

## Recent Developments in Polarized Line Formation in Magnetic Fields

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**Summary.** The nature of solar surface magnetism has been an open problem in solar physics. In this paper we address three frontline problems of spectropolarimetry of the Sun. We first review the theoretical formulation and numerical solutions of Zeeman absorption and then the Hanle scattering phenomena in 'turbulent magnetic fields'. We show that the mean emergent Stokes profiles can not be obtained by simply averaging the scattering and absorption opacities respectively, over a given distribution of the random field (except when the micro-turbulence prevails). A new formulation of the transfer equation is necessary to study the astrophysically interesting meso-turbulence case. Such formulations of the stochastic polarized radiative transfer problems for absorbing and scattering media are developed only in recent years. We review them and show some results computed by our new formulations.

Until recent years the solution of the polarized line radiative transfer equation in LTE (Zeeman absorption in strong fields), and its NLTE counterpart (Hanle scattering in weak fields), were treated as two disparate problems. The reason for this artificial division was more due to the theoretical and numerical difficulties encountered in the solution of the combined Hanle-Zeeman radiative transfer equation. A very general form of the transfer equation was formulated only a decade ago, for the case of complete frequency redistribution. A more difficult case of partial frequency redistribution is explored by us recently. We review these developments through a study of the Hanle-Zeeman effect in arbitrary strength magnetic fields.

### 1 Introduction

The radiative transfer equation represents one of the most fundamental equations in astrophysics. It is an energy balance equation that describes the transmission and reflection (scattering) of radiation in a stellar atmosphere. In a general form it describes the non-equilibrium thermodynamic processes responsible for the net outflow of radiant energy from the stellar atmosphere.

In this review we focus on the processes responsible for the 'generation and transfer' of polarized radiation in an astrophysical plasma. Polarization is produced by (a) scattering of anisotropic radiation on atoms and/or (b) emission/absorption of radiation by atoms in an external magnetic field. Scattering

can give rise to polarization even in the absence of magnetic fields (‘resonance scattering’). A modification to this scattering brought about by the weak magnetic fields is called ‘Hanle-effect’ (Hanle 1924). Scattering in arbitrary fields is termed Hanle-Zeeman effect (Stenflo 1998; Sampurna et al. 2007a).

The paper is organized into three parts. In the first part (cf. Sect. 2.2) we review our recent work on polarized Zeeman line formation in turbulent magnetic media. In the second part we discuss the effects of magnetic turbulence (stochasticity) on the Hanle scattering polarization (see Sect. 2.3). The relevant radiative transfer equations are formulated and solved. Stochastic polarized line formation theory has a diagnostic potential in stellar spectroscopy. The third part of the review (cf. Sect. 3) concerns our recent formulation of the theory of Hanle-Zeeman effect, and solution of the relevant transfer equation.

## 2 Line formation in magnetically turbulent media

### 2.1 Historical context

It is now well established that 99% of the solar photospheric volume is filled with magnetic fields that are hidden from direct measurements. What actually has been measured so far through Zeeman effect (that diagnoses only strong kG fields), constitutes just 1% accounting for the deterministic part of the total field. The enigmatic nature of the hidden field (which is proposed to be turbulent) is still an unsolved problem in solar physics.

Turbulence causes formation of eddies of different length scales. The average eddy size  $l_{\text{eddy}}$  is called the ‘correlation length’. Magnetic fields in the stellar atmosphere become turbulent as the field lines get tangled up by the turbulent fluid motion (under the frozen-in condition : gas pressure larger than the magnetic pressure). The photon mean free path  $l_{\text{photon}}$  ( $= 1/\kappa_{\nu}$  where  $\kappa_{\nu}$  is absorption coefficient) defines the radiative transfer length scale. When  $l_{\text{photon}} \gg l_{\text{eddy}}$  the effects of turbulence in non-scattering problems can be included by averaging the Zeeman line absorption matrix over a probability density function (PDF) of the random magnetic field. This is called the micro-turbulent limit, which is widely used in line formation theory. The form of Zeeman line transfer equation in the micro-turbulent limit was first written by Stenflo (1971). The effect of micro-turbulent magnetic field on Zeeman line absorption matrix was first studied by Dolginov & Pavlov (1972), and Domke & Pavlov (1979), which was revisited in greater detail by Frisch et al. (2005).

The extension of micro-turbulence to absorbing-scattering media involves averaging over both the absorption matrix and the scattering phase matrix. Here we confine our attention to the averaging over the weak field Hanle phase matrix. Such a calculation was first presented by Stenflo (1982) who proposed turbulent Hanle effect as a diagnostic tool to determine the solar hidden fields. His simplified approach (based on single scattering) gave a lower limit of 10 G

for this general turbulent field. Through a realistic 1D radiative transfer modeling of the scattering polarization in Sr I 4607 Å line, Faurobert et al. (2001) estimated this photospheric general field to be less than 20 G. The recent work of Trujillo Bueno et al. (2006) based on 3D radiative transfer modeling of the solar atmosphere shows that the photospheric turbulent fields could be in the range of 60-130 G. All these authors use a micro-turbulent magnetic field distributed according to an angularly isotropic PDF, with constant given mean field  $B_0$ , or an exponential field strength distribution.

The macro-turbulent limit ( $l_{\text{photon}} \ll l_{\text{eddy}}$ ) consists of performing an averaging of the emergent specific intensity  $I_\nu$  over a PDF. A deterministic field radiative transfer problem is first solved for each realization of the magnetic field in a macro-structure. The emergent solution is then averaged to obtain the mean solution. Multi-component models to represent unresolved macro-structures were introduced by Stenflo (1971, 1994).

