SOME RECENT DEVELOPMENTS IN STELLAR PULSATION THEORY

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Abstract

We review recent developments in the following areas: the interaction between stellar pulsations and convection; the instability mechanism for β Cephei stars; Cepheid masses and beat Cepheids; nonlinear Cepheid pulsation calculations; oscillations of crystallizing white dwarfs; oscillations of rotating stars; solar oscillations; oscillations of R CrB stars; the causes of small period changes in some RR Lyrae stars; and linear, adiabatic, nonradial stellar oscillations.

1. INTRODUCTION

The main purpose of the present paper is to survey some of the more exciting recent work on the theory of stellar pulsation. Clearly, the way a star pulsates or oscillates depends on its internal structure and on other things as well, such as the presence or absence of a magnetic field, rotation, etc. Thus, observations of stellar pulsations serve as an additional "probe" of stellar internal structure (and possibly of other things as well), and may reveal information over and above that which may be gleaned from the static characteristics of a star alone.

In §§2-7 we shall survey recent work on, respectively, convection calculations, the instability mechanism for β Cephei stars, Cepheid masses and beat Cepheids, nonlinear Cepheid pulsation calculations, oscillations of crystallizing white dwarfs, and oscillations of rotating stars. Certain other items will be briefly discussed in §8. Finally, a section (§9) containing a summary and conclusions completes the paper.

2. CONVECTION CALCULATIONS

The interaction between pulsations and convection remains one of the outstanding problems of stellar pulsation theory. Thus, it is commonly believed (as was suggested, e.g., by Baker and Kippenhahn 1965) that convection in the stellar envelope is what terminates pulsations on the red side of the Cepheid instability strip. Perhaps the most ambitious attempt to come to grips with the problem was made by Deupree (1975 a,b; 1976 a,b,c; 1977a,b,c,d). He adopted the fundamental approach of actually solving numerically the nonlinear mass, momentum, and energy equations in two spatial dimensions and time in a convectively unstable region in a star. The turbulent "cascading" of energy to smaller and smaller scales was treated by the introduction of an "eddy viscosity." While some of Deupree's assumptions may (perhaps legitimately) be criticized (particularly that two spatial dimensions in the calculations are adequate), still his calculations have yielded a red edge to the Cepheid instability strip, not dependent on any kind of phenomenological convection theory, such as the "mixing-length theory" (see, e.g., Vitense 1953; Böhm-Vitense 1958; Cox and Giuli 1968, Chap. 14; Gough 1977a). Also, his work has provided us with the first physical picture as to how convection may "throttle" pulsations on the red side of the Cepheid instability strip.

According to Deupree's results, convection is most efficient approximately when the stellar material is most compressed, i.e., near the instant of minimum stellar radius (as for a fundamental mode). Thus, convection at about this time lets out the energy which has been "dammed up" in the He^+ ionization zone by the action of the kappa and gamma mechanisms (e.g., Cox 1974). Hence, convection essentially "undoes" the effects of the above two driving mechanisms, and so terminates instability at the red side of the Cepheid instability strip.

Recently, linearized pulsation calculations have been reported by Baker and Gough (1978) which include convection treated by means of Gough's (1977b) theory. These calculations have confirmed the above result that convection indeed terminates pulsational instability on the red side of the Cepheid instability strip.

3. THE INSTABILITY MECHANISM FOR THE BETA CEPHEI STARS

In the three-quarters of a century that have elapsed since the β Cephei (or β Canis Majoris) stars were first discovered, no viable and generally accepted destabilizing mechanism has been found. (Reviews of the mechanisms proposed to date have been provided by Cox 1976 and Kato 1976). These are perhaps the hottest variables known. They are of spectral types in the early B's, and occupy a narrow region just to the right and about 1^m above the upper main sequence on a Hertzsprung-Russell (H-R) diagram, roughly parallel to the main sequence (e.g., Lesh and Aizenman 1978).

