Viscous damping of Alfvén surface waves at a magnetic interface

G David Rathinavelu[1], M Sivaraman[1] and A Satya Narayanan[2]

[1]Department of Physics, Gandhigram Rural University, Gandhigram – 624 302, Tamil Nadu, India

[2]Indian Institute of Astrophysics, Koramangala, Bangalore – 560 034, India

E-mail: sivaram_gri@yahoo.com

Abstract. The dispersion relation for the propagation of viscous Alfvén surface waves along viscous plasma-plasma interface has been derived. Two modes of Alfvén surface waves are found to propagate with their characteristics depend on the interface parameters like magnetic field, density ratio, viscosity….etc. The viscous damping of Alfvén surface waves has been studied in the astrophysical point of view. The damping length of Alfvén surface waves due to viscosity in the solar atmosphere has been estimated.

1. Introduction

The observed inhomogeneous nature of the solar atmosphere has motivated many researchers to study the wave propagation in a magnetically structured atmosphere. In the solar atmosphere, discontinuities in magnetic field and density is observed on the edges of sun spots, flux tubes, prominences, coronal holes…etc which may lead to the existence of surface waves. The magnetohydrodynamic (MHD) surface waves at a single interface in the incompressible medium have been studied by several authors [1, 2, 3].

Recently, Alfvén waves are detected in the solar corona [4] and many researchers have explored the presence of Alfvén waves in the solar chromosphere [5], in the solar prominence [6], and in the x-ray jets [7] and confirmed through Hinode observations. The ubiquitous presence of Alfvén waves in the solar atmosphere has been revealed and accepted by the scientific community. Hence, the study of Alfvén waves in the structured atmosphere namely the Alfvén surface waves (ASW) may unveil many interesting phenomena occurring in the solar atmosphere.

The investigation on viscous damping of MHD waves in the solar corona was started much earlier [8] and significant works have been done later [9, 10]. The viscous damping of surface MHD waves at a magnetic interface with cold plasma approximation has been studied in the solar corona [11] and in the solar chromosphere [12] by including the thermal conductivity as an additional damping mechanism. However, it is suggested that viscosity plays a crucial role in the damping of waves over the other existing mechanisms such as phase mixing, ohmic heating, resonance absorption…etc [13]. The viscous MHD spectra for cylindrical plasma has been analysed by Santiago et al. [14] and confirmed the viscosity as the dominant mechanism for coronal loop heating. In this work, we study

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[1] Address for corresponding author. Email: sivaram_gri@yahoo.com;
Phone: +91 451 2452371-75; Fax: +91 451 2454466.

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the viscous damping of ASW at a single magnetic interface separating two incompressible viscous plasma media and estimate the damping length in the solar atmosphere.

2. Basic equations and derivation of dispersion relation

 In the highly structured solar atmosphere, in order to describe the propagation of ASW , we consider a single interface separating two magnetised media. It is assumed that the magnetised media occupying above (x>0) and below (x<0) the interface (x=0) are incompressible and viscous in nature with different magnitudes of magnetic field and density.

In the MHD approximation the linearized equations governing the electromagnetic and hydrodynamic properties of an incompressible viscous medium of density with magnetic field have been adopted from Uberoi and Somasundaram [15]

Applying a small perturbation of the form $f(x, y, z, t) = f(x) \exp[i(ly + kz - \omega t)]$ and solving the linear MHD equations, the field components can easily be derived. Applying the boundary conditions that the normal and tangential component of the velocity, tangential viscous stress and total pressure are continuous, we obtain the dispersion relation for ASW as,

$$
i\left[\rho_{01}\left(\omega^2 - k^2 V_{A1}^2\right) + \rho_{02}\left(\omega^2 - k^2 V_{A2}^2\right)\right]\left[\rho_{01}\left(\omega^2 - k^2 V_{A1}^2\right)(\tau_2 - K) + \rho_{02}\left(\omega^2 - k^2 V_{A2}^2\right)(\tau_1 - K)\right]
$$
$$
+ i4K\rho_{01}\left(\omega^2 - k^2 V_{A1}^2\right)\rho_{02}\left(\omega^2 - k^2 V_{A2}^2\right) - 4K^2\omega(v_1 - v_2)\left[\rho_{01}\left(\omega^2 - k^2 V_{A1}^2\right)(\tau_2 - K) - \rho_{02}\left(\omega^2 - k^2 V_{A2}^2\right)(\tau_1 - K)\right]
$$
$$
+ i4K^3\omega^2(v_1 - v_2)^2(\tau_1 - K)(\tau_2 - K) = 0
$$

