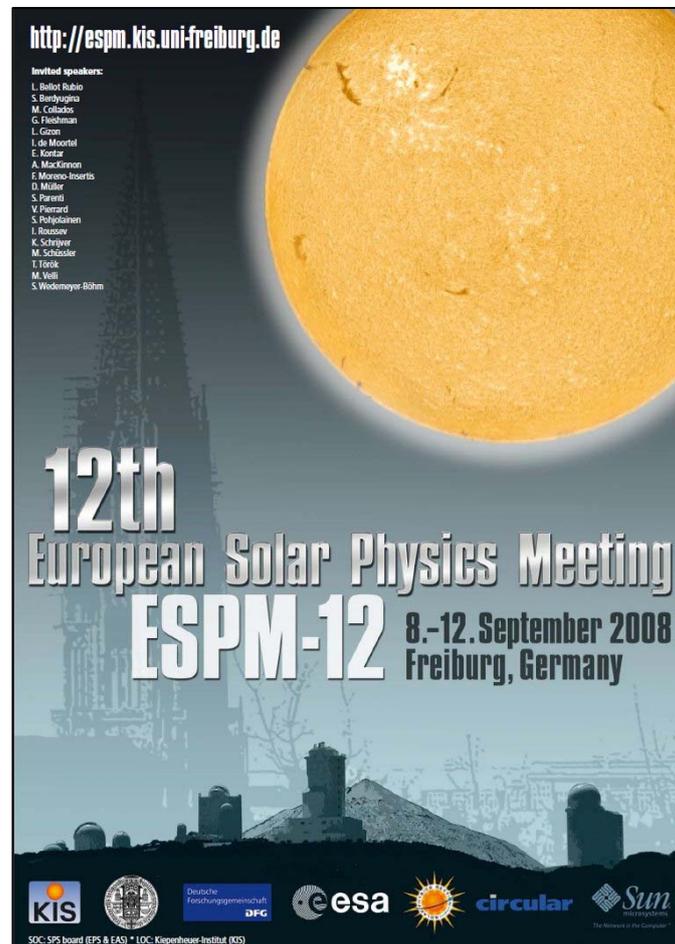


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# 3.3-37

## **MHD Waves at a Tangential Discontinuity with Inclined Magnetic Fields and Flows**

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Propagation of magnetohydrodynamic (MHD) waves in inhomogeneous magnetic plasmas is interesting from space and astrophysical point of view. In this study, the combined effect of non-parallel propagation, steady flow and inclined magnetic fields on either side of the tangential discontinuity is examined, with change in the field strength of the magnetic field, though uniform in each layer. The density is also assumed to be different on both sides of the interfacial layer. It is assumed that the fluid is perfectly conducting, infinite in extent with an interface that supports both body waves as well as surface waves. This model will also support fast, Alfvén modes depending on the parametric values of the system. Some special cases are discussed briefly. These modes have interesting applications in the solar corona and solar wind

## MHD Waves at a Tangential Discontinuity with Inclined Magnetic Fields and Flows

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**Abstract.** Propagation of magnetohydrodynamic (MHD) waves in inhomogeneous magnetic plasmas is interesting from space and astrophysical point of view. In this study, the combined effect of non-parallel propagation, steady flow and inclined magnetic fields on either side of the tangential discontinuity is examined, with change in the field strength of the magnetic field, though uniform in each layer. The density is also assumed to be different on both sides of the interfacial layer. It is assumed that the fluid is perfectly conducting, infinite in extent with an interface that supports both body waves as well as surface waves. This model will also support fast, Alfvén and slow modes depending on the parametric values of the system. Some special cases are discussed briefly. These modes may have interesting applications in the solar corona and solar wind.

*Keywords :* Hydromagnetic Waves, Corona, Solar Wind

### 1. Introduction

Magnetohydrodynamic (MHD) waves in inhomogeneous magnetic plasmas is interesting for laboratory plasma physicists, as well as researchers working in space or astrophysical plasmas. Hydromagnetic Waves play an important role in the transfer of energy in the atmosphere of the Sun. The physical structure of the solar corona is very complicated. The interaction between the magnetic field, shear flows and the plasma plays a central role in the physical properties of this medium and is responsible for many phenomena occurring in the corona. Alfvén wave heating in fusion plasma (Chen and Hasegawa

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1974), heating of solar and stellar corona (Nakariakov et al 1999), transport of magnetic energy in solar and stellar winds (Cranmer 2002), the geomagnetic pulsations (McPherron 2002) are some of the important applications of MHD waves in wide varieties of plasma systems. A more recent discussion of Hydromagnetic waves on a tangential discontinuity can be found in Joarder and Nakariakov (2006).