A new destabilizing mechanism, called the "bump mechanism," has recently been proposed by Stellingwerf (1978, 1979). It arises from a local "wiggle," or "bump," in the opacity, i.e., a local change in the slope of the opacity as a function of temperature at a given density. This bump is caused, in turn, approximately, by the coincidence of the peak of the Planck function with the ionization edge of He^+ (at 54.4 eV). Consequently, the temperature at which the wiggle occurs is about 1.5×10^5 °K, only very weakly dependent on the density. This proposed destabilizing agent is seen to be an " envelope ionization mechanism," and the pulsations should accordingly be mostly radial (although nonradial oscillations are not ruled out). Interestingly enough, the above temperature is close to that required for an envelope ionization mechanism to account for the instability of these stars, as has been pointed out by Cox (1967).

It has been argued by Cox and Stellingwerf (1979) that, if these stars are destabilized by an envelope ionization mechanism, they should occupy an " instability strip " on the H-R diagram and they should also exhibit a period-luminosity (Π-L) relation. More interestingly, the effects of radiation pressure are found to have profound consequences in regard to these two features. In particular, the instability strip is caused by these effects to be almost vertical, and its slope can even be of the same sign as that of the main sequence—in contrast to the Cepheid instability strip—for only modest amounts of radiation pressure. Also, the Π-L relation is steepened, i.e., L increases more strongly with increasing Π than if radiation pressure is ignored. Both features—an instability strip which slopes " backward " and a relatively steep Π-L relation—are observed properties of the β Cephei stars (e.g., Lesh and Aizenman 1978).

Whether or not this proposed destabilizing mechanism will be found wanting in some respect, is a question for the future. Also, it would be interesting to know whether or not this kind of mechanism could excite, in addition, certain nonradial modes.

4. CEPHEID MASSES AND BEAT CEPHEIDS

It is well known that the (necessarily) indirectly inferred masses of Cepheids and of " beat (or double-mode) Cepheids " appear, usually, to be smaller than expected, for given luminosity, from conventional stellar evolution theory. The " Cepheid mass problem " has been reviewed in two recent papers by A. Cox (1978) and J. P. Cox (1978), where many references to the literature are given.

Briefly, we may divide the above types of pulsating stars into two broad groups : the single-mode (non-beat), ordinary Cepheids; and the beat Cepheids. The " mass discrepancy " for the stars in the former group is far less severe than for those in the latter group. According to J. P. Cox (1978), ordinary Cepheids appear to be undermassive, as compared with expectations based on conventional stellar evolution theory, by some 20-40%, which may be comparable with the sizes of the uncertainties involved in these indirect mass estimates. Perhaps smallest are the masses derived from the requirement that the " bumps " in the calculated velocity curves agree with the bumps often observed in the light curves at periods around 7-10 days, in accordance with the Hertzsprung relation (Payne-Gaposchkin 1951 ; Cox 1974). Christy (1966) was the first to suggest such a comparison, and his nonlinear calculations of velocity curves implied masses about one-half those expected for similar luminosities from conventional stellar evolution theory. Somewhat similar conclusions have been reached by other workers (summarized in Fischel and Sparks 1975).

Recently, new nonlinear pulsation calculations have been carried out (Adams, Castor and Davis 1978; Davis and Davison 1978; Adams, Davis and Keller 1978) using a non-Lagrangian, moving mesh which follows the hydrogen ionization zone during the pulsations (see §5) and permits good , continuous resolution of this zone. This method was devised by J. Castor (Castor, Davis and Davison 1977) and makes possible, for the first time, the calculation of accurate and reliable light curves for pulsating stars; these calculated light curves may be compared directly with observed light curves. However, the results of this kind of calculation seem to agree, at least qualitatively, with results of previous workers obtained with more conventional computing schemes in which computed velocity curves were compared with observed light curves. In order for the calculations to show the proper details in the light curves in the appropriate period range, the models appear to require smaller masses than evolutionary calculations would suggest, perhaps two-thirds of the " evolutionary masses." Another result, revealed by this new method of calculation (Adams, Davis and Keller 1978), is that the exact relation between light and velocity curves is more complicated than heretofore assumed.