(1)

Where,

$$
\tau_{1,2} = \left[K^2 - \frac{i\rho_{01,2}}{v_{1,2}\,\omega}\left(\omega^2 - k^2 V_{A1,2}^2\right)\right]^{\frac{1}{2}} \quad \text{and } K^2 = l^2 + k^2.
$$

$$
V_{A1,2} = \frac{\mathbf{B}_{01,2}}{\sqrt{\mu\rho_{01,2}}}, \quad \mathbf{B}_{01,2}, \quad \rho_{01,2} \text{ and } v_{1,2} \text{ are respectively the Alfvén velocity, magnetic field,}
$$

density and viscosities in medium 1 and 2. Equation (1) is normalized by introducing the following non-dimensional interface parameters,

$$
T_1 = \frac{\tau_1}{K} = \left[1 - \frac{(x^2 - 1)}{V_1 z}\right]^{\frac{1}{2}} \quad , \quad T_2 = \frac{\tau_2}{K} = \left[1 - \frac{(x^2\eta - \alpha^2)}{\alpha^2 V_2 z}\right]^{\frac{1}{2}} \text{ and } z = 1 + \tan^2\theta
$$

Where,

$$
x = \frac{\omega}{kV_{A1}}, \quad V_{1,2} = \frac{v_{1,2}\omega}{\rho_{01,2}V_{A1,2}^2}, \quad \alpha = \frac{\mathbf{B}_{02}}{\mathbf{B}_{01}} \text{ and } \eta = \frac{\rho_{02}}{\rho_{01}} \text{ is the normalized phase velocity, viscosity}
$$

in medium 1 and 2, magnetic filed ratio and density ratio respectively. θ -is the angle between the background magnetic field and propagation wave vector.

The normalized dispersion relation is obtained as,

$$
\left[\left(x^2 - 1\right) + \left(x^2\eta - \alpha^2\right)\right]\left[\left(x^2 - 1\right)(T_2 - 1) + \left(x^2\eta - \alpha^2\right)(T_1 - 1)\right] + 4\left(x^2 - 1\right)\left(x^2\eta - \alpha^2\right)
$$
$$
+ i4z\left(V_1 - \alpha^2 V_2\right)\left[\left(x^2 - 1\right)(T_2 - 1) - \left(x^2\eta - \alpha^2\right)(T_1 - 1)\right] + 4z^2\left(V_1 - \alpha^2 V_2\right)^2(T_1 - 1)(T_2 - 1) = 0
$$

(2)

Since the equation (2) is irrational, solving this equation as such becomes very cumbersome. Hence, by employing the binomial approximation, T_1 and T_2 can be expanded as,

$$T_1 = 1 - \frac{i(x^2-1)}{2V_1 z} + \frac{(x^2-1)^2}{8V_1^2 z^2} + \frac{i(x^2-1)^3}{16V_1^3 z^3} + \dots\dots \tag{3}$$

$$T_2 = 1 - \frac{i(x^2\eta-\alpha^2)}{2\alpha^2 V_2 z} + \frac{(x^2\eta-\alpha^2)^2}{8\alpha^4 V_2^2 z^2} + \frac{i(x^2\eta-\alpha^2)^3}{16\alpha^6 V_2^3 z^3} + \dots\dots \tag{4}$$

