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Stability of cool flux tubes in the solar chromosphere. Linear analysis*

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Summary. We examine the linear stability of cool flux tubes in the solar chromosphere which are initially in radiative equilibrium. Owing to the presence of carbon monoxide, there exists a narrow region near the temperature minimum where the temperature gradient becomes steep enough to drive a convective instability. We use the thin flux tube equations and include in a simple manner radiative heat exchange with the ambient medium. Initial states of constant β (where β is the ratio of gas to magnetic pressure) are considered. We present results for various values of β. It is found that for $\beta < 5.7$ the tube is overstable with periods in the range 300–600 s. At $\beta = 5.7$ a bifurcation occurs into two purely growing modes. We calculate growth rates, eigen-vectors of the fundamental modes and examine phase relationships. It is suggested that overstable oscillations should invariably be associated with cool flux tubes. These oscillations transport energy and can thus change the thermodynamic structure of flux tubes. We conjecture that the CO overstability may be responsible for spicules.

Key words: hydromagnetics – convection – Sun – chromosphere – magnetic fields – flux tubes – overstability – oscillations

1. Introduction

The observation by Noyes and Hall (1972) and Ayres and Testerman (1981) of low radiation temperatures ($T_R \approx 3500$ K) in the infrared rotation-vibration bands of carbon monoxide (CO) have reinvoked a discussion on the structure of the solar atmosphere and its energy budget. The emperical model of Ayres and Testerman (1981) based on low spatial resolution in CO possesses at low temperature at the top of the chromosphere and no temperature rise there, thus markedly differing from standard models of the solar atmosphere (e.g. HRSA, Gingerich et al., 1971 or VALC, Vernazza et al., 1981). Recently, Deming et al. (1984), from observations of the hydroxyl radical (OH) in the quiet Sun, found very low temperatures as well. They also conclude like Ayres (1981, 1984), that the solar atmosphere is inhomogeneous laterally i.e., it possesses a cool component with little chromospheric heating which emits molecular lines and a hot component, the chromospheric network, with strong non-radiative heating, but without molecular lines.

Estimates of the radiative energy involved in the CO bands were made by Ayres (1981). He demonstrated that the cooling due to CO lines outweights energy losses in all other spectral ranges and is of the order of the chromospheric heating requirements estimated so far (e.g. Vernazza et al., 1981). It is thus obvious that models describing the energy budget of the chromosphere of the Sun and of late type stars in general must include the effects of CO and possibly of other molecules as well.

Early attempts to treat energy balance in atmospheres in radiative equilibrium, including CO lines, failed (Gustafsson et al., 1975; Kneer, 1983). Near the temperature minimum of standard emirical models, where the $\Delta v = 1$ vibration transitions of CO reach optical depth unity, the RE calculations were destabilised. Kneer (1983)conjectured that this is not a mere coincidence, but rather that the formation of CO molecules prevents atmospheres from reaching RE and is thus responsible for the onset of motions and ultimately for the heating of chromospheres. Muchmore and Ulmschneider (1985), on the other hand, criticized Kneer's argument and demonstrated that RE can be reached in the presence of CO. They also constructed mechanically heated chromospheres using time dependent solutions of the hydrodynamic equations in plane parallel geometry. They concluded that insufficient heating leads to cool chromospheres dominated by CO cooling, whereas strong heating leads to chromospheres too hot for the formation of CO.

Furthermore, Kneer (1984a, b) obtained a variety of RE models with some interesting properties due to the presence of CO: (a) CO molecule formation is of autocatalytic nature, i.e., self amplifying because it leads to the formation of more molecules and to enforced cooling due to line radiation. (b) The dissociation energy of CO is high and thus the opacity in the CO lines is extremely temperature sensitive. (c) Consequently, once CO has formed, the temperature drops very steeply to about 2000 K and all models become *convectively unstable* i.e., $\nabla_{\rm rad} > \nabla_{\rm ad}$. (d) Many of the atmospheres are bistable i.e., for the same parameters two solutions can be found.

In the present investigation, we examine the stability of a magnetically structured atmosphere in radiative equilibrium, We neglect mechanical and all other external non-radiative forms of heating in the magnetic structure or flux tube. However, we include heat exchange between the tube and the ambient medium. We shall limit ourselves to a linear analysis, deferring a nonlinear treatment to subsequent work.

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For purposes of mathematical tractability we shall work within the framework of the thin flux tube approximation, which has been extensively applied to model intense flux tubes in the photosphere and convection zone (Defouw, 1977; Roberts and Webb, 1978; Spruit, 1979a; Webb and Roberts, 1980a, b; Hasan, 1984a, b; Hasan and Schüssler, 1985). Recently, this approximation has also been applied to flux tubes in the chromosphere (Herbold et al., 1985). Admittedly, this approximation becomes tenuous in the upper layers of the atmosphere where the horizontal dimension of a flux tube exceeds the local pressure scale height. Nevertheless, we retain it as it leads to considerable mathematical simplicity in that it permits a one dimensional treatment of the problem.

We shall allow for radiative heat exchange in the horizontal direction using Newton's law of cooling with Spiegel's expression (in the optically thin approximation) for the cooling time constant and a Planck mean opacity.

This optically thin treatment is not correct in those layers where we have strong contributions from CO to the opacity i.e., at low temperatures where the optical depth in the CO lines can become large. However, a rigorous treatment would require an inclusion of the radiative transfer equations for the relevant opacities, which would render the system of equations as complicated as the nonlinear equations to be treated in the future.

In the next section we describe the equilibrium model atmosphere, the stability of which we shall investigate in Sect. 3. In Sect. 4 we present results for the eigenfrequencies of the flux tube along with some of the associated eigenvectors, which is followed by a discussion. Finally, some of the observational implications of the study are pointed out.

2. The equilibrium model

Parts of the model calculations have been described elsewhere (Kneer, 1984b) and a detailed description will be given in a separate paper. In this section we shall briefly describe the assumptions and methods used to calculate the equilibrium.