All of the above remarks are based on conventional, chemically homogeneous stellar envelopes. Nonlinear pulsation calculations of helium-enriched stellar envelopes (such helium enrichment has been suggested by A. Cox and collaborators, e.g., Cox, Deupree, King and Hodson 1977; see below) have not yielded definitive results (Adams 1978).

It has been suggested by Simon and Schmidt (1976) that the appropriate bumps occur on the velocity curves only when the ratio Π_2/Π_0 of second overtone to fundamental pulsation period is close to 0.5. Cox, Deupree, King, and Hodson (1977) showed on the basis of linearized pulsation calculations that models with helium-rich outer layers and expected " evolutionary " masses indeed achieve the above value of this ratio at fundamental periods around 7-10 days.

The " mass discrepancy " for the beat Cepheids* is far worse than for ordinary Cepheids. The masses as inferred for these stars on the basis of radial pulsation theory may be 2-4 times smaller than expected on the basis of conventional evolution theory.

A number of attempts to explain this mass discrepancy have been made, and it has given rise to a large number of papers (see the detailed references contained in the above review papers). Most such attempts have been aimed at a modification of the " static " envelope structure. The presence of a fair amount of convection in the envelope has been suggested (Cogan 1977; Saio, Kobayashi and Takeuti 1977), but this suggestion has not been universally accepted (e.g., Deupree 1977c).

Perhaps the boldest assumption has been that of A. Cox and collaborators (e.g., Cox, Deupree, King and Hodson 1977) which asserts that the outer layers of the

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*The beat Cepheids—of which 11 are presently known—are variable stars whose light curves are not periodic. However, the light curves can be decomposed into (in nearly all cases) two periodic curves whose sum gives back the original, nonperiodic curve. These two component light curves are usually assumed to be the radial fundamental and first overtone modes. The longer of these two periods normally lies between about 2 and 6 days. See Stobie (1977).

star are, for some reason, enriched in helium. This assumption apparently resolves the mass discrepancy, at least for the beat Cepheids, and leads to " normal," expected masses for these stars. However, this assumption raises other problems that may be as severe as the mass discrepancy itself. Thus, how did it happen that the outer layers of these stars came to be enriched in helium? To answer this question, Cox, Michaud and Hodson (1978) have postulated the existence of a " Cepheid wind " that is hydrogen-rich and that therefore leaves the helium behind. Also, it is well known that an " inverted μ gradient" such as a helium-rich mixture overlying hydrogen-rich material is unstable against mixing (Ulrich 1972; Kippenhahn 1974). According to Cox, Michaud and Hodson (1978), the postulated Cepheid wind results in a helium-enrichment of these outer layers in a time presumably short compared with the time required for the above mixing process to mix the helium down to deeper layers. However, a fairly efficient wind-separation mechanism would be required for this purpose. Estimates based on stellar wind theory show that such efficiency is not likely to be realized, at least on the basis of our current understanding of stellar winds (Castor 1978). Finally, to the best of this reviewer's knowledge, there is no independent evidence for (or against) helium enrichment in the outer layers of Cepheids.

It was suggested by J. P. Cox (1978) that strictly radial pulsation theory may not be applicable to the beat Cepheids, as usually assumed, because their pulsations might be contaminated with a small admixture of nonradial oscillations. However, this suggestion would be very difficult to test.

The fairest appraisal would be to say that the mass discrepancy of the beat Cepheids (and perhaps of ordinary Cepheids) is still with us. None of the various resolutions that has been proposed so far is completely satisfactory.

Indeed, it has been pointed out by Cogan (1978) that there are severe difficulties with both alternatives—chemically homogeneous envelopes and small masses, or helium-enriched envelopes and conventional masses.