Applying the above approximation, the equation (2) becomes,

$$\left[(x^2-1)+(x^2\eta-\alpha^2)\right]\left[-\frac{i}{2\alpha^2 V_1 z} + \frac{(x^2\eta-\alpha^2)}{8\alpha^4 V_2^2 z^2} + \frac{i(x^2\eta-\alpha^2)^2}{16\alpha^6 V_2^3 z^3} - \frac{i}{2V_1 z} + \frac{(x^2-1)}{8V_1^2 z^2} + \frac{i(x^2-1)^2}{16V_1^3 z^3}\right]$$

$$4 + i4z(V_1 - \alpha^2 V_2)\left[-\frac{i}{2\alpha^2 V_1 z} + \frac{(x^2\eta-\alpha^2)}{8\alpha^4 V_2^2 z^2} + \frac{i(x^2\eta-\alpha^2)^2}{16\alpha^6 V_2^3 z^3} + \frac{i}{2V_1 z} - \frac{(x^2-1)}{8V_1^2 z^2} - \frac{i(x^2-1)^2}{16V_1^3 z^3}\right]$$

$$+ \frac{(V_1-\alpha^2 V_2)^2}{\alpha^2 V_1 V_2}\left[\begin{array}{l} -1 - \frac{i(x^2\eta-\alpha^2)}{4\alpha^2 V_2 z} - \frac{i(x^2-1)}{4V_1 z} + \frac{(x^2\eta-\alpha^2)}{8\alpha^4 V_2^2 z^2} + \frac{(x^2-1)}{8V_1^2 z^2} + \frac{(x^2-1)(x^2\eta-\alpha^2)}{16\alpha^2 V_1 V_2 z^2} \\ + \frac{i(x^2-1)(x^2\eta-\alpha^2)^2}{32\alpha^4 V_1 V_2^2 z^3} + \frac{i(x^2-1)^2(x^2\eta-\alpha^2)}{32\alpha^4 V^2_1 V_2 z^3} - \frac{(x^2-1)^2(x^2\eta-\alpha^2)^2}{64\alpha^4 V_1^2 V_2^2 z^4} \end{array}\right] = 0$$

$$\tag{5}$$

3. Results and discussion

The equation (5) is solved numerically and drawn curves for the interface parameters $\alpha = \sqrt{0.2}$, $\eta = 0.02$ and 1.2, $V_2 = 0.2$, 1.0 and 2.0 and V_1 ranging from 0 to 1.0. The dependence of k_r and k_i (in units of ω/V_{A1}) on the normalized viscosity (V_1) for various magnitudes of V_2 (=0.2, 1.0 and 2.0) have been plotted. The values of k_r and k_i can be calculated from the obtained values of x ($= \omega/kV_{A1}$), where k is the complex wave number $(k = k_r + i k_i)$.

Figures 1a, 1b and 1c are plotted for $\alpha = \sqrt{0.2}$, $\eta = 0.02$ with various magnitudes of V_2. It can be seen from the figures that there are two modes of ASW. It is interesting to note that in all the figures (1a-1c), for a typical critical value of $V_1 (V_{1c})$, k_{r1} has a down hump and k_{i1} has a raising hump in the first mode, which is vice versa in the second mode. This critical V_1 value increases as V_2 increases. From the figures 1a-1c, V_{1c} for V_2 =0.2, 1.0 and 2.0 are obtained as 0.1, 0.2 and 0.4 respectively. In the figures 2a-2c, we observe the same characteristics as that of figures 1a-1c. These graphs are drawn for $\alpha = \sqrt{0.2}$, $\eta = 1.2$ with various V_2 values. The amplitudes of the observed peaks are significantly higher than observed in figures 1a-1c. The critical values of $V_1 (V_{1c})$ are same for figures 1b and 2b, 1c and 2c, except that the V_{1c} value in figure 1a and 2a has been measured to be 0.1 and 0.05 respectively. However, from the figures 1c and 2c, it seemed that a second critical V_1 value (=0.1) appears for V_2 >1, which is less than V_{1c} (=0.4).
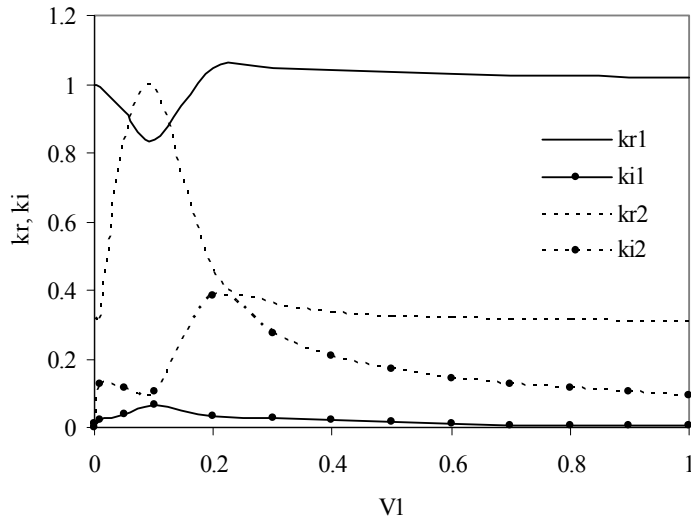
Figure 1a. Plots are drawn for k_r and k_i (in units of ω/V_{A1}) against V_1 with $\alpha = \sqrt{0.2}$, $\eta = 0.02$ and $V_2 = 0.2$
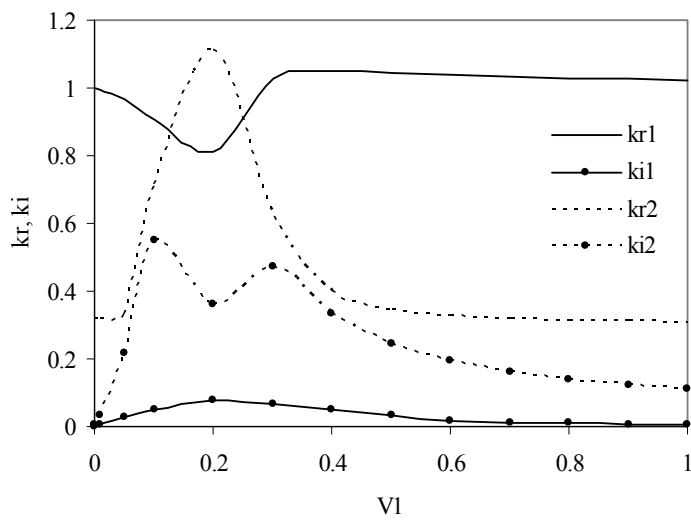