The combined effect of non-parallel propagation, steady flow and inclined magnetic field on the propagation characteristics of hydromagnetic surface waves is examined. The dispersion relation is derived for a plasma which is infinite in extent, perfectly conducting and having an interface wherein the plasma parameters like density and pressure have discontinuities. The number of parameters characterizing the present model is large and hence one is forced to restrict the analysis to specific parametric values only. Depending on the values of the parameters, the model supports Alfvén as well as fast magnetosonic waves. These modes have interesting application in the solar corona and solar wind.

## 2. Magnetic Interface with Flow

We assume the equilibrium such that

$$\frac{d}{dx}\left(p_0 + \frac{B^2}{2\mu}\right) = 0$$

where  $p_0$ ,  $B$  are the gas pressure and magnetic field, respectively. Assuming small perturbations about the basic flow fields such as density, pressure, magnetic field and shear flow, as given below

$$\bar{\rho} = \rho_0(x) + \rho, \quad \bar{\mathbf{v}} = \mathbf{U}(x) + \mathbf{v}, \quad \bar{p} = p_0(x) + P, \quad \bar{\mathbf{B}} = \mathbf{B}(x) + \mathbf{b}$$

where  $\mathbf{U} = (0, U_y, U_z)$ ,  $\mathbf{B} = (0, B_y, B_z)$ . We can simplify the basic equations of MHD to get a single differential equation for the velocity component  $v_x$  as

$$\hat{v}_x'' + (m_0^2 + k_y^2)\hat{v}_x = 0, \quad m_0^2 = \frac{\Omega^4 + k_x^2 c_f^2 (\omega_T^2 - \Omega^2)}{c_f^2 (\omega_T^2 - \Omega^2)}$$

where  $\Omega$ ,  $\omega$ ,  $k_x$ ,  $k_y$ ,  $c_f$  are the Doppler shifter frequency, frequency, wavenumbers and sound speed, respectively.

The solution to this equation is of the form

$$\hat{v}_x = Ae^{(m_0^2+k_y^2)^{1/2}x} - Be^{-(m_0^2+k_y^2)^{1/2}x} = 0$$

Introducing suitable boundary conditions and simplifying the dispersion relation can be written as

$$\rho_0(\omega_{A0}^2 - \Omega_0^2)(m_e^2 + k_y^2)^{1/2} + \rho_e(\omega_{Ae}^2 - \Omega_e^2)(m_0^2 + k_y^2)^{1/2} = 0$$

Using the trigonometric expressions for the wavenumber  $\mathbf{k}$  and magnetic field  $\mathbf{B}$  as follows :

$$\mathbf{k} = (\mathbf{0}, k\sin\theta, k\cos\theta), \quad \mathbf{B} = (\mathbf{0}, B\sin\gamma, B\cos\gamma)$$

the final dispersion relation can be written as

$$\rho_0(k^2 c_{A0}^2 \cos^2(\theta - \gamma) - \Omega_0^2)(m_e^2 + k_y^2)^{1/2} + \rho_e(k^2 c_{Ae}^2 \cos^2(\theta - \gamma) - \Omega_e^2)(m_0^2 + k_y^2)^{1/2} = 0$$

$$m_{0,e}^2 = \frac{\Omega_{0,e}^4 + k^2 c_{f0,e}^2 \cos^2\theta (k^2 c_{T0,e}^2 \cos^2(\theta - \gamma) - \Omega_{0,e}^2)}{c_{f0,e}^2 (k^2 c_{T0,e}^2 \cos^2(\theta - \gamma) - \Omega_{0,e}^2)}$$

The case of  $\mathbf{U} = \mathbf{0}$ ,  $\mathbf{k} = (\mathbf{0}, k\sin\theta, k\cos\theta)$  and  $\mathbf{B} = (\mathbf{0}, B_{0,e}\cos\gamma_{0,e}, B_{0,e}\sin\gamma_{0,e})$  was discussed by Uberoi and Satya Narayanan (1986). For the case  $\gamma = 0$ , the above relation reduces to that derived by Joarder and Satya Narayanan (2000). When both  $\theta = 0$  and  $\gamma = 0$ , the above reduces to the one derived by Nakariakov and Roberts (1995).