For the external atmosphere we assume a plane parallel RE model in hydrostatic equilibrium i.e.

$$\int \kappa_\nu (J_\nu - S_\nu)\, d\nu = 0\,, \tag{1}$$

$$dp/dz = -\varrho g\,, \tag{2}$$

where κ_ν is the absorption coefficient at frequency ν, J_ν is the mean intensity, S_ν is the source function (we assume LTE, thus $S_\nu = B_\nu(T)$), ϱ is the density, p is the gas pressure and $g = 2.736\ 10^4\,\mathrm{cm\,s^{-2}}$ is the gravitational acceleration. As absorbers we take H^- and CO. For the H^- bound-free transitions we adopt the cross-section from Geltman (1962) and for the free-free transitions the polynomial approximation given in Auer et al. (1972). For the CO lines at $4.6\,\mu$, opacity distribution functions were calculated with approximations to the oscillator strengths given by Kirby-Docken and Liu (1978) and with CO energy levels from Roh and Rao (1974). The abundances $A_c = 4\ 10^{-4}$ and $A_0 = 8\ 10^{-4}$ were adopted from Ayres and Testerman (1981). The integral in Eq. (1) was approximated by a sum of 5 frequency points below $1.64\,\mu$, 4 frequencies between $1.64\,\mu$ and $4.6\,\mu$ and 5 frequencies for the CO line opacities.

To calculate the ionisation equilibrium, we adopt the Saha formula for He and the metals with standard abundances and an analytic non-LTE solution for the hydrogen Lyman continuum (Kneer and Mattig, 1978). In addition, we allow for the LTE formation of H_2 and H_2^+ molecules according to Mihalas' (1967) procedure. H_2 is abundant at low temperatures and is taken into account also in the calculation of the specific heats (Mihalas, 1967; Cox and Giuli, 1968).

The mean intensities J_ν are obtained from the differential equations in the Eddington approximation

$$\frac{1}{3}\frac{d^2 J_\nu}{d\tau_\nu^2} = J_\nu - B_\nu(T) \tag{3}$$

with the upper and lower boundary condition

$$\frac{1}{\sqrt{3}}\frac{dJ_\nu}{d\tau_\nu} = J_\nu \tag{4}$$

and

$$\frac{1}{\sqrt{3}}\frac{dJ_\nu}{d\tau_\nu} = \sqrt{3}\,H_\nu = -J_\nu + I_\nu^+ \tag{5}$$

respectively. Here, I_ν^+ is the radiation from below and is given in the diffusion limit by

$$I_\nu^+ = B_\nu(T) + \frac{1/\sqrt{3}}{\kappa_\nu}\frac{\partial B_\nu}{\partial T}\left|\frac{dT}{dz}\right| \tag{6}$$

The temperature gradient is obtained from frequency integration of Eq. (5) as

$$\left|\frac{dT}{dz}\right| = \frac{[\sqrt{3}H - \int (B_\nu(T) - J_\nu)\, d\nu]}{\int (1/\kappa_\nu)\dfrac{\partial B_\nu}{\partial T}\, d\nu} \tag{7}$$

With the Eddington flux $H = \sigma T_{\mathrm{eff}}^4/4\pi$ Eq. (7) ensures that the correct amount of energy is transported through the lower boundary (Auer, 1971). We use $T_{\mathrm{eff}} = 5950$ K for reasons that follow.

The difference equations corresponding to Eqs. (2)–(5) are solved together with Eq. (1) and the upper boundary condition for the pressure

$$p_{\mathrm{top}} = 2.74\ 10^{-2}\,\mathrm{dyn\,cm^{-2}} \tag{8}$$

on the z scale with a mesh size $\Delta z = 10$ km by means of a partial linearization scheme. Once we have obtained a self-consistent model, we calculate the specific heats C_p and C_v, the radiative and adiabatic temperature gradients, ∇_{rad} and ∇_{ad} respectively, and the Planck mean opacity as

$$\bar{\kappa}_p = \frac{\int \kappa_\nu B_\nu(T)\, d\nu}{\int B_\nu(T)\, d\nu}\,. \tag{9}$$

Figure 1 shows the dependence of temperature and density with height in the atmosphere outside the tube, where $z = 0$ corresponds approximately to $\tau_{5000} = 1$ in the VAL C model (Vernazza et al., 1981). With $T_{\mathrm{eff}} \approx 5800$ K the solution above the temperature minimum would drop below 2000 K and stay there up to the top of the model. To have a temperature above 4000 K at the top, we would require non-radiative heating, which we wish to avoid in the present investigation. The present solution possesses a jump in temerature at $z = 760$ km of $\Delta T \approx 2700$ K which reflects the bistable nature of the system (Kneer, 1984b). We have restricted the model to heights larger than $z = 80$ km, because below this height the temperature gradient steepens and becomes ultimately superadiabatic in the subphotospheric layers. These layers are not of interest in the present study. We assume that at the initial instant the temperature inside the flux tube equals that outside at each z