Another problem with the beat Cepheids is that it has not as yet been possible to calculate true double (or multiple) mode behavior to everyone's satfisfaction in any stellar model (e.g., Cox, Hodson and Davey 1976; Hodson and Cox 1976).

This phenomenon of double-mode behavior has been discussed by Simon (1978a). The ratio of first overtone to fundamental period Π_1 / Π_0 for the beat Cepheids is known to lie in the narrow range 0.696-0.711 (Stobie 1977). On the basis of this fact, Simon has made the very interesting suggestion that double-mode behavior results from a kind of " resonance," in the sense that such behavior occurs in a star only when the " combination " period, say $\Pi_{01} = 2\pi / (\omega_0 + \omega_1)$, where ω_0 and ω_1 are the angular frequencies of, respectively, the fundamental and first overtone radial modes, coincides with the period of one of the higher overtones. In the case of the beat Cepheids it is the third overtone that is presumably resonant with Π_{01}, because a resonance with either the second or fourth overtone would occur only at effective temperatures well outside the instability strip.

This " resonance theory " corroborates the small masses of the beat Cepheids yielded by other aspects of pulsation theory as applied to chemically homogeneous envelopes. However, with the models that have thus far been proposed having helium-enriched outer layers and conventional masses, there is apparently no resonance at the appropriate period with the third overtone (or, for that matter, with any overtone) as required by the " resonance theory " (Simon 1978b).

Simon has also applied this " resonance theory " to the double-mode AI Velorum stars, whose period ratio Π_1 / Π_0 lies in the narrow range 0.768-0.778 (Π_0 between about 0.05 and 0.22 days). He finds that this ratio is realized when the *fourth* overtone period is coincident with Π_{01}, provided that these stars are assumed to have low masses, say less than one solar mass.

5. NONLINEAR CEPHEID PULSATION CALCULATIONS

Important recent work in this area concerns the development of methods of obtaining periodic solutions of the nonlinear equations of hydrodynamics and heat flow. Such methods were pioneered by Baker and von Sengbusch (1969); see also von Sengbusch (1973) and Baker (1973). Somewhat later, a variant of the Baker-von Sengbusch method was developed and used by Stellingwerf (1974a,b, 1975a,b), in connection with stellar envelopes. Still later, Stellingwerf's methods were adapted to much deeper stellar envelopes by A. Cox and collaborators (e.g., Cox, Hodson and Davey 1976).

Another important recent development, referred to in §4, concerns the development of methods of calculating accurate and reliable light curves of pulsating stars. The main difficulty in the past has been the very large temperature gradient in the hydrogen ionization zone of cool stars such as Cepheids (e.g., Cox 1974, p. 630). This very steep temperature gradient makes calculations of the luminosities of such stars difficult when this region of large temperature gradient is moving about in mass during the pulsations, as is the case with models of Cepheids and RR Lyrae variables. Because of this motion, this region of steep temperature gradient will occasionally jump from one mass zone to another in finite-mass-zoned model envelopes, perhaps yielding " bumpy" results (e.g., Keller and Mutschlecner 1971). In other words, adequate " resolution " of this region cannot be maintained at all times during the pulsations (with present computer time limitations) because of this finiteness of zone thicknesses in conventional Lagrangian zoning schemes.

Recently, a method has been developed by Castor, Davis and Davison (1977) which overcomes the above difficulties. This method employs a non-Lagrangian, moving mesh which " follows " this region of steep temperature gradient during the pulsations. Consequently, fine zoning can be used in this critical region, and coarse zoning elsewhere. This technique permits fine zoning to be used—where necessary—at all times during the pulsations, without excessive computing times. Hence, such calculations should yield accurate and reliable light curves. These calculations have been used to good advantage by Davis and Davison (1978), Adams, Castor and Davis (1978), and Adams, Davis and Keller (1978). Some of these results have been described in §4.