To begin with, let us consider the simple case of low  $\beta$  plasma. The pressure balance condition in this case is

$$p_0 + \frac{B_0^2}{2\mu} = p_e + \frac{B_e^2}{2\mu}$$

so that at the interface

$$\rho_0 c_{A0}^2 \approx \rho_e c_{Ae}^2$$

Here 0 and e represent both sides of the interface. Define the following quantities :

$$\alpha = \frac{\rho_e}{\rho_0}, \quad \delta = \frac{U_e}{U_0}, \quad \epsilon = \frac{U_0}{c_{A0}}, \quad x = \frac{\omega}{kc_{A0}}$$

It can be shown that

$$\frac{\Omega_0}{kc_{A0}} = x - \epsilon \quad \frac{\Omega_e}{kc_{A0}} = x - \delta\epsilon$$

$$(m_0^2 + k_y^2)^{1/2} = k(1 - (x - \epsilon)^2)^{1/2} \quad (m_e^2 + k_y^2)^{1/2} = k(1 - \alpha(x - \delta\epsilon)^2)^{1/2}$$

The dispersion relation can be simplified as

$$\{\cos^2(\theta - \gamma) - (x - \epsilon)^2\} \{1 - \alpha(x - \delta\epsilon)^2\}^{1/2} + \{\cos^2(\theta - \gamma) - \alpha(x - \delta\epsilon)^2\} \{1 - (x - \epsilon)^2\}^{1/2} = 0$$

The special case wherein  $\mathbf{U} = \mathbf{0}$ , i.e.  $\epsilon = 0$  was considered by Satya Narayanan (1997).

The relation for the case when  $\mathbf{U} = \mathbf{0}$  and  $\gamma = 0$  reduce to

$$(1 - \alpha x^2)^{1/2} (\cos^2 \theta - x^2) + (1 - x^2)^{1/2} (\cos^2 \theta - \alpha x^2) = 0$$

which can be simplified to yield

$$\alpha x^4 - (1 + \alpha)x^2 + \cos^2 \theta (1 + \sin^2 \theta) = 0$$

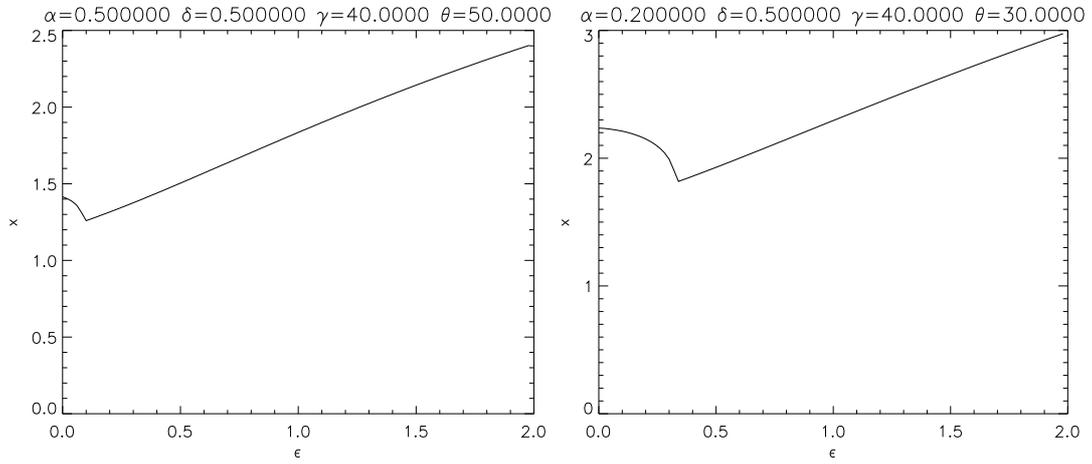
so that

$$x^2 = (1 + \alpha) \pm [(1 - \alpha)^2 + 4\alpha \sin^4 \theta]^{1/2} / 2\alpha$$

The above relation is the same as given in Jain and Roberts (1991).