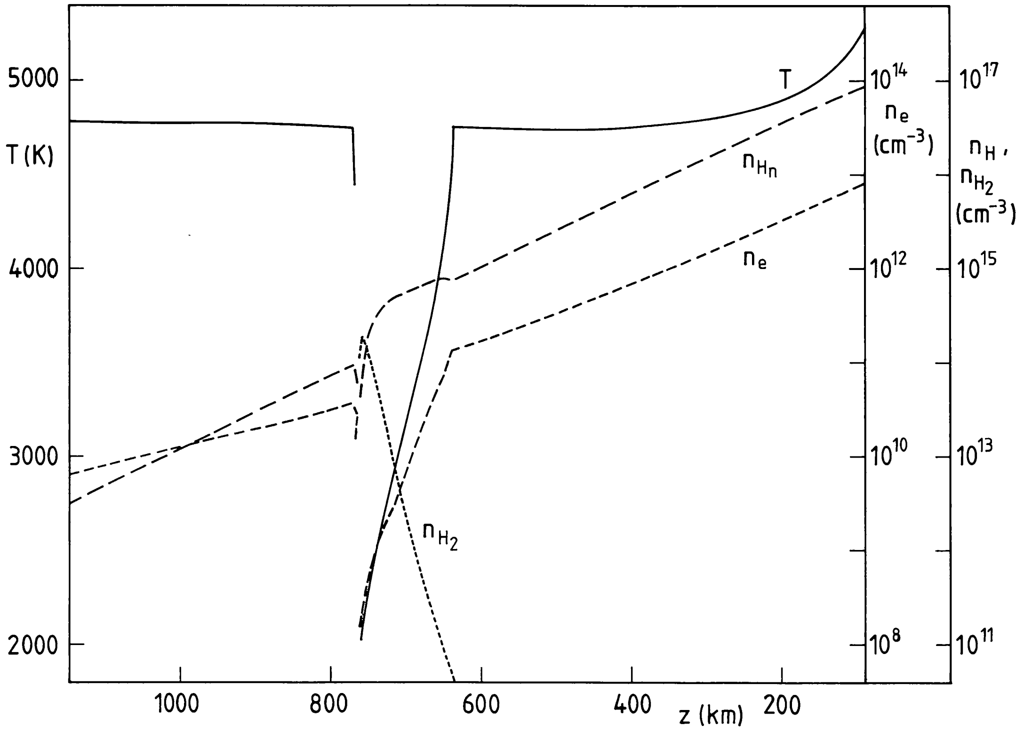

Fig. 1. The variation of T, n_e, n_H, and n_{H_2} with z

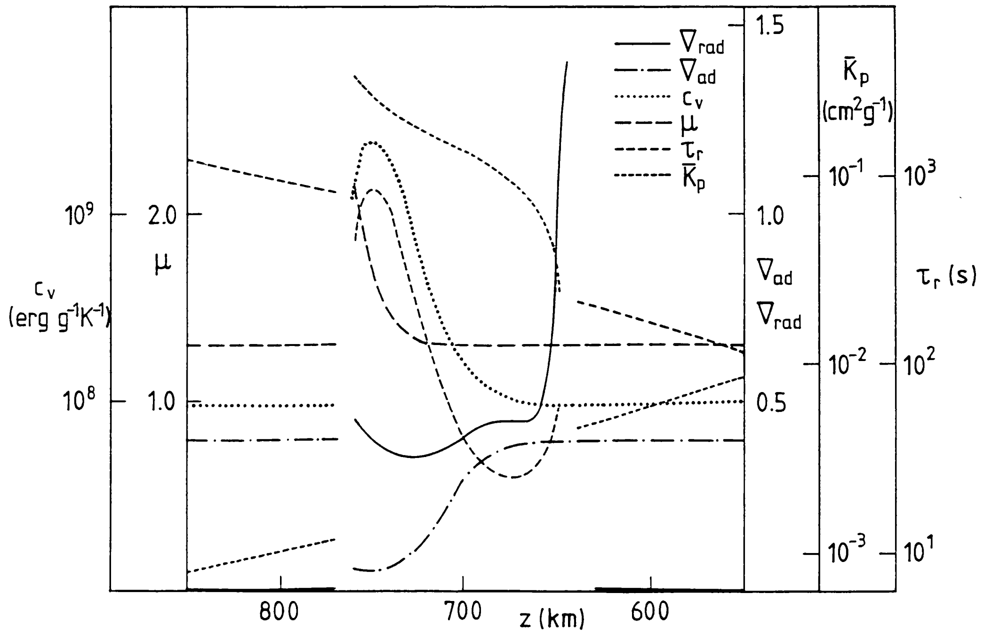

Fig. 2. The variation of C_v, μ, ∇_{rad}, ∇_{ad}, $\bar{\kappa}_p$ and τ_r with z in the superadiabatic layer

value. Assuming that the mean molecular weight is approximately the same inside and outside, the density and pressure inside the flux tube are lower than the value outside by a factor $\beta/(\beta+1)$, where $\beta = 8\pi p/B^2$ is constant (for more discussion see Hasan, 1984b).

In Fig. 2 we also depict for the interesting height range, in which the strong temperature depression occurs, the variation of μ, the mean molecular weight, of C_v, of ∇_{rad}, of ∇_{ad}, of κ_p and of the radiation exchange time constant τ_r. It is clearly seen that the temperature gradient is superadiabatic. There are two layers of strong superadiabaticity: one at $z \approx 650$ km where the radiative temperature gradient is high and the other at $z \approx 750$ km where the specific heats become large due to the formation of H_2 and thus ∇_{ad} approaches zero. We also see that in the superadiabatic layer τ_r drops quite rapidly so that radiative heat exchange may become quite important here.

3. Equations

We shall examine the stability of the model atmosphere, described in the previous section, by solving the MHD equations in the thin flux tube approximation. Let us consider a flux tube extending vertically into the chromosphere. Adopting a cylindrical coordinate system (r, θ, z) and assuming axial symmetry, the thin flux tube equations are obtained by making an expansion about $r=0$. To zeroth order they are (Roberts and Webb, 1978)

$$\frac{\partial}{\partial t}\left(\frac{\varrho}{B}\right)+\frac{\partial}{\partial z}\left(\frac{\varrho v}{B}\right)=0, \tag{8}$$

$$\varrho\left(\frac{\partial v}{\partial t}+\frac{v\,\partial v}{\partial z}\right)=-\varrho g-\frac{\partial p}{\partial z}, \tag{9}$$

$$p+\frac{B^2}{8\pi}=p_e, \tag{10}$$

$$\frac{\partial p}{\partial t}+\frac{v\,\partial p}{\partial z}=\frac{\gamma p}{\varrho}\left(\frac{\partial \varrho}{\partial t}+\frac{v\,\partial \varrho}{\partial z}\right)+\frac{p\chi_T}{C_v T}\left.\frac{dQ}{dt}\right|_{\rm rad} \tag{11}$$

where B is the magnetic field, v is the vertival velocity, T is the temperature, $dQ/dt|_{\rm rad}$ is the amount of radiative energy per gram per unit time lost to the ambient medium, χ_T and χ_ϱ are defined as follows

$$\chi_T \equiv \left(\frac{\partial \ln p}{\partial \ln T}\right)_\varrho = 1-\left(\frac{\partial \ln \mu}{\partial \ln T}\right)_\varrho$$