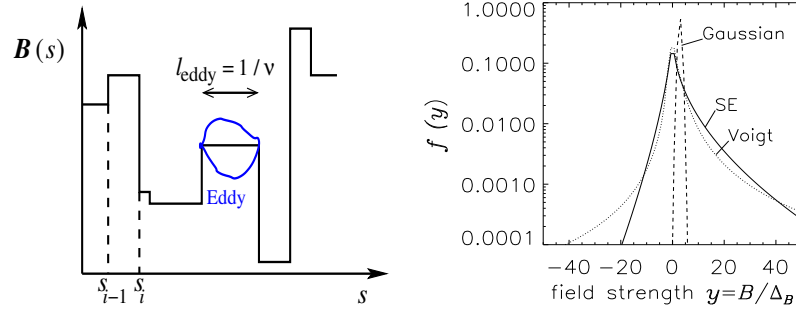
The general regime of meso-turbulence ( $l_{\text{photon}} \approx l_{\text{eddy}}$ ) requires the solution of ‘stochastic polarized transfer equation with random coefficients’. For the Zeeman effect, an exact solution method was first given by Landi Degl’Innocenti (1994). A more sophisticated version of this theory was proposed by Frisch et al. (2006). A qualitatively similar approach was taken by Carroll & Staude (2006, and references therein). The general theory of line transfer for a meso-turbulent Hanle scattering problem was developed recently by Frisch (2006). A numerical method to solve this newly formulated transfer equation is given in Frisch et al. (2009, see also Anusha et al. 2009).

## 2.2 Stochastic line radiative transfer in an absorbing medium

### Theoretical formulation :

In this section we restrict ourselves to the simpler case of Stokes vector ( $\mathbf{I} = (I, Q, U, V)^T$ ) transfer equation in a truly absorbing medium (LTE). We consider the specific case of Zeeman line formation in a 1D semi-infinite constant property media with a linear variation of the source vector (the Milne-Eddington model). We model random magnetic field  $\mathbf{B}(s)$  by a KAP (Kubo-Anderson Process). A KAP is a step-wise stationary Markov process. The idea of using KAP is to describe the atmosphere in terms of several eddies (see left panel of Fig. 1) having lengths distributed according to a Poisson’s law with a given density  $\nu$  (number of jumps/unit optical depth). We assume that in each eddy (say  $s_{i-1} < s < s_i$ ) the magnetic field remains constant ( $\mathbf{B}(s) = \mathbf{B}_i$ ), and it takes values according to a chosen PDF, denoted  $P(\mathbf{B})$ .

The mean Stokes vector  $\mathbf{I}$  is calculated by averaging over the length distribution of the eddies, and  $P(\mathbf{B})$ . A KAP is thus characterized by a correlation length ( $1/\nu$ ) and a PDF, the selection of which is guided by the structure that is being modeled. The transfer equation for the particular case of the rays propagating outwards along the surface normal is written as



**Fig. 1.** Left panel: A typical Kubo-Anderson process  $B(s)$  with density  $\nu$  (mean correlation length is  $1/\nu$ ). Right panel: Three different field strength PDFs are used. The rms fluctuation (full width)  $\Delta_B = 6$  G and mean field  $y_0 = B_0/\Delta_B = 2.9$ .

$$\frac{d\mathbf{I}}{d\tau_c} = \left( \hat{E} + \beta \hat{\Phi} \right) (\mathbf{I} - \mathbf{S}), \quad (1)$$

where  $\hat{\Phi}$  is the Zeeman line absorption matrix,  $\mathbf{S}$  is the source vector given by  $\mathbf{S}(\tau_c) = (C_0 + C_1\tau_c)\mathbf{U}$  with  $\mathbf{U} = (1\ 0\ 0\ 0)^T$ ,  $\tau_c$  being the continuum optical depth, and  $\beta = \kappa_0/\kappa_c$  with  $\kappa_0$  and  $\kappa_c$  being frequency integrated line and continuum opacity. Following Landi Degl'Innocenti & Landolfi (2004), the formal solution of Eq. (1) at  $\tau_c = 0$  is

$$\mathbf{I}(0) = \left[ C_0 \hat{E} + C_1 \int_0^\infty \hat{O}(0, \tau_c) d\tau_c \right] \mathbf{U}, \quad (2)$$

where  $\hat{O}(0, \tau_c)$  is the evolution operator. We are actually interested in the calculation of the mean emergent Stokes vector  $\langle \mathbf{I}(0) \rangle_{\text{KAP}}$  defined as

$$\langle \mathbf{I}(0) \rangle_{\text{KAP}} = \left[ C_0 \hat{E} + C_1 \int_0^\infty \langle \hat{O}(0, \tau_c) \rangle_{\text{KAP}} d\tau_c \right] \mathbf{U}. \quad (3)$$

We recall that the notation  $\langle \dots \rangle_{\text{KAP}}$  means that we need to perform two averages, one over the length distribution of the eddies and the other over a PDF. The first averaging can be analytically performed, as we consider a simple Milne-Eddington atmospheric model. The averaging over the PDF is performed in general numerically (analytical averaging is possible in the micro-turbulent regime - see Frisch et al. 2005).

Following Brissaud & Frisch (1974), we establish a convolution type integral equation for  $\langle \hat{O}(0, \tau_c) \rangle_{\text{KAP}}$  that is solved by a Laplace transform method to obtain  $\langle \mathbf{I}(0) \rangle_{\text{KAP}}$ . We prefer to calculate the residual Stokes vector  $\mathbf{r}(0) = [\mathbf{I}_c(0) - \mathbf{I}(0)]/C_1$ , where the continuum intensity  $\mathbf{I}_c(0) = (C_0 + C_1)\mathbf{U}$ . After averaging over all the realizations of the random fields we obtain

$$\langle \mathbf{r}(0) \rangle_{\text{KAP}} = (1 + \nu) \hat{R}_{\text{macro}} \left( \frac{\beta}{1 + \nu} \hat{\Phi} \right) \left[ \hat{E} + \nu \hat{R}_{\text{macro}} \left( \frac{\beta}{1 + \nu} \hat{\Phi} \right) \right]^{-1} \mathbf{U}, \quad (4)$$

for an arbitrary value of  $\nu$ . Here  $\hat{R}_{\text{macro}}(\lambda\hat{\Phi}) \equiv \langle \lambda\hat{\Phi}[\hat{E} + \lambda\hat{\Phi}]^{-1} \rangle$ , with  $\lambda$  a scalar. The macro and micro turbulent limits can be recovered by setting  $\nu = 0$  and  $\infty$  respectively in Eq. (4).