Figure 1b. Plots are drawn for k_r and k_i (in units of ω/V_{A1}) against V_1 with $\alpha = \sqrt{0.2}$, $\eta = 0.02$ and $V_2 = 1.0$
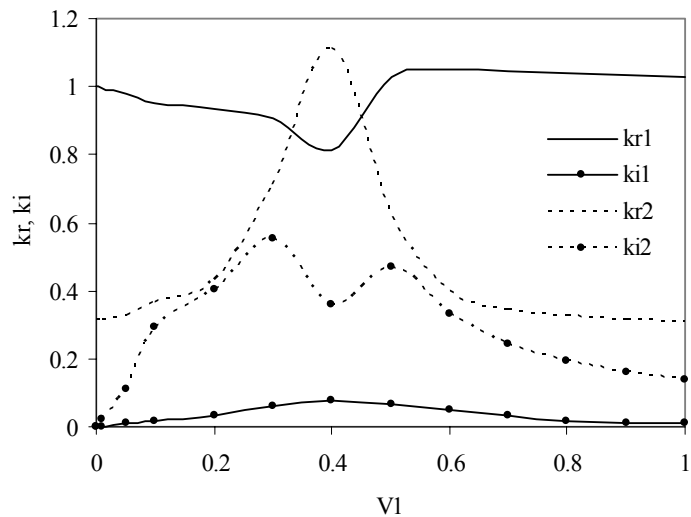