$$\chi_\varrho \equiv \left(\frac{\partial \ln p}{\partial \ln \varrho}\right)_T = 1-\left(\frac{\partial \ln \mu}{\partial \ln \varrho}\right)_T$$

and $\gamma=\chi_\varrho C_p/C_v$. Equation (11) is a generalization of the energy equation given by Webb and Roberts (1980), who generalized their previous work to include radiative losses. In addition to radiative losses, we also include the effect of varying ionization. The subscript e denotes quantities in the external medium. For $dQ/dt|_{\rm rad}$ we use Newton's law of cooling and write

$$\left.\frac{dQ}{dt}\right|_{\rm rad}=\frac{C_v(T_e-T)}{\tau_r}, \tag{12}$$

where τ_r is given in the optically thin approximation by Spiegel (1957)

$$\tau_r=\frac{C_v}{16\sigma\bar{\kappa}_p T^3}. \tag{13}$$

In Eq. (13) $\bar{\kappa}_p$ is themean Planck opacity and σ is the Stefan-Boltzmann constant.

Linearizing Eqs. (8)–(11) about the equilibrium state and assuming a perturbation with a time dependence of the form $\hat{f}(z)\,e^{i\omega t}$, where $\hat{f}$ is the amplitude of the perturbation, Eqs. (8) and (12) yield

$$\frac{\hat{\varrho}}{\varrho_0}=\frac{-1}{i\omega s}\left[\hat{v}'+\left(\frac{\varrho_0'}{\varrho_0}-\frac{B_0'}{B_0}\right)\hat{v}-\frac{i\Omega}{i\Omega+\gamma}\frac{\gamma\beta}{2}\frac{N_0^2}{g}\hat{v}\right], \tag{14a}$$

$$\frac{\hat{p}}{p_0}=\frac{-\gamma}{i\omega s}\frac{i\Omega+\chi_\varrho}{i\Omega+\gamma}\left[\hat{v}'+\left(\frac{\varrho_0'}{\varrho_0}-\frac{B_0'}{B_0}\right)\hat{v}+\frac{i\Omega}{i\Omega+\chi_\varrho}\frac{N_0^2}{g}\hat{v}\right], \tag{14b}$$

where

$$\Omega=\gamma\omega\tau_r,\ c_0^2=\gamma p_0/\varrho_0,\ N_0^2=-\frac{\varrho_0' g}{\varrho_0}-\frac{g^2}{c_0^2}$$

and

$$s=1+\frac{\gamma\beta}{2}\frac{i\Omega+\chi_\varrho}{i\Omega+\gamma}$$

The subscript 0 denotes quantities inside the flux tube in the unperturbed state and the primes denote derivatives with respect to z. The symbols N_0 and c_0 denote the Brunt-Väisälä frequency and sound speed in the equilibrium state. We also assume that the perturbation in the external medium is zero.

Substituting Eqs. (14a) and (14b) in the linearized form of Eq. (9), we obtain after some algebra the following differential equation for $\hat{v}$ (Hasan, 1985)

$$\hat{v}''+\frac{L}{\Lambda_0}\hat{v}'+\left(\frac{M}{\Lambda_0}+\frac{N}{\Lambda_0^2}\right)\hat{v}=0, \tag{15}$$

where

$$L=-\frac{1}{2}+\frac{\gamma'}{\gamma}\Lambda_0 X-\frac{1}{i\Omega+\chi_\varrho}\left[\Lambda_0'\chi_\varrho+(\chi_\varrho-1)\right]$$

$$M=\frac{\gamma'}{\gamma}\left[\frac{1}{\gamma}+X\left(\frac{1}{2}-\frac{1}{\gamma}\right)\right]+\frac{\omega^2}{\gamma g}\left(1+\frac{\gamma\beta}{2}\right)+\frac{1}{i\Omega+\chi_\varrho}\cdot\left[-\Lambda_0''\chi_\varrho-\frac{\gamma'}{\gamma}\chi_\varrho\left(\frac{1}{\gamma}+n^2X\right)+\frac{(\gamma-\chi_\varrho)}{\gamma}\frac{\omega^2}{g}\right]+\frac{n^2 i\Omega}{(i\Omega+\chi_\varrho)^2}\chi_\varrho\left(\frac{i\Omega'}{i\Omega}-\frac{\chi_\varrho'}{\chi_\varrho}\right)$$

$$N=-\frac{n^2}{2}(1+\beta)+\frac{1}{i\Omega+\chi_\varrho}\left[\chi_\varrho\Lambda_0'\left(1+\frac{\beta}{2}\right)+\Lambda_0'^2\chi_\varrho+\left(\frac{\gamma-\chi_\varrho}{2\gamma}\right)(1+\beta)+(\chi_\varrho-1)\left(1+\frac{\beta}{2}+\Lambda_0'\right)\right]$$

$$X=\frac{W-\gamma/\gamma' W'}{W+\gamma\beta/2}$$

$$\Lambda_0=p_0/\varrho_0 g,\ n^2=(\gamma-1)/\gamma+\Lambda_0' \text{ and } W=\frac{i\Omega+\gamma}{i\Omega+\chi_\varrho}.$$

In the limit of constant μ and constant τ_r, Eq. (15) goes over to Eq. (10) of Webb and Roberts (1980b).