### Numerical results on turbulent Zeeman effect :

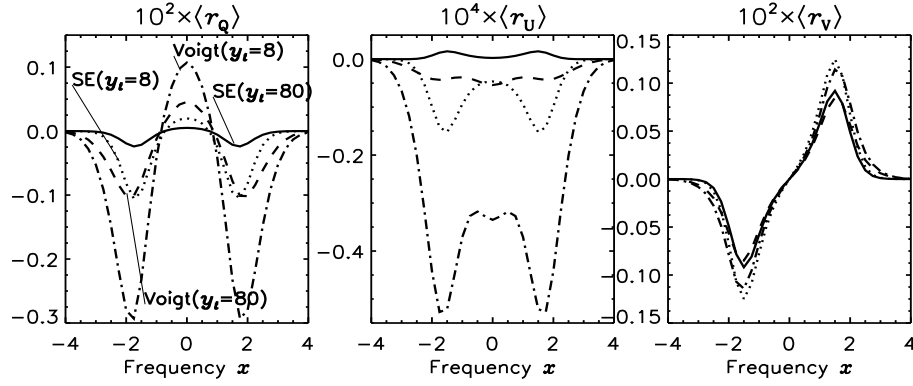
We illustrate the sensitivity of  $\langle \mathbf{r}(0) \rangle_{\text{KAP}} = (\langle r_I \rangle, \langle r_Q \rangle, \langle r_U \rangle, \langle r_V \rangle)^T$  to the choice of the PDFs. The model used has a damping parameter  $a = 0$ , line strength  $\beta = 10$ , and correlation length  $1/\nu = 1/5$ . Further the line has a wavelength around  $5000 \text{ \AA}$ , a Landé factor of 3 and a Doppler width of  $1.5 \text{ Kms}^{-1}$ .

In the solar atmosphere, both the field strength and its orientation fluctuate. Hence we need a PDF that can represent the combined fluctuations. We propose such ‘composite PDFs’, generally guided by the nature of field distribution in solar atmospheric structures. Examples of such PDFs are :

$$P(y, \mu_B, \chi_B) d\mu_B d\chi_B dy = \frac{(p+1)}{2\pi} \begin{cases} P_V(y, a_B) \mu_B^p d\mu_B d\chi_B dy, \\ C e^{-|y|^k} \mu_B^p d\mu_B d\chi_B dy. \end{cases} \quad (5)$$

Here  $\theta_B$  and  $\chi_B$  are the random field orientation with respect to the atmospheric normal, and  $C$  is the normalization constant.  $P_V(y, a_B)$  is the Voigt function (that now depends on the non-dimensional field strength  $y$  and magnetic damping parameter  $a_B$ ), and  $e^{-|y|^k}$  is the stretched exponential (SE) distribution with  $k$  being the stretching parameter. The angular fluctuations are taken care by a cosine power law term  $\mu_B^p$  where  $p = |y|/y_t$  with  $y_t = B_t/\Delta_B$ , and  $B_t$  is the transition field strength between isotropic (in the weak field regime), and peaked (in the strong field regime) distribution. Such composite PDFs are expected to describe the general behavior of observed solar magnetic field to some degree of realism - a large volume filled by weak isotropic fields, and a very small volume filled by strong well directed fields.

For illustrations we choose  $B_t = 500 \text{ G}$  and  $50 \text{ G}$ . The  $\langle r_{Q,U,V} \rangle$  computed using these composite PDFs are shown in Fig. 2 for  $\mu = 0.1$ . We use asymmetric Voigt and SE functions (Sampoorna et al. 2008) with a mean field  $y_0 = 2.9$ , to represent the fluctuations only in the field strength. The width  $\Delta_B$  (rms fluctuation) of the PDF is found to be  $6 \text{ G}$  by an analysis of the solar magnetogram data (see Stenflo & Holzreuter 2003).  $\gamma_B$ , which represents  $\Delta_B$  converted to Zeeman shift in Doppler width units, is  $0.0084$  for this choice of  $\Delta_B$ . As  $\gamma_B \ll 1$ , the  $\langle r_I \rangle$  profiles remain insensitive to the turbulent field related parameters (not shown here). For  $y_t = 8$  the PDF becomes more and more anisotropic (larger contribution from wings), resulting in larger degree of linear polarization. For  $y_t = 80$ , a situation opposite to the above prevails, causing  $\langle r_{Q,U} \rangle$  to decrease considerably (solid and dashed lines).  $\langle r_U \rangle$  is very small in magnitude as it is generated purely by magneto-optic effects. The sensitivity of  $\langle r_V \rangle$  to  $y_t$  is similar to that of  $\langle r_Q \rangle$  but weaker.



**Fig. 2.** Effect of choice of the composite PDFs. Larger the transition field strength  $y_i$ , smaller is the polarization, as the composite PDFs become more and more isotropic.

### 2.3 Stochastic radiative transfer equation in a scattering medium

#### Theoretical formulation :

In this section we address a more involved case of polarized line scattering in a stochastic medium. We restrict our attention to the so called complete frequency redistribution (CRD : meaning complete non-coherence during scattering process within a line profile), and ignore the continuum contribution.

