distance R in case 1 i.e., ($x_2 = R$, $y_2 = 0$, $z_2 = 0$); In case 2, the secondary component is placed between X and Z axis as the line making 45° with X-axis i.e., ($x_2 = R\sin\frac{\pi}{4}$, $y_2 = 0$, $z_2 = R\cos\frac{\pi}{4}$); and in case 3, the secondary component is placed on the Z-axis with distance of R i.e., ($x_2 = 0$, $y_2 = 0$, $z_2 = R$).

In figures 2 to 4 we have plotted the self radiation represented by dash dot line, reflected radiation represented by dash dash line and total radiation is represented by continuous line for K= 8, 14 in a scattering medium along Y-axis which is a line of sight. In all the above three cases we observe that the self radiation is same in all the cases, this means that radiation is uniformly scattered in all directions from the centre of the star.

Case 1: Reflected radiation is reduced in figure 2(b) when compared to K= 8 in figure 2(a). This is due to the fact that, irradiation from the secondary component is reduced in the deeper layers of the primary component. We also can observe that the maximum radiation is occurring at Y= 0 ie., the middle layers of the atmosphere.

Case 2: In figure 3(a, b) we see the reflected radiation is almost constant in the range -0.3 to +0.3. The total radiation also behaves as the self radiation in the range -0.3 to +0.3.

Case 3: In figure 4(a, b) it is interesting to see that K= 8 and K= 14 the radiation is same in both the cases.

4. Conclusion

We have calculated the radiation field when irradiation is a point source in a close binary system. We notice that the intermediate regions of the primary component shows more radiation than extreme regions.

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