We solve Eq. (15) numerically by approximating the derivatives by finite differences. This leads to a homogeneous system of equations, which along with the boundary conditions constitutes a generalized eigenvalue problem. For simplicity we assume closed boundary conditions i.e. zero velocity at the boundaries. The results are not sensitive to the precise choice of boundary conditions since the amplitude of the perturbation, as we shall see later on, becomes vanishingly small at the boundaries. The "eigenfrequencies" ω can be found by locating the roots of a determinantal equation and the eigenvectors can then be determined using the method of inverse iteration (Wilkinson and Reinsch, 1971, for details on the actual numerical procedure followed see Hasan, 1985). In general ω and $\hat{v}$ are complex, but in the adiabatic limit ($\Omega\to\infty$), ω^2 and $\hat{v}$ are real. Once $\hat{v}$ is determined, $\hat{\varrho}$ and $\hat{p}$ can be calculated using Eqs. (13) and (14). The temperature perturbation can be determined using the relationship

$$\frac{\hat{T}}{T_0}=\frac{1}{\chi_T}\left(\frac{\hat{p}}{p_0}-\chi_\varrho\frac{\hat{\varrho}}{\varrho_0}\right). \tag{16}$$

4. Results

4.1. *Adiabatic case* ($\Omega \to \infty$)

In the adiabatic case, the frequency appears only as ω^2 (thus ω and $-\omega$ are both eigenvalues). Equation (15) possesses only solutions for which ω^2 is real, which corresponds to either purely growing or purely oscillatory modes. In Fig. 3 the variation of the growth rate $\eta = i\omega$ with β is plotted for the fundamental mode. For weak magnetic fields (high β), η is real, which means that the system is unstable. Decreasing β (i.e. increasing the magnetic field strength) leads to a decrease in the growth rate. For β less than a critical value β_c, η becomes purely imaginary and the flux tube is therefore stable. We find $\beta_c \simeq 5.15$.

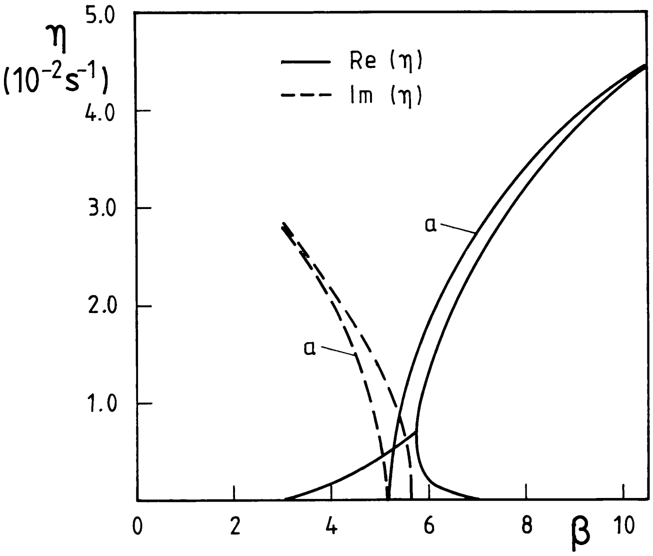

Fig. 3. The dependence of η on β for the adiabatic case and for the case when radiative heat exchange is included

4.2. *Non-adiabatic case: radiative exchange included*

In the general case, η is complex. In Fig. 3, the real and imaginary parts of η, η_r, and η_i respectively, are plotted as functions of β for the fundamental mode. For $\beta < \beta_c$, with $\beta_c \simeq 5.7$, the system is overstable with growth rates typically in the range 150–300 s and periods 5–10 min. At $\beta = \beta_c$, the system bifurcates into two purely growing modes. The frequency of the slow mode diminishes very rapidly with β, whereas the frequency of the fast mode increases with β. Similar behaviour also occurs for flux tubes in the convection zone (Spruit, 1979b; Hasan, 1985).

Figure 4 shows the variation of the normalized velocity amplitude with height. It can be seen that the largest values of $\hat{v}$ occur in the height range 650–750 km (shown by vertical arrows and henceforth to be referred to as layer A). Outside this region, $\hat{v}$ drops fairly rapidly becoming negligibly small close to the boundaries. The reason why the dominant contribution to $\hat{v}$ comes from layer A is that the temperature gradient is large here and $\nabla_{\rm ad}$ is reduced owing to the formation of H_2 molecules so that $\nabla > \nabla_{\rm ad}$. The large superadiabatic gradient is responsible for driving the modes of the system. As we move away from this layer, the temperature gradient becomes small and the amplitude of the perturbation decreases. In the figure several curves, corresponding to different β, are shown. It should be borne in mind that the amplitude in each curve is normalized with respect to the maximum value for that particular case. What, however, can be discerned is that the spatial extent of the region, where $\hat{v}$ is large, increases with decreasing β, although the absolute magnitude of $\hat{v}$ is likely to decrease with β (Hasan, 1984a, b). The extent of the region into which the disturbance generated in layer A penetrates is larger for smaller β. This extent depends roughly upon the tube speed which is inversely proportional to β.

Figures 5 and 6 show the variation of $\hat{T}$ and $\hat{\varrho}$ as functions of height. The amplitudes are largest where the degree of superadiabaticity is largest. The sign of the perturbation depends upon the sign of the initial velocity perturbation. Let us assume that the latter is positive (i.e., we have an upflow in the tube). We consider first the convectively unstable case $\beta = 7$, for which $\hat{T}$ and $\hat{\varrho}$ are real. In this case, $\hat{T}$ and $\hat{\varrho}$ are positive and negative respectively in