#### The polarized radiative transfer equation for the Hanle effect :

The relevant transfer equation in standard form is (see Nagendra et al. 1998) :

$$\mu \frac{\partial \mathbf{I}(\tau, x, \mathbf{n})}{\partial \tau} = \varphi(x) [\mathbf{I}(\tau, x, \mathbf{n}) - \mathbf{S}(\tau, \mathbf{n})], \quad (6)$$

where  $\mathbf{I} = (I, Q, U)^T$  and the Stokes source vector  $\mathbf{S}$  is defined as

$$\mathbf{S}(\tau, \mathbf{n}) = \mathbf{G}(\tau) + (1 - \epsilon) \int_{-\infty}^{+\infty} dx' \oint \frac{d\Omega'}{4\pi} \varphi(x') \hat{P}_H(\mathbf{n}, \mathbf{n}'; \mathbf{B}) \mathbf{I}(\tau, x', \mathbf{n}'). \quad (7)$$

Here  $\epsilon$  is the thermalization parameter,  $\mathbf{G}(\tau) = \epsilon B_{\nu_0} \mathbf{U}$  with  $\mathbf{U} = (1, 0, 0)^T$  and  $B_{\nu_0}$  the Planck function, and  $\hat{P}_H(\mathbf{n}, \mathbf{n}'; \mathbf{B})$  is the Hanle phase matrix. The non-axisymmetry of the radiation is built into the Stokes vector form of the transfer equation. It is advantageous to work with the ‘reduced form’ of Eq. (6) that is obtained through the expansion of  $\mathbf{I}$  and  $\mathbf{S}$  in the form :

$$I_i(\tau, x, \mathbf{n}) = \sum_{KQ} \mathcal{T}_Q^K(i, \mathbf{n}) \mathcal{I}_Q^K(\tau, x, \mu); \quad S_i(\tau, \mathbf{n}) = \sum_{KQ} \mathcal{T}_Q^K(i, \mathbf{n}) S_Q^K(\tau), \quad (8)$$

where  $i = 0, 1, 2$ . The ‘reduced intensity’  $\mathcal{I}_Q^K$  is now independent of the azimuth angle of the radiation field (axi-symmetric). Notice that the reduced

source vector  $\mathcal{S}_Q^K$  is even angle independent! By substituting Eq. (8) in to Eq. (6), we can derive a standard form of the transfer equation for the 6-component reduced intensity vector  $\mathcal{I}$  :

$$\mu \frac{\partial \mathcal{I}(\tau, x, \mu)}{\partial \tau} = \varphi(x) [\mathcal{I}(\tau, x, \mu) - \mathcal{S}(\tau)], \quad (9)$$

where the 6-component reduced source vector is now written as

$$\mathcal{S}(\tau) = \mathcal{G}(\tau) + \hat{N}(\mathbf{B}) \int_{-\infty}^{+\infty} dx \frac{1}{2} \int_{-1}^{+1} d\mu' \varphi(x) \hat{\Psi}(\mu') \mathcal{I}(\tau, x, \mu'). \quad (10)$$

The matrix  $\hat{N}(\mathbf{B})$  contains  $\mathbf{B}$  dependence coming from the Hanle effect, and  $\hat{\Psi}(\mu)$  describes angular dependence coming from the Hanle phase matrix. For details on this expansion procedure and the definition of  $\mathcal{T}_Q^K$  etc., we refer to Frisch (2007). In the presence of a random magnetic field of finite correlation length Eq. (9) becomes a stochastic Hanle transfer equation. To solve such an equation we need to first specify the model for the random field.

### Representation of random (stochastic) magnetic field :

In the case of line transfer with pure absorption, the photons do not change the direction. Therefore it is easy to represent the randomness of the field by a Markov process. However in scattering problems, the photons change the direction randomly (multiple scattering). Hence defining the field randomness (both in the strength and orientation), together with the random walk of the photons in a consistent manner is a difficult task. The difficulty is the following: Suppose the magnetic field is a Markov process (meaning, future depends only on the present and is independent of the past), taken to be along the atmospheric normal. In this case the radiation field  $\mathcal{I}$  at a point  $\mathbf{r}$  on a line-of-sight (LOS) in the direction  $\mathbf{n}$  depends on magnetic field values below and above the point  $\mathbf{r}$  along  $\mathbf{n}$ . As a result the joint process  $\{\mathbf{B}(\mathbf{r}); \mathcal{I}(\mathbf{r})\}$  is not a Markov process and hence the stochastic transfer problem cannot be solved by the standard Fokker-Planck techniques (Brissaud & Frisch 1974). A way out of this situation is to assume that  $\mathbf{B}$  is random in time domain. Such an approach was used by Frisch & Frisch (1976) for random velocities with a finite correlation length. Again we use KAP to model the magnetic field randomness, but defined with respect to time. Thus,  $\mathcal{I}$  should also be time-dependent (i.e., solution of a time-dependent transfer equation). Now the joint process  $\{\mathbf{B}(t); \mathcal{I}(t)\}$  becomes a Markov process, and hence one can apply the standard techniques of solving the stochastic differential equation.

### Transfer equation for the conditional mean Stokes parameters :

We are interested in calculating the 6-component mean Stokes parameters

$$\langle \mathcal{I}_Q^K \rangle = \int \int \mathcal{P}(\mathbf{B}, \mathcal{I}; t) \mathcal{I}_Q^K d\mathcal{I}_0^0 d\mathcal{I}_0^2 \dots d\mathcal{I}_2^2 d^3\mathbf{B}, \quad (11)$$

where  $\mathcal{P}(\mathbf{B}, \mathcal{I}; t)$  is the probability density of the joint process  $\{\mathbf{B}(t); \mathcal{I}(t)\}$ . It is a priori not known, because  $\mathcal{I}$  itself depends on random field  $\mathbf{B}$  and  $\mathcal{I}$  has to be obtained from a solution of the stochastic transfer equation. To overcome this implicit nature of the problem we can work with an intermediate quantity called ‘‘conditional mean Stokes vector’’ with components

$$\mathcal{I}_Q^K(t, \tau, x, \mu | \mathbf{B}) = \frac{1}{P(\mathbf{B})} \int \mathcal{P}(\mathbf{B}, \mathcal{I}; t) \mathcal{I}_Q^K d\mathcal{I}_0^0 d\mathcal{I}_0^2 \dots d\mathcal{I}_2^2. \quad (12)$$