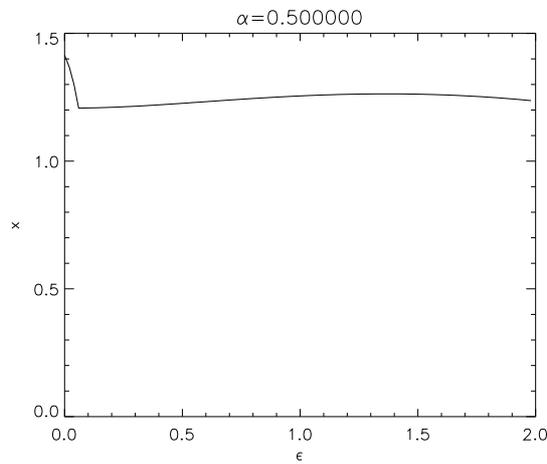
### 3. Discussion

The dispersion relation has been solved for the relevant parameters of the model. The number of parameters is large that it is impossible to give a single criterion for the



**Figure 1.** Dispersive Characteristics for non-dimensional phase velocity as a function of flow and specific parametric values

existence of waves. Moreover, depending on the value of the basic flow, negative energy instabilities may appear which was discussed in the earlier paper by Joarder and Satya Narayanan (2000).



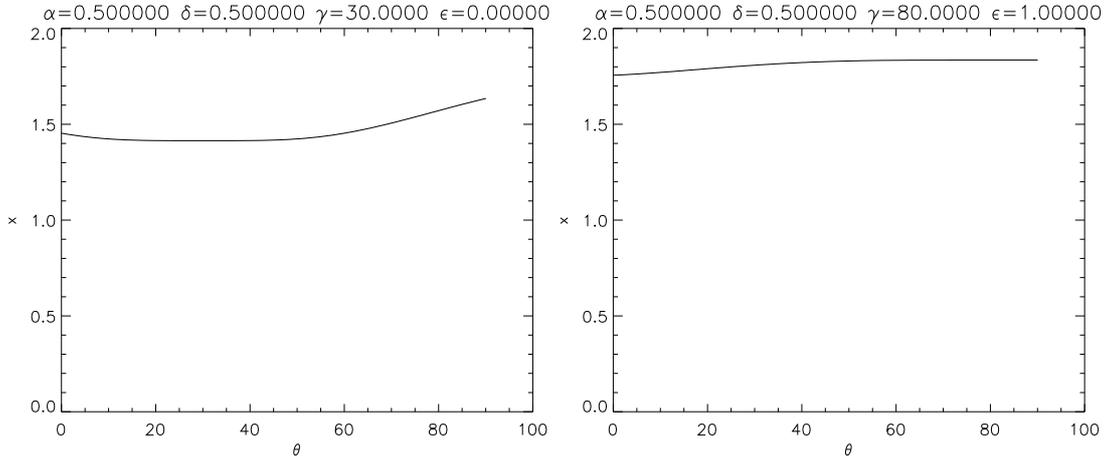
**Figure 2.** Dispersive Characteristics for non-dimensional phase velocity as a function of flow and specific parametric values

The dispersive characteristics of the mode for specific parametric values describing the model and various values of the basic flow is presented in Figures 1 and 2. It is interesting to note that the variation in the phase velocity is significant only for small values of the

ratio of the flow velocity to that of the Alfvén velocity  $V_{A1}$ . Also the normalised phase velocity is larger than  $V_{A1}$  for values of  $\epsilon$  varying from 0.0 to 2.0.

Figure 3 presents the normalised phase velocity as a function of different angles of the magnetic field for specific values of the model parameters. The normalised phase speed of the mode (left panel) has an interesting trend, namely, a decrease in the phase speed up to a certain angle and then a steady increase.

The present study does not cover more parametric values to get a clear picture of the nature of the modes. We are in the process of solving the dispersion relation for more values of the relevant parameters of the model. We also plan to apply this study for situations that may be found in coronal loops and also solar wind.



**Figure 3.** Dispersive Characteristics for non-dimensional phase velocity as a function of angles and specific parametric values

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