6. OSCILLATIONS OF CRYSTALLIZING WHITE DWARFS

In recent years, a few (12, according to McGraw 1977 and Nather 1978), otherwise apparently normal, variable DA white dwarfs have been discovered (e.g., Robinson and McGraw 1976), called " ZZ Ceti stars " by McGraw (1977). These stars have colours in the narrow range $0.20 < \text{B-V} < 0.30$ (effective temperatures close to 10,000 K), and exhibit somewhat irregular light variations with periodicities of, typically, a few hundred up to about a thousand seconds. Van Horn (1978) and McGraw (1977) have pointed out that these variable white dwarfs lie on an H-R diagram exactly on a downward extension of the Cepheid instability strip, and may represent the hottest variable stars in this strip. These long periods have been difficult to account for theoretically (of course, such long periods could be interpreted as high-order nonradial *g* modes, but such high orders would be required that this interpretation is not very satisfactory (see Dziembowski 1977a).

Van Horn and Savedoff (1976) have pointed out that these stars occupy just the part of the H-R diagram where cooling white dwarfs should be developing crystalline cores. Moreover, within this narrow range of colours the crystalline cores may contain anywhere from no mass up to nearly the entire mass of the white dwarf. These authors have therefore raised the very pertinent question of the oscillatory properties of such structures. Also, the authors made some preliminary estimates of periods, etc., of such structures.

A detailed analysis of the oscillatory properties (in linear, adiabatic theory) of white dwarf models with crystalline cores has been begun by Hansen and Van Horn (1978). Just as in purely gaseous stars, the system of equations is of the sixth order in the linear, adiabatic approximation, before any assumptions are made about the angular dependences of the perturbation variables. Again just as for purely gaseous stars, there are two possible classes of oscillations : the *spheroidal* and *toroidal* modes. Each of these classes is characterized, in general, by certain assumptions regarding the relation between the radial and tangential components of the displacement vector, and the relation of these components to spherical harmonics (see, e.g., Ledoux and Walraven 1958, §75; and Aizenman and Smeyers 1977). In the case of spheroidal modes of purely gaseous stars (on which almost all of stellar oscillation theory is based), these two components are related by only an algebraic equation, so that the above system of equations becomes only of the fourth order; this system comprises the usual *p*, *g*, and *f* modes of nonradial stellar oscillation theory. However, when part (or all) of the star can sustain shear, as in a partially crystalline star, these two components are related by a second-order differential equation. Hence, the system of equations remains of the sixth order for such stars. We may therefore expect the possibility of new modes of oscillation. These new modes should be revealed by Hansen and Van Horn's investiagation , the results of which are eagerly anticipated.

Toroidal modes are characterized, in general, by purely horizontal motions of mass elements. In the case of purely gaseous (and nonrotating) stars the toroidal modes have zero frequency (e.g., Aizenman and Smeyers 1977). However, for partially crystalline stars, in which shear stresses can be maintained, these toroidal modes possess non-vanishing frequencies (just as for a rotating star, but for a different physical reason (see §7). These toroidal modes are called "torsional" modes by geophysicists. Hansen and Van Horn's preliminary results make it appear that these frequencies may be too short to account for the relatively long periods that are observed, but a definitive statement must await the completion of their work. Also, it may be noted that, without viscosity, these effects of a finite shear on the oscillatory properties of such objects are in any event entirely confined to the crystalline core, and never show up on the surface of the star.

7. OSCILLATIONS OF ROTATING STARS

The question of the oscillations of a rotating star is, not surprisingly, vastly more difficult and complicated then for a nonrotating star. Thus, during the oscillations each mass element is subject to two additional forces besides the usual ones arising from gravity and pressure gradients : Coriolis and centrifugal forces. Even the "static" (nonpulsating) structure of a rotating star is a highly nontrivial problem, and considerable work has been devoted to this area (see e.g., Ostriker 1978 and references therein ; see also the review papers by Mestel 1965; Lebovitz 1967; and Fricke and Kippenhahn 1972).