The conditional mean Stokes vector satisfies a time-dependent transfer equation (Frisch 2006). Here  $\mathbf{B}$  plays the role of an additional independent variable of the problem. In particular we look for stationary solution  $\mathcal{I}(\tau, x, \mu | \mathbf{B})$  only, which can be obtained by setting  $t \rightarrow \infty$  in the time-dependent transfer equation. The stationary solution satisfies the following transfer equation:

$$\begin{aligned} \mu \frac{\partial \mathcal{I}(\tau, x, \mu | \mathbf{B})}{\partial \tau} &= \varphi(x) [\mathcal{I}(\tau, x, \mu | \mathbf{B}) - \mathcal{S}(\tau | \mathbf{B})] \\ &+ \nu \left[ \mathcal{I}(\tau, x, \mu | \mathbf{B}) - \int \mathcal{I}(\tau, x, \mu | \mathbf{B}') P(\mathbf{B}') d^3\mathbf{B}' \right]. \end{aligned} \quad (13)$$

The conditional source and mean intensity vectors respectively are:

$$\mathcal{S}(\tau | \mathbf{B}) = \mathcal{G}(\tau) + \hat{N}(\mathbf{B}) \mathcal{J}(\tau | \mathbf{B}), \quad (14)$$

$$\mathcal{J}(\tau | \mathbf{B}) = \int_{-\infty}^{+\infty} \frac{1}{2} \int_{-1}^{+1} \varphi(x) \hat{\Psi}(\mu) \mathcal{I}(\tau, x, \mu | \mathbf{B}) d\mu dx. \quad (15)$$

The mean Stokes vector is now given by

$$\langle \mathcal{I} \rangle(\tau, x, \mu) = \int P(\mathbf{B}) \mathcal{I}(\tau, x, \mu | \mathbf{B}) d^3\mathbf{B}. \quad (16)$$

Clearly, the use of conditional mean Stokes vector cleverly avoided an explicit knowledge of  $\mathcal{P}(\mathbf{B}, \mathcal{I}; t)$ . Applying the above averaging to Eq. (13), we obtain the transfer equation for  $\langle \mathcal{I} \rangle(\tau, x, \mu)$  as

$$\mu \frac{\partial}{\partial \tau} \langle \mathcal{I} \rangle(\tau, x, \mu) = \varphi(x) [\langle \mathcal{I} \rangle(\tau, x, \mu) - \langle \mathcal{S} \rangle(\tau)]. \quad (17)$$

It is not possible to write an integral equation for  $\langle \mathcal{S} \rangle(\tau)$  (except in the micro-turbulent limit). We first calculate  $\mathcal{S}(\tau | \mathbf{B})$  and then average it over  $P(\mathbf{B})$ .

By substituting the formal solution of Eq. (13) into Eq. (15) (see Frisch et al. 2009, for details) we obtain an integral equation for  $\mathcal{S}(\tau | \mathbf{B})$  as

$$\mathcal{S}(\tau | \mathbf{B}) = \mathcal{G}(\tau) + \hat{N}(\mathbf{B}) \Lambda[\mathcal{S}], \quad (18)$$

where  $\Lambda[\mathcal{S}]$  and  $\hat{\mathcal{L}}$  operator are defined as



$$A[\mathcal{S}] = \int_0^T d\tau' \left\{ \hat{\mathcal{L}}(\tau - \tau'; \nu) \mathcal{S}(\tau' | \mathbf{B}) + \right. \\ \left. [\hat{\mathcal{L}}(\tau - \tau'; 0) - \hat{\mathcal{L}}(\tau - \tau'; \nu)] \int P(\mathbf{B}') \mathcal{S}(\tau' | \mathbf{B}') d^3 \mathbf{B}' \right\}, \quad (19)$$

$$\hat{\mathcal{L}}(\tau; \nu) = \int_{-\infty}^{+\infty} \int_0^1 \frac{1}{2\mu} \hat{\Psi}(\mu) e^{-\frac{|\tau|}{\mu}(\varphi(x)+\nu)} \varphi^2(x) d\mu dx. \quad (20)$$

A polarized approximate lambda iteration method can be devised to solve Eq. (18) (see Frisch et al. 2009, for details).

### Numerical results on turbulent Hanle effect :

Here we present some results on the turbulence effects in a Hanle scattering media. The atomic and atmospheric models are defined by a set of parameters  $(T, a, \epsilon, B_{\nu_0})$  where  $T$  is the optical thickness of the slab. In all the calculations we assume a normal Zeeman triplet, an electric-dipole transition and no depolarizing collisions. For the magnetic field, the free parameters are the Hanle efficiency factor  $\Gamma_B = eg_u B / (2mcA_{ul})$  in standard notation; the polar angles  $(\theta_B, \chi_B)$ ; the density  $\nu$  of jumping points; and the PDF  $P(\mathbf{B})$ .

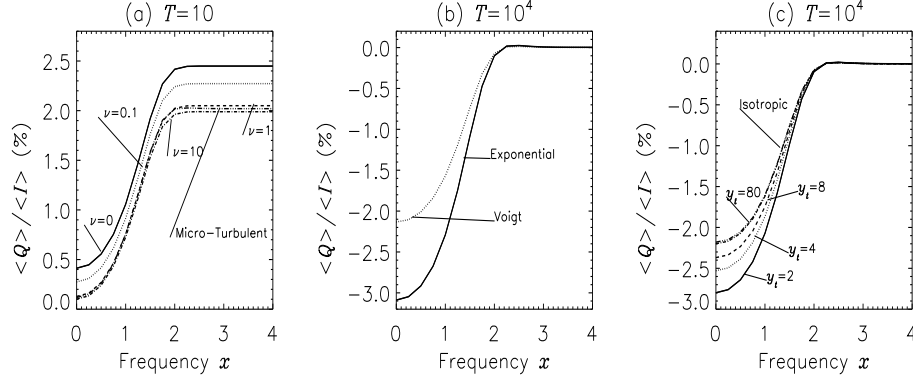
Fig. 3a shows the dependence of linear polarization on the correlation length. An isotropic angular distribution with an exponential field strength distribution is used. Since the PDF is azimuthally symmetric,  $\langle U \rangle = 0$ . Further, as we consider scattering in weak fields Stokes  $\langle I \rangle$  remains insensitive to the changes in the free parameters of  $\mathbf{B}$ , and hence not shown.