Figure 1c. Plots are drawn for k_r and k_i (in units of ω/V_{A1}) against V_1 with $\alpha = \sqrt{0.2}$, $\eta = 0.02$ and $V_2 = 2.0$
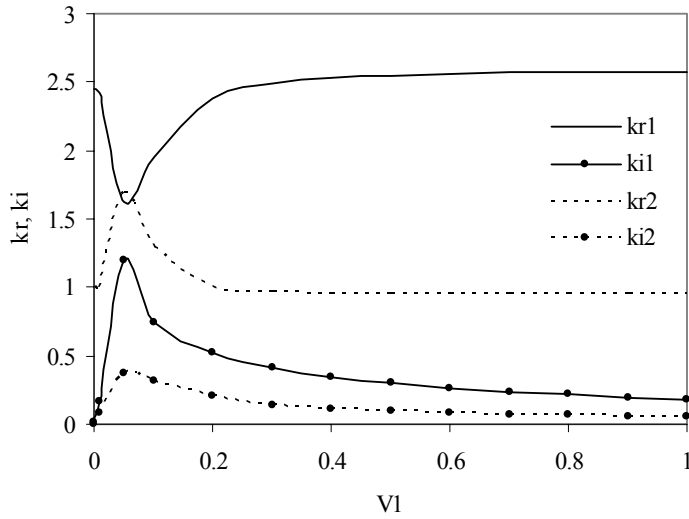
Figure 2a. Plots are drawn for k_r and k_i (in units of ω/V_{A1}) against V_1 with $\alpha = \sqrt{0.2}$, $\eta = 1.5$ and $V_2 = 0.2$
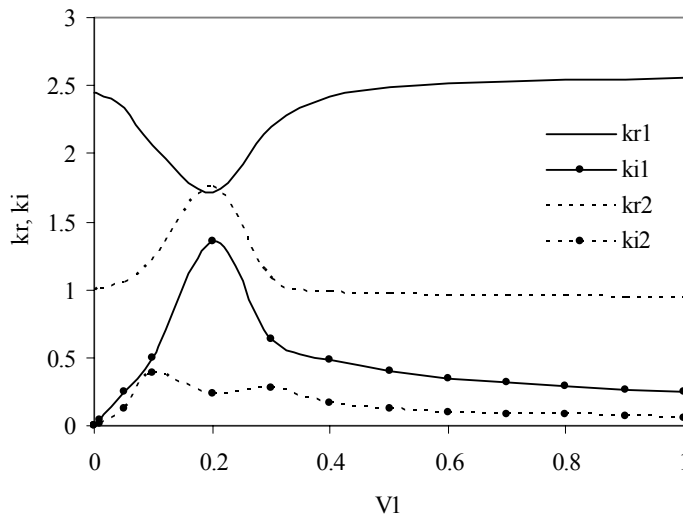

Figure 2b. Plots are drawn for k_r and k_i (in units of ω/V_{A1}) against V_1 with $\alpha = \sqrt{0.2}$, $\eta = 1.5$ and $V_2 = 1.0$
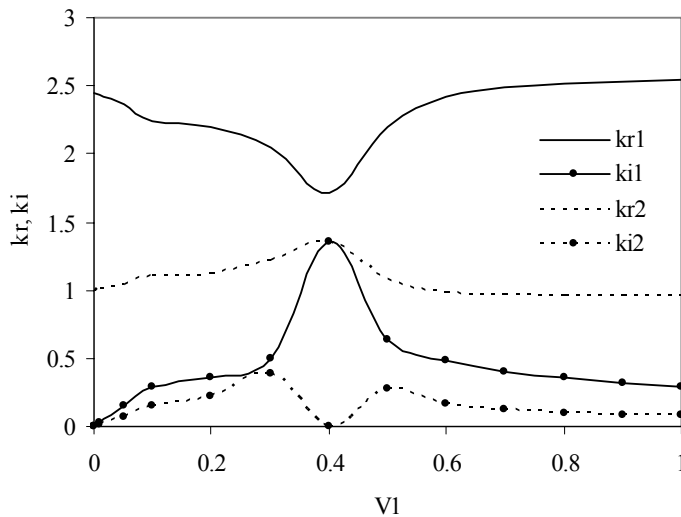

Figure 2c. Plots are drawn for k_r and k_i (in units of ω/V_{A1}) against V_1 with $\alpha = \sqrt{0.2}$, $\eta = 1.5$ and $V_2 = 2.0$