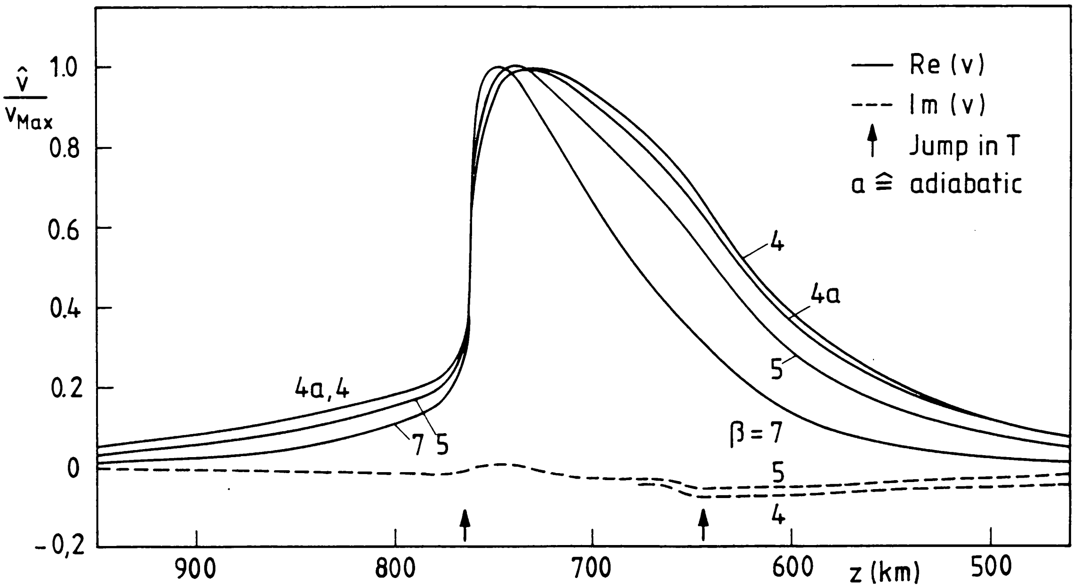

Fig. 4. The variation of the normalized velocity amplitude $\hat{v}$ with height. The vertical arrows denote the superadiabatic region

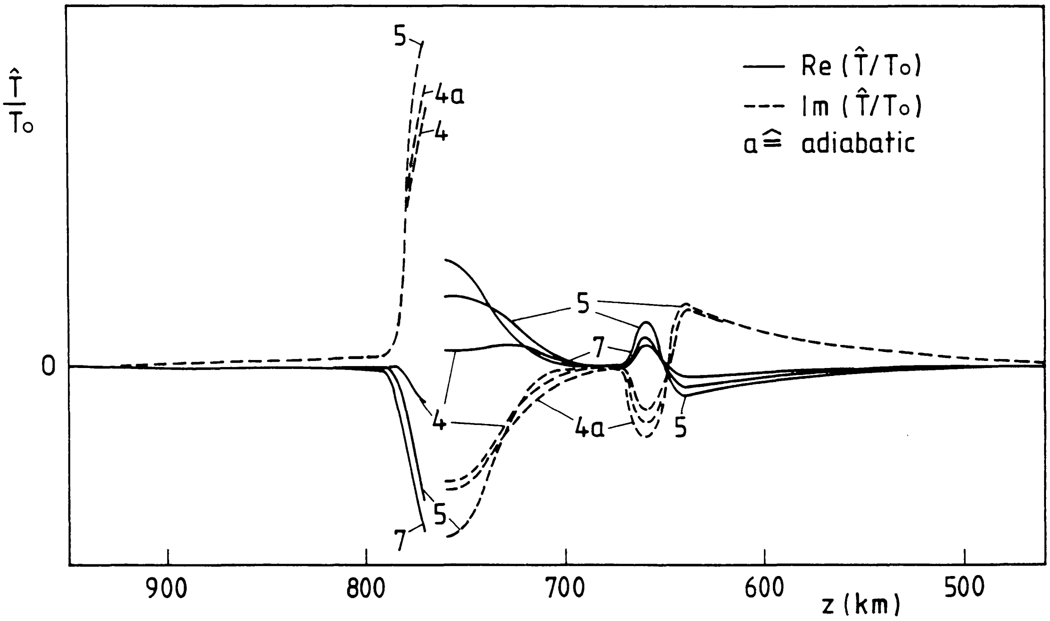

Fig. 5. The variation of $\hat{T}/T_0$ (arbitrary units) with z

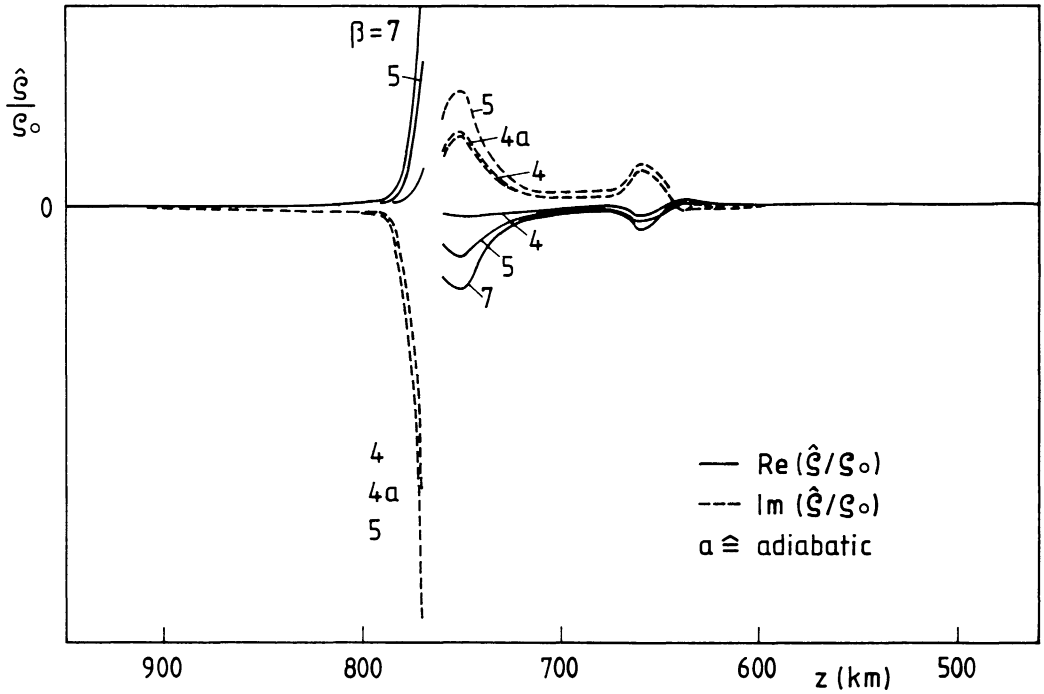

Fig. 6. The variation of $\hat{\varrho}/\varrho_0$ (arbitrary units) with z

layer A and have opposite signs outside this layer with a discontinuity at $z=750$ km and $z=650$ km. The velocity and temperature perturbations are in phase in A and 180° out of phase outside A. In the stable case (i.e. in the adiabatic limit), $\hat{T}$ and $\hat{\varrho}$ are pure imaginary with opposite signs in layer A and the outside region. Velocity and temperature perturbations for this case are 90° out of phase. Let us now turn our attention to the overstable case, for which $\hat{T}$ and $\hat{\varrho}$ are in general complex. We see that the real and imaginary parts of the perturbations have different signs in A and the outside layers. Furthermore, in each layer the real and imaginary parts have opposite signs. The phase relationships are not so straightforward and are plotted in Fig. 7. In the layers outside A, the phase difference between velocity and temperature remains fairly constant around $-90°$, but in A varies markedly with z.