For optically thin ( $T \ll 1$ ) and thick ( $T \gg 1$ ) lines the  $\langle Q \rangle / \langle I \rangle$  profiles can be explained well in terms of micro-turbulence. For intermediate values of  $T$ , the  $\langle Q \rangle / \langle I \rangle$  profiles sensitively depend on  $\nu$ . The micro limit is reached for  $\nu = 10$  itself, because  $\nu$  occurs in an exponential term (see Eq. (20)). In Frisch et al. (2009) it is shown that single scattering is a good approximation for both optically thin, and very thick slabs. In the former the number of scatterings are too small (1-2). In the later the photons may suffer a large number of scatterings inside the slab, but it is the last few ones near the free boundary that contribute to the emergent polarization. In other words polarization is a surface phenomenon. It can be shown that just 4-5 scatterings suffice to reach the final value of  $\langle Q \rangle / \langle I \rangle$  in very thick slabs. For slabs of intermediate thickness a full-scale solution of the transfer equation is essential, as the single scattering approximation breaks down. The accuracy of the single scattering approximation progressively improves as one goes from Rayleigh scattering to deterministic field Hanle scattering, and finally to micro-turbulent Hanle scattering. As the single scattering approximation does not depend on  $\nu$ , the emergent  $\langle Q \rangle / \langle I \rangle$  for optically thin and very thick slabs also do not depend on  $\nu$ . The same thing cannot be said of the lines with intermediate thickness (say  $T = 10$ ), where correlation length effects become important.

In Fig. 3b we compare the micro-turbulent  $\langle Q \rangle / \langle I \rangle$  computed using two different field strength distributions with an isotropic angular distribution. A

large difference is seen in the line core region, because the exponential PDF is strongly peaked in the weak field region, compared to the Voigt PDF.

In Fig. 3c we present  $\langle Q \rangle / \langle I \rangle$  computed with composite PDFs given in Eq. (5) with the asymmetric Voigt function. As the transition field strength  $y_t$  (between the isotropic and peaked angular distribution) increases, larger domains of the field strength PDF become isotropic. This results in a decrease in  $\langle Q \rangle / \langle I \rangle$ , finally reaching isotropic distribution for large values of  $y_t$  ( $= 80$ ).



**Fig. 3.** Effect of turbulence in a Hanle scattering media. Model parameters:  $(a, \epsilon, B_{\nu 0}) = (10^{-3}, 10^{-4}, 1)$ . (a) shows the influence of correlation length. (b) and (c) show the effects of different PDFs. The field parameter  $\Gamma_{\sigma} = ge\Delta_B / (2mcA_{ul}) = 1.588$  when  $g = 3$ ,  $\Delta_B = 6$  G and  $A_{ul} = 10^8$  s $^{-1}$ . In (b) the angular distribution is isotropic, whereas field strength PDF are different. For (c) we use a composite PDF with a coupling between angular and field strength distribution.

### 3 Scattering polarization in arbitrary strength magnetic fields - the Hanle-Zeeman effect

#### 3.1 Historical context

The solar atmosphere with its magnetically active regions harbor a whole range of field strengths (milligauss to kilogauss fields). Hence we need a general form of the radiative transfer equation that is valid in this entire range. Such a transfer equation was formulated by Stenflo (1994).

Until recent years, polarized line transfer equation in the presence of deterministic magnetic fields has been solved in two extreme limits, namely, true absorption in strong fields (Zeeman line transfer in LTE) neglecting the scattering effects, and resonance scattering in weak fields (Hanle line transfer in NLTE) neglecting Zeeman absorption. In a correct treatment of NLTE line

formation in arbitrary strength magnetic fields, one has to take into account the ‘Zeeman absorption matrix’ and ‘scattering redistribution matrix (RM)’ together. The latter describes the correlations in frequency, angle and polarization between the incoming and outgoing photons in a scattering event. In the early years Hanle RM for weak fields was used in the line transfer computations (Faurobert-Scholl 1991; Nagendra et al. 1998; Manso Sainz & Trujillo-Bueno 1999). In these papers a hybrid approximation for RM was used, where the frequency correlation part was assumed to be CRD, and the angular correlation to be the Hanle phase matrix (Stenflo 1978; Landi Degl’Innocenti & Landi Degl’Innocenti 1988). However partial frequency redistribution (PRD) is a better description of the frequency correlation. Such PRD computations retaining the hybrid approximation were performed by Faurobert-Scholl (1991) and Nagendra et al. (1999).

A derivation of RM with a proper treatment of PRD dates back to Omont et al. (1973). A correct form of the RM suitable for radiative transfer computation had to wait until the work of Bommier (1997a,b). The laboratory frame RMs for weak fields were written in a factorized form taking frequency and polarization correlations properly into account through appropriate branching ratios. Such factorization takes place in the two dimensional  $(x, x')$  frequency space. This domain based RMs were used in transfer computation by Nagendra et al. (2002), who used a time consuming perturbation method.

To diagnose the solar magnetic fields of arbitrary strength we need RMs that are valid in the entire field strength regime (milligauss to kilogauss). We call such a scattering matrix as Hanle-Zeeman RM. A QED theory of scattering in arbitrary fields was given by Bommier (1997b). The classical analogue of the same is presented in Bommier & Stenflo (1999). In these papers the RM was derived in the rest frame of the atom. The corresponding laboratory frame expressions which are essential for line transfer computations were derived in Sampoorna et al. (2007a). It was shown in Sampoorna et al. (2007b) that for the specific case of  $J = 0 \rightarrow 1 \rightarrow 0$  transition the classical and quantum theories give exactly identical RMs. These RMs were later included by Sampoorna et al. (2008b) in line transfer computations using a simple perturbative approach. No high speed iterative methods (like the one based on approximate lambda operator) has so far been developed to solve this problem, which remains a challenge to numerical radiative transfer.