In this study, we have taken typical numerical values in the solar corona, the field aligned viscosity $\nu \rho^{-1} = 6.9 \times 10^7 \, T^{5/2} \, n^{-1} \, cm^2 \sec^{-1}$ [16], temperature $T = 10^5 - 10^6 \, K$, number density $n = 10^9 \, cm^{-3}$, Alfvén velocity $V_{A1} = 10^8 \, cm \sec^{-1}$ and frequency $\omega = 1 \sec^{-1}$ [17]. For the above values, the calculated value of normalized viscosity lies between $2.18 \times 10^{-5} - 6.9 \times 10^{-3}$. Damping length of ASW due to viscosity has been estimated at the critical values of V_1 observed in figures 1a-1c for different values of V_2 when $\eta = 0.02$. At $V_{1c} = 0.1$ for $V_2 = 0.2$, the first mode in figure 1a has the values $k_{r1} = 0.836698 \, \omega / V_{A1}$, $k_{i1} = 0.063515 \, \omega / V_{A1}$. The propagation wavelength (λ_{r1}) and damping length (λ_{i1}) can be calculated from the formulae, $\lambda_{r1} = 2\pi / k_{r1}$ and $\lambda_{i1} = 2\pi / k_{i1}$. Hence, $\lambda_{r1} = 7.5 \times 10^3 \, km$, $\lambda_{i1} = 9.88 \times 10^4 \, km$ the ratio $\lambda_{i1} / \lambda_{r1} = 13.17 \approx 13$ can be calculated. Similarly for the second mode, $\lambda_{r2} = 6.36 \times 10^3 \, km$, $\lambda_{i2} = 5.90 \times 10^4 \, km$ and $\lambda_{i2} / \lambda_{r2} = 9.28 \approx 9$, which reveals that the first mode damps slowly since the damping length is 13 times that of propagation wavelength, whereas, the second mode gets damped shortly than first mode as the damping length is 9 times that of propagation wavelength. Hence, first mode experiences slow damping which can propagate outwards to longer distance while second mode suffers heavy damping. The damping length calculation for $V_2 \geq 1$ (figures 1b and 1c) shows that the first mode damps slowly as the damping length is 10 times that of propagation wavelength and second mode suffers heavy damping since damping length to propagation length ratio is just 3.

From the figure 2a, for $V_2 = 0.2$, $\eta = 1.5$ at $V_{1c} = 0.05$, for the first mode $\lambda_{r1} = 3.88 \times 10^3 \, km$, $\lambda_{i1} = 5.22 \times 10^3 \, km$, the ratio $\lambda_{i1} / \lambda_{r1} = 1.35 \approx 1$, for the second mode $\lambda_{r2} = 3.70 \times 10^3 \, km$, $\lambda_{i2} = 1.66 \times 10^4 \, km$, the ratio $\lambda_{i2} / \lambda_{r2} = 4.486 \approx 4$. In this case, for the first mode, the damping length and the propagation wavelength are nearly equal ($\lambda_{r1} \approx \lambda_{i1}$), hence it suffers a very heavy damping and the second mode can propagate to a distance more than first mode before damping since the ratio between damping length to propagation length ratio is 4. This change is observed due to the influence of η, because the already reported damping length to propagation wavelength ratio at viscous plasma-vacuum interface has been 14 [15]. Hence the density plays a major role in damping of surface waves. However, when $V_2 \geq 1$, from figures 2b and 2c, for the first mode, $\lambda_{i1} / \lambda_{r1} \approx 1$. The damping length and the propagation wavelength are nearly equal for the first mode and hence it suffers a very heavy damping. For the second mode, $\lambda_{i1} / \lambda_{r1} \approx 7$, therefore, the second mode propagates 7 times of the propagation wavelength before being damped. Hence, the first mode in figures 2a-2c experiences heavy damping and dies out shortly due to the influence of density enhancement since this mode has the ratio ($\lambda_{i1} / \lambda_{r1}$) ≥ 10 in the calculations from figures 1a-1c. The damping length of second mode increases as η increases when $V_2 \geq 1$, while decreases for $V_2 < 1$.

It is clear from the numerical calculation that among the two modes of ASW modified by viscosity, one of them is damped normally while the other is heavily damped. From the figures 1a-1c, first mode is normally/weakly damped and the second mode is heavily damped. The calculation from figure 2a-2c shows that the first mode suffers very heavy damping and dies out shortly while the second mode exhibits normal damping.

4. Conclusion

In this work, the viscous damping of ASW at a magnetic interface in the solar atmosphere has been studied. Two modes of ASW modified by viscosity are observed. The damping length has been calculated in the solar coronal situations. The results show that one of the modes is normally damped

and the other is heavily damped depending on the interface parameters. Interestingly, for $\alpha = \sqrt{0.2}$, $\eta = 1.2$, for all values of V_2 (0.2, 1.0 and 2.0), one of the modes suffer rapid damping since the propagation wavelength and damping length are almost equal. Hence, the density plays a vital role in the damping of surface waves in addition to the viscosity.

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