Figure 8 shows the variation of $\hat{p}$ (normalized), the amplitude of the pressure perturbation, with height. Unlike $\hat{T}$ and $\hat{\varrho}$, $\hat{p}$ exhibits a continuous behaviour. In the convectively unstable case $\hat{p}$ is pure real, in the stable case it is pure imaginary and in the overstable case it is complex.

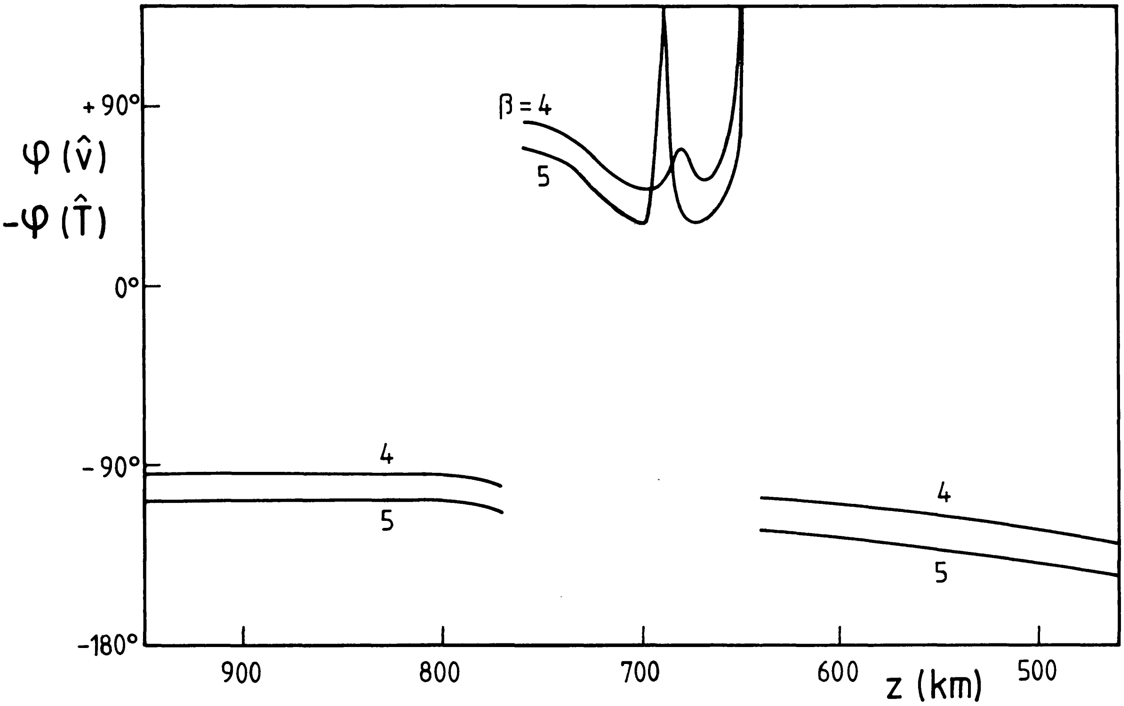

Fig. 7. The variation of the phase difference between velocity and temperature $\phi(v)-\phi(T)$ as a function of z

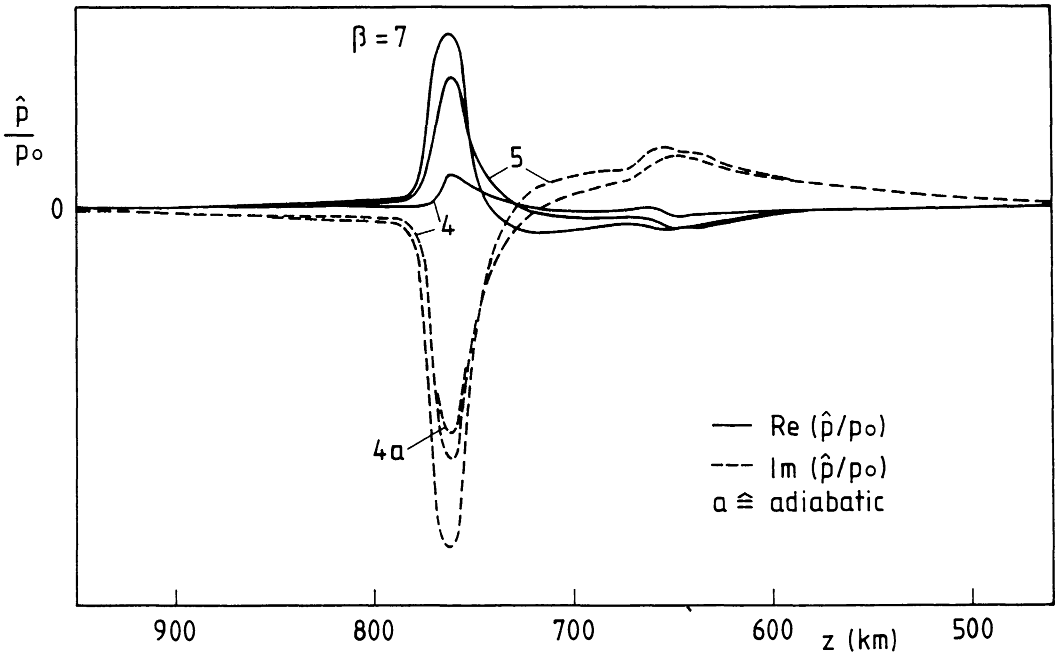

Fig. 8. The variation of $\hat{p}/p_0$ (arbitrary units) with z

A general remark may be in order here. We find that the absolute maximum amplitudes for $\hat{T}$, $\hat{\varrho}$ and $\hat{p}$ decrease with β. For the velocity, the dependence on β cannot be deduced, as the solution of Eq. (15) does not give $\hat{v}$ uniquely, since any constant multiple of $\hat{v}$ is also a solution of the equation. However, as already pointed out, ot seems reasonable to expect that the maximum amplitude should decrease with β.