It is interesting to note that the Coriolis forces are proportional only to the first power of Ω, the angular rotation velocity of the star, whereas the centrifugal forces are proportional to Ω^2. Consequently, if only terms linear in Ω are retained both as regards the "static" structure of the star and the oscillations, then centrifugal forces are being neglected. The star can accordingly be considered spherical. However, any study of the effects of rotation on the oscillations of such a system must necessarily consider nonradial oscillations with m (the azimuthal order of a spherical harmonic) $\neq 0$, because axially symmetric oscillations (of which radial oscillations are a special case) are affected only at least to order Ω^2 (see , e.g., Ledoux and Walraven 1958, §82).

A number of studies have exploited the fact that for suficiently slow rotation only first powers of Ω need be considered. Thus, for example, the splitting of nonradial modes with $m \neq 0$ in slowly, differentially rotating stars was investigated by Hansen, Cox and Van Horn (1977 and references therein). The vibrational stability in the quasiadiabatic approximation of these rotationally split "m sublevels" was investigated by Hansen, Cox and Carroll (1978). Interestingly enough, it was found in this last work that those sublevels corresponding to azimuthal running waves traveling in the same direction as the star's rotation and most slowly were the least stable of all the sublevels.

Rotational distortion in stellar pulsations has been taken into account by, among others, Simon (1969), Smeyers and Denis (1971), and Mohan and Singh (1978).

In a very important and fundamental paper dealing with the oscillations of arbitrarily rapidly rotating stars, Papaloizou and Pringle (1978) called attention to a new type of mode, which they called "*r* modes." These *r* modes are somewhat similar to Rossby waves on the earth's surface, are essentially transverse, and they correspond to toroidal modes in nonrotating stars (see, e.g., Lebovitz 1965; Aizenman and Smeyers 1977). These toroidal modes have been called "trivial" by Lebovitz (1965), as they have zero frequency in a nonrotating star. However, in a rotating star these r modes acquire non-zero frequencies. The angular frequency of these r modes in a sufficiently slowly rotating star (rotating so slowly that rotational distortion may be neglected) was shown by Papaloizou and Pringle to be given by $2m\Omega/[l(l+1)]$, where l is the "latitudinal" index of a spherical harmonic. (Somewhat surprisingly, this value of the angular frequency of what are now called r modes was first obtained by Ledoux 1949.) It is interesting to note that the above value of the angular frequency of r modes is independent of the structure of the star (a fact first pointed out by Aizenman 1978).

Unfortunately, the notation employed in the part of the Papaloizou-Pringle paper dealing with rotation is extremely unconventional. Consequently, anyone who wishes to follow the mathematical development in this paper in detail should proceed with suitable caution.

8. MISCELLANEOUS

In this section we shall discuss a few subjects that do not fit cleanly into any of the foregoing sections. These subjects are : possible solar oscillations (§8a); pulsation properties of models of R Coronae Borealis stars (§8b); the Sweigart-Renzini suggestion about the causes of small period changes in RR Lyrae stars (§8c); and the work by Wolff and Shibahashi (separately) on linear, adiabatic, nonradial stellar oscillations (§8d).