### 3.2 Theoretical formulation

The transfer equation for the Stokes vector  $(I, Q, U, V)^T$  in a magnetically absorbing and scattering medium is given by

$$\mu \partial \mathbf{I}(\tau, x, \mathbf{n}) / \partial \tau = (\hat{\Phi} + r \hat{E}) \mathbf{I}(\tau, x, \mathbf{n}) - [(r \hat{E} + \epsilon \hat{\Phi}) B_{\nu_0} \mathbf{U} + \mathbf{S}_{\text{scat}}(\tau, x, \mathbf{n})], \quad (21)$$

where the scattering integral is defined as

$$\mathbf{S}_{\text{scat}}(\tau, x, \mathbf{n}) = \int_{-\infty}^{+\infty} \oint \hat{R}(x, \mathbf{n}; x', \mathbf{n}'; \mathbf{B}) \mathbf{I}(\tau, x', \mathbf{n}') \frac{d\Omega'}{4\pi} dx', \quad (22)$$

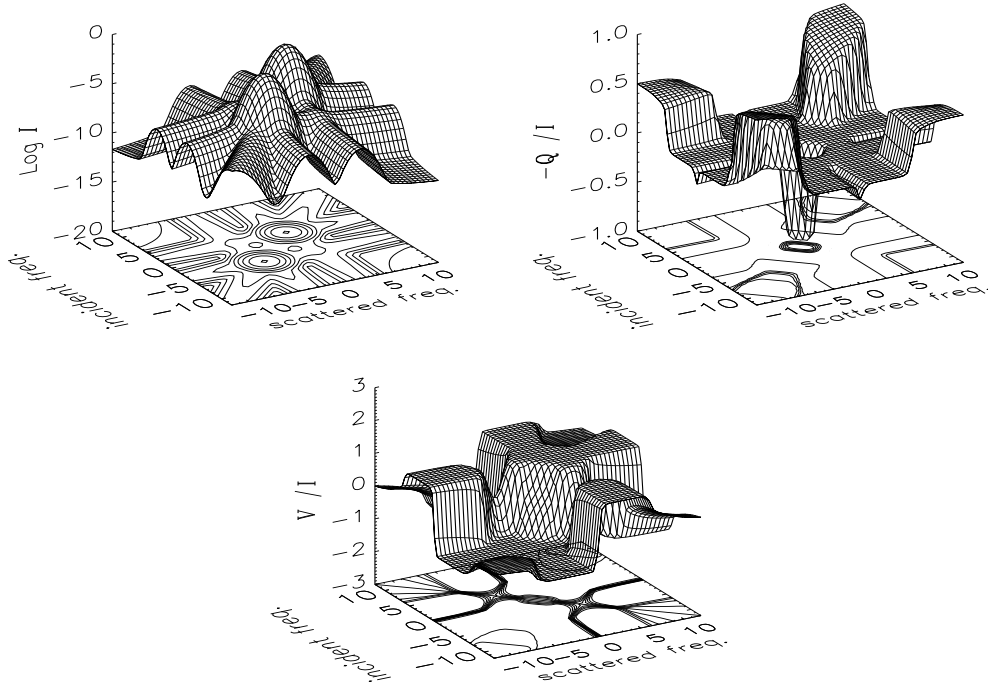
in standard notation. Here  $r = 1/\beta$  is the ratio of continuum to the line center opacity. The Hanle-Zeeman redistribution matrix  $\hat{R}$  is the heart of the problem. It represents PRD in line scattering, namely an appropriately weighted combination of frequency coherent and frequency non-coherent (respectively Hummer's type II and III) scattering. The weighting factors are the branching ratios which contain the physics of various types of collisions (Domke & Hubeny 1988; Nagendra 1994; Bommier 1997b) in polarized line formation theory. It is worthwhile to note that elastic collisions (denoted by  $\Gamma_E$ ) are responsible for destroying the frequency correlation between the incoming and outgoing photons, while the depolarizing collisions  $D^{(K)}$  cause destruction of the atomic alignment and orientation. As in Nagendra et al. (2002) we solve Eqs. (21) and (22) using a perturbation method (see Sampoorna et al. 2008b)

To illustrate the nature of the RM we perform a single scattering experiment in which an unpolarized monochromatic ray is incident on the atom, and undergoes a  $90^\circ$  scattering. The magnetic field is chosen to be parallel to the scattered ray, so that there is maximum impact of the Hanle effect on line scattering. Fig. 4 shows the scattered Stokes profiles in  $(x, x')$  space for a stronger field of  $v_B = g\nu_L/\Delta\nu_D = 3$ . The contour plot below exhibits the 2D domain structure in the  $(x, x')$  space. This figure illustrates the interesting case of Rayleigh scattering in strong fields. In a  $90^\circ$  scattering event only two  $\sigma$ -components are scattered. In the  $\log I$  profile they are displaced on both sides of the line center and are well separated because the Zeeman shift is 3 Doppler width. Far from the line center and Zeeman components, scattering is Rayleigh like and hence the linear polarization of scattered ray is 100% (see along the first diagonal of the  $Q/I$  panel in Fig. 4). Also at line center (far from the Zeeman component), Rayleigh scattering applies and hence the scattered radiation is again 100% but with opposite sign (see Bommier 1997b, Sect. 6).  $U/I = 0$  for two reasons: (i) the field is strong enough to cause Hanle saturation (no rotation of the polarization plane); (ii) the choice of geometry, namely magnetic field along the LOS. Further for the same reasons,  $V/I$  is large, and reaches a maximum of unity at the position of Zeeman components.

### 3.3 Numerical results for the Hanle-Zeeman effect

Here we discuss line formation with the angle-averaged Hanle-Zeeman RM, for the case of a medium that is self emitting and effectively thin ( $\epsilon T \ll 1$ ).