5. Discussion

The results in the preceding section support earlier conjectures (Kneer, 1983, 1984) that the presence of CO can produce extremely steep gradients which can drive a convective instability. When radiative heat exchange is omitted, the flux tube is either stable or monotonically unstable depending upon how strong the magnetic field in the initial state is. Decreasing β, or increasing B, has a stabilizing effect as is well known (for more details see Spruit, 1979a and Hasan, 1984a). In the general case, the flux tube is either unstable for large enough β or overstable. Overstability is due to radiative heat exchange which permits the transfer of energy from the external medium into the oscillations. An interesting feature of the solutions is the existence of a critical value of β at which a bifurcation occurs from overstability to instability with two branches. The latter do not represent different harmonics of the same mode, but are rather two different types of modes. This can be clearly seen by looking at the corresponding eigenvectors, which show the absence of nodes within the flux tube. Comparison with the adiabatic case allows us to identify the top branch as a convective mode modified by radiative exchange. The lower branch appears to correspond to a new mode introduced by the inclusion of radiative exchange (more details will appear in Hasan, 1985).

Another interesting feature of the results is the fact that the temperature and density perturbations have different signs in layer A and in the outside region. In region A where $dT/dz < 0$, an upflow brings hotter material upwards, and if there is approximate pressure balance with the surroundings, this material should be less dense. At the left interface between A and the outside region where the temperature gradient reverses sign, the opposite situation prevails. This is also the cause of the discontinuity in the temperature and density perturbations.

Let us now consider energy transport in the flux tube. To second order, the kinetic energy flux is zero. The time-average (over the period of an oscillation) enthalpy flux at some height $\approx \bar{\varrho} C_p \langle vT \rangle$, where $\bar{\varrho}$ is an average mass density and the brackets denote a time average. (There is no first order contribution as $\langle v \rangle = 0$). In the adiabatic limit for $\beta < \beta_c$, $\hat{v}$ and $\hat{T}$ are out of phase by 90° and $\langle vT \rangle = 0$. However, in the general case, $\langle vT \rangle = \hat{v}\hat{T} \cos[\phi(v) - \phi(T)]$ is finite and the oscillations can transport energy.

We would briefly like to dwell on some of the limitations of our theoretical analysis. For mathematical convenience we used a thin flux tube approximation and modelled radiative exchange with the ambient medium using a mean Planck opacity. Although these approximations are not strictly valid, they still allow us to gain insight into the physical processes that occur in the flux tube. Furthermore, they also give us approximate information on the periods and growth rates of the overstable oscillations and also on the phase relationships between the different variables.

6. Observational implications

We have found that the existence of a superadiabatic temperature gradient in the flux tube can drive a convective instability provided that the equilibrium magnetic field is not too strong. The critical value of the field (in the general case) can be determined from the condition $\beta_c \approx 5.7$ to be approximately 20–40 G in the region of interest (about 150 km above the temperature minimum). Thus, flux tubes initially in radiative equilibrium will collapse if the initial field is not strong enough. Collapse will lead to field intensification, which tends to quench the instability. The final state that is likely to result is one with overstable oscillations similar to intense flux tubes in the convection zone (Hasan, 1984b; Spruit, 1979a). Owing to redistribution of energy by overstable oscillations, the thermodynamic structure in the final state is likely to be different from the initial state. On qualitative grounds we would expect the temperature profile to be smoother and the temperature minimum to be raised compared to the cool material outside the flux tube. A quantitative examination of this problem is currently under progress and the results will be communicated in a forthcoming paper.

The periods of the overstable oscillations found by us (5–10 min) lie in the range of typical lifetimes and periods of (sub-)photospheric and chromospheric periodic and non-periodic processes, but an order of magnitude shorter than those found by Muchmore and Ulmschneider (1985). This apparent discrepancy could be due to the fact that the latter authors used a purely one-dimensional analysis, thus neglecting the effects of a convective instability.

We have found that the phase difference between velocity and temperature oscillations in general varies with height. In the stable layers, the phase difference is around 90° i.e., close to the adiabatic value, since radiative exchange is not so dominant in these layers. However, in layer A it differs markedly from 90°, varying from about 25° to 140°. Unfortunately, there are at present no observations with which a comparison is possible. The observations of phase lags in the chromosphere by Lites and Chipman (1979) refer to the average atmosphere and not to discrete structures of the type we have considered. Thus, a comparison of our theoretical results with observations must await a future date.

It is tempting to speculate here on the relevance of the CO overstability to the spicule phenomenon. Recently, it has been suggested (Hollweg, 1984; Parker, private communication) that spicules may be produced by oscillations that strikes the botton of flux tubes and are then trapped within them like in a resonance cavity. We conjecture that spicules may be the result of a resonance between the overstable oscillation produced in subphotospheric layers and the oscillations related to the CO overstability. The periods of the two oscillations appear to be comparable. However, a detailed quantitative analysis is needed to substantiate this claim and is deferred to a subsequent paper.

7. Conclusions

The aim of the present investigation was to examine the linear stability of cool flux tubes in radiative equilibrium. We found that the stability of the tube depends upon β, i.e. on the strength of the magnetic field in the equilibrium state. For purely adiabatic perturbations, the tube is stable for $\beta < 5.15$. However, when heat exchange is included the flux tube is overstable for $\beta < 5.7$ and for larger β a bifurcation into two purely growing modes occurs. The faster of the two modes is an ordinary convective mode modified by heat exchange, whereas the slower one appears to be a new mode owing its existence to the inclusion of radiative exchange. We have calculated eigenvectors of the fundamental modes and also derived phase relationships between different physical quantities. We suggest that cool flux tubes should invariably be associated with overstable oscillations, which lead to a redistribution of energy and to a modification of the thermal structure of the tubes. It is conjectured that spicules may be related to the CO overstability.

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