8a) Solar oscillations

Are there oscillations of the whole Sun? (Here we do not refer to the "five-minute oscillation," as there is little doubt about its existence and nature as high-order acoustic modes; see, e.g., Rhodes, Ulrich, and Simon 1977; also, it is reasonably well understood theoretically—see, e.g., Ando and Osaki 1975.) There have been several reports of periodic changes in the solar diameter with periods ranging from some 10 minutes up to about 70 minutes, by Hill and collaborators (e.g., Hill, Stebbins, and Brown 1975; Brown, Stebbins and Hill 1976, 1978). Also, a longer period oscillation with a duration of 160 minutes ($2^{h}40^{m}$) has been reported by Severny, Kotov and Tsap (1976); Brookes, Isaak and van der Raay (1976); and Snider, Kearns and Tinker (1978); see also Brookes, Isaak, McLeod, van der Raay and Roca Cortes (1978). However, these apparent solar oscillations have not been confirmed by all observers (e.g., Musman and Nye 1976; Grec and Fossat 1977; Livingston, Milky and Slaughter 1977). Possible reasons for the apparent discrepancy are given by Hill (1978), and a review (as of early 1977) has been given by Goldreich and Keeley (1977). Doubt has been expressed about the statistical significance of the reported solar oscillations by Dittmer (1978), who also questions the results of Hill, Rosenwald and Caudell (1977) based on theoretical considerations. It has been suggested by Grec and Fossat (1977) that the apparently observed solar oscillations might be due to variability of the transparency and refractive index of the earth's atmosphere. More recently, Clarke (1978) has also suggested that at least some of the solar variability observations might be due to influences of the earth's atmosphere. The earth's magnetosphere has also been implicated by Toth (1977). Lanzerotti and Maclennan (1978) have concluded that such a magnetospheric cause is unlikely. In general, terrestrial factors do not appear at present to be mainly responsible for the observations, i.e., they appear to be caused by actual solar oscillations, but the matter is not settled yet (Harvey 1978); see also Gough (1978).

8b) Pulsations of models of R Coronae Borealis stars

Wheeler (1978, 1979) has suggested that the progenitors of Type I supernovae may be hydrogen-deficient stars that, as one possibility, have for some reason lingered near the main sequence for an unusually long time and have consumed essentially all of their hydrogen. The basic reason underlying this suggestion is that Type I supernovae are observed to occur in elliptical systems, which are presumably $> 10^{10}$ years old. However, according to conventional stellar evolution theory the "turn-off point" mass for such old systems is probably $< 1\ M_\odot$ ($M_\odot$ = solar mass). However, for a star to explode, its mass should be of the order of or greater than the "Chandrasekhar's limit" (e.g., Chandrasekhar 1939), $\sim 1.4\ M_\odot$.

Wheeler suggests that the "hydrogen-deficient carbon stars" seem to fulfill the conditions required by observation and theory for the progenitors of Type I supernovae. The R Coronae Borealis stars represent one of the three types of known hydrogen-deficient carbon stars. These stars (of which about 30 are known) occasionally exhibit abrupt and large (several magnitudes) decreases in luminosity. However, three are known to be fairly periodic variables : RY Sgr, with a period of 39 days ; R CrB itself, with a period of 44 days; and UW Cen with a period of 42 days (Feast 1975). It is generally assumed that the variations are caused by pulsation.

In order to test Wheeler's suggestion, and also because of the intrinsic interest in such pulsators, King, Wheeler, Cox Cox and Hodson (1978) have begun a theoretical investigation of the pulsation properties of models for R CrB stars. Hopefuly, one result of such a study would be the derivation of limiting values of some of the parameters of these stars. For example, if it should turn out that a mass of, say, $2\ M_\odot$ is required for the models to match the observed properties of such stars, then Wheeler's suggestion would be vindicated. However, preliminary results indicate that such studies may not yield definitive limits such as the above. One difficulty is that certain important parameter values for the R CrB stars are not very well established. For example, it is known only that such stars are very luminous, with absolute bolometric magnitudes of -4 to -6 (some 10^4 times as luminous as the Sun); but exact values are not available. With masses only of order $1\ M_\odot$, the luminosity/mass ratio for these stars is very large. This fact has the consequence that the pulsations of such stars are extemely nonadiabatic. The "thermal" or" Kelvin" times for such starts are not greatly different from their "dynamical" times (e.g., Cox and Giuli 1968, Chap. 1), whereas, for most stars, the former time is much larger than the later. Studies of the pulsations of such stars have previously been carried out by Trimble (1972) and Wood (1976).