Figure 5 shows the emergent profiles for  $\mu = 0.11$  and  $\varphi = 0^\circ$ . The Hanle strength parameter  $\Gamma_B$  is used as a free parameter. In panels (a) and (b) we compare the solutions obtained by solving Eq. (6) (using scalar absorption coefficient) and Eq. (21) (using the Zeeman absorption matrix) respectively. For weak fields  $\Gamma_B \leq 10$  (which corresponds to  $v_B \leq 0.01$ ), the solutions with and without Zeeman absorption do not differ. This shows that in weak fields one may neglect the contributions from the off-diagonal elements of the polarized Zeeman absorption matrix (as done in the past : see e.g. Faurobert-Scholl 1991;

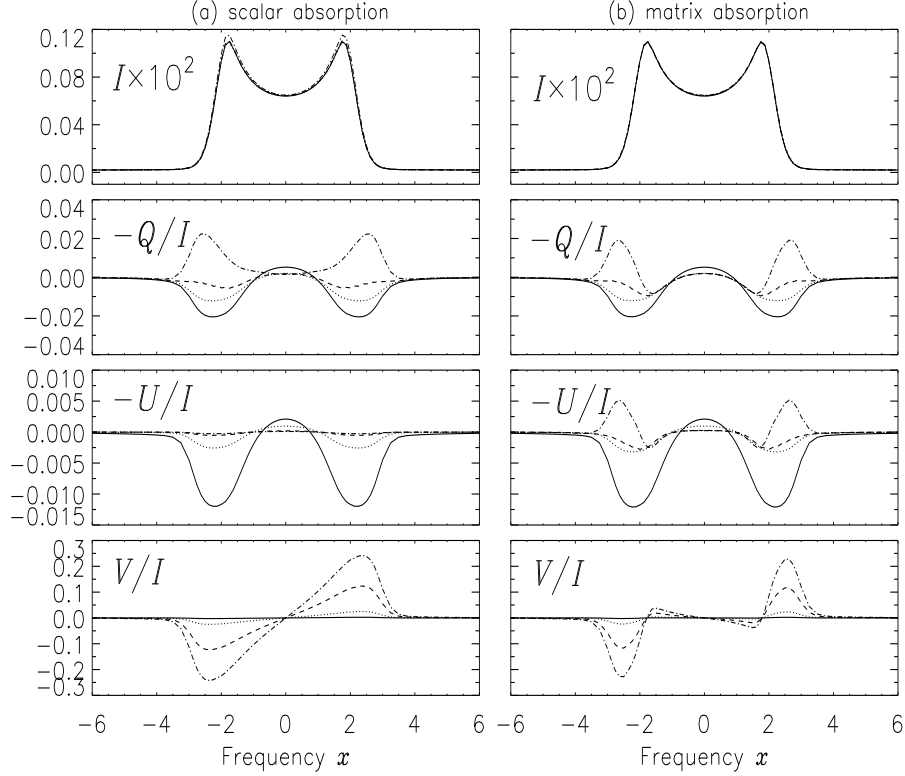


**Fig. 4.** Stokes parameters vs. incident frequency  $x'$  and scattered frequency  $x$ , in a  $90^\circ$  single scattering event involving a pencil of unpolarized incident radiation. The model parameters are  $[a, v_B, \Theta; \epsilon, \Gamma_E/\Gamma_R, D^{(K)}/\Gamma_R] = [10^{-3}, 3, 90^\circ; 0, 1, 0]$ . Such a model corresponds to an equal mix of both type II and III redistribution.  $\Gamma_R$  denotes the radiative rate.  $V/I$  stays within the range  $(-1, +1)$ , although we use a z-scale  $(-3, +3)$  in order to avoid overlapping of surface and contour plots.

Nagendra et al. 1998). For  $(\Gamma_B > 10, v_B > 0.01)$  there are considerable differences in the  $Q/I$  and  $U/I$  profiles between the two cases. The regime of field strengths covered in this figure (2.5 - 250 G) is the domain which was studied traditionally ignoring the anisotropic elements of the absorption matrix. From the figure it is clear that one should not neglect the anisotropic absorption in the transition regime of weak  $\rightarrow$  hectogauss fields in the solar atmosphere, where both Zeeman anisotropic absorption and Hanle anisotropic scattering contribute significantly and simultaneously. The general form of transfer equation given in this paper takes care of this transition regime between weak to hectogauss magnetic fields in the solar photosphere.

## 4 Conclusions

In this review we highlight three important developments in the polarized line formation theory applied to the solar atmosphere. The first problem concerns



**Fig. 5.** Comparison of the solutions computed using a scalar absorption coefficient (panel a) and the full Zeeman absorption matrix (panel b), for  $\mu = 0.11$  and  $\varphi = 0^\circ$ . Model parameters:  $(T, a, \epsilon, r, \Gamma_E/\Gamma_R, D^{(K)}/\Gamma_R) = (20, 10^{-3}, 10^{-4}, 10^{-7}, 1, 0.5)$ . Magnetic field orientation:  $(\theta_B, \chi_B) = (30^\circ, 0^\circ)$ . Line types: solid ( $\Gamma_B = 1, v_B = 0.001$ ); dotted (10, 0.01); short-dashed (50, 0.05); dash-dotted (100, 0.1).

a revisit to the Zeeman line formation in turbulent fields. We show that finite correlation length effects (meso-turbulence) are important for Zeeman lines. In the second part we discuss Hanle transfer equation in turbulent fields, and show that Hanle scattering is insensitive to the correlation length - and therefore micro-turbulence is a good approximation. Both Zeeman absorption and Hanle scattering sensitively depend on the choice of vector PDF - which describes how magnetic field fluctuates in both strength and direction. The third problem we address is the polarized line transfer with PRD, for arbitrary strength deterministic magnetic fields. This problem could not be attempted in the past as the relevant PRD redistribution matrices were not available. Using our new derivation of this Hanle-Zeeman redistribution matrix, we now solve the concerned transfer equation. We present a few interesting results.

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