8c) Period changes in RR Lyrae variables

Small period changes ($\sim$1 part in 10^5 over an interval of $\sim$100 yrs) have been observed in some RR Lyrae variables (Rosino 1973 and references therein). These period changes are too large to be accounted for in terms of slow, continuous stellar evolution (Rosino 1973). Sweigart and Renzini (1979) have suggested that these small period changes, which are observed to be of either sign, are caused by small mixing events occurring in the semi-convective zones of horizontal branch stars. Although little is known about the physics of semi-convection, Sweigart and Renzini's suggestion appears quite plausible. Percy, Matthews and Wade (1978) have suggested that these mixing events may also be the cause of the small period changes observed in some Delta Scuti—like stars.

8d) Some recent work on linear, adiabatic, nonradial stellar oscillations

Wolff (1979) has noted that, for all except the lowest modes of linear, adiabatic, nonradial oscillations, there are simple properties of such pulsations; these properties have apparently not been noticed before because of the attention previously devoted to the lower modes. By considering primarily higher modes, Wolff has shown that the equations for such oscillations can be put into a simple form which makes manifest many of these properties. For example, he shows in a very brief and rather elegant way that high frequencies are generally associated with nearly vertical, irrotational motions of fluid elements, whereas low frequencies are generally associated with nearly horizontal motions and that the fluid behaves as if it were nearly incompressible in this limit. He has also introduced a new mode classification scheme based more on the physical pulsation properties of pulsating stars than the conventional scheme (Cowling 1941). Moreover, he has discussed the mode "trapping" phenomenon (Osaki 1975; Cox 1979, §17.10) from the standpoint of his simplified equations. Finally, he has suggested some possibilities for mode coupling.

In an investigation somewhat similar in spirit to that of Wolff but more mathematical (and independent!) Shibahashi (1978) examined some features of high-order linear, adiabatic, nonradial stellar oscillations. He discussed the above 'trapping" phenomenon (Osaki 1975), the sometimes "dual" nature of nonradial oscillations in complicated, evolved stellar models (Osaki 1975; Cox 1979, §17.10), "avoided crossings" occasionally encountered in numerical calculations (Osaki 1975; Hansen, Aizenman and Ross 1976; Aizenman, Weigert and Smeyers 1977), and the "progressive" character of certain nonradial oscillations as found by Shibahashi and Osaki (1976). In this high-order, "asymptotic" limit there are many similarities between stellar pulsation theory and the Bohr-Sommerfeld quantization rule of quantum mechanics.

9. SUMMARY AND CONCLUSIONS

In this paper we have reviewed some recent work in pulsation theory. This work is extremely exciting, and some old and long-standing problems in pulsation theory are coming to be solved, or at least the solutions are more-or-less in sight. Also, stellar pulsation is seen to be a fairly ubiquitous phenomenon : thus, even the Sun may be a pulsating star (§8a); some white dwarft are pulsating (§6); and many, if not most, supergians stars may be slightly variable (see, e.g., Abt 1957; Lucy 1976; Osaki 1977; Dziembowski 1977b).

The very impressive work of Deupree may have revealed how convection manages to terminate pulsation on the red side of the instability region (§2). A new, and relatively simple, destabilizing mechanism for the Beta Cephei stars has been proposed (§3). The problem of the anomalously small Cepheid masses is still with us, but some intriguing suggestions as to its resolution have been made (§4). New nonlinear computing techniques now permit accurate and reliable light curves for pulsating star models to be computed (§5). Calculations of the oscillatory properties of white dwarf models with crystalline cores may reveal new types of pulsation modes and may solve a vexing problem connected with the observed variable white dwarfs (§6). New types of pulsation modes are apparently associated with rotating stars (§7).

Finally, in §8 we have briefly reviewed the current situation with respect to solar oscillations (§8a), preliminary calculations of the oscillatory properties of models for the very luminous R Coronae Borealis stars, initiated in part in an attempt to answer certain questions involving Type I supernovae (§8b), a recently suggested mechanism to explain the small period changes that have been observed to occur in some RR Lyrae variables (§8c), and, finally, two recent and very insightful studies of high-order nonradial stellar oscillations (§8d).

Certainly, the subject of stellar pulsation is an exciting field of investigation with many applications to diverse areas of modern astronomy and astrophysics.

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