To appear in "Magnetic Coupling between the Interior and the Atmosphere of the Sun", eds. S. S. Hasan and R. J. Rutten, Astrophysics and Space Science Proceedings, Springer-Verlag, Heidelberg, Berlin, 2009.

The Hanle Effect as Diagnostic Tool for Turbulent Magnetic Fields

L. S. Anusha[1,2], M. Sampoorna[1], H. Frisch[2], and K. N. Nagendra[1]

[1] Indian Institute of Astrophysics, Bangalore, India
[2] Laboratoire Cassiopée, Université de Nice and Observatoire de la Côte d'Azur, France

Summary. The Hanle effect is calculated for a random magnetic field characterized by a finite correlation length and a probability density function of the magnetic field vector. It is shown that linear polarization is essentially independent of the magnetic field correlation length, but strongly depends on the distribution of the field strength.

1 Introduction

The Hanle effect provides a powerful diagnostic for the detection of weak turbulent solar magnetic fields (Stenflo 1982). Micro-turbulence, a magnetic field with an isotropic distribution and a single-valued field strength, are usually assumed in the interpretation of the spectro-polarimetric data. Due to very high magnetic Reynolds number, the energy spectrum of the solar magnetic field covers a very wide range of scales. In the present paper, we study the Hanle effect due to a random magnetic field with a finite correlation length (comparable to a typical photon mean free path) and algebraic spectrum. An iterative method of solution of the ALI (Approximate Lambda Iteration) type is used to calculate the mean Stokes parameters and examine their dependence on the correlation length of the magnetic field. We also investigate the sensitivity of the polarization to the shape of magnetic field PDF (Probability Density Function).

2 Transfer equations

The random magnetic field vector $\boldsymbol{B}$ is represented by a Kubo–Anderson process (KAP). It is a Markov process, stationary, discontinuous, piecewise constant. A KAP is characterized by a correlation length $1/\nu$ (where ν is the number of jumping points per unit optical depth) and a probability density function $P(\boldsymbol{B})$ (Frisch 2006). The choice of this process allows one to write a transfer equation for a mean radiation field, still conditioned by the value of the vector magnetic field $\boldsymbol{B}$ (Frisch 2006; see also Frisch et al. 2009).

For the 6-component vector $\boldsymbol{\mathcal{I}}$, constructed with the six irreducible components I^K_Q describing linear polarization (Frisch 2007), this transfer equation may be written as

$$\mu \frac{\partial \boldsymbol{\mathcal{I}}(\tau, x, \mu|\boldsymbol{B})}{\partial \tau} = \varphi(x) \left[\boldsymbol{\mathcal{I}}(\tau, x, \mu|\boldsymbol{B}) - \boldsymbol{\mathcal{S}}(\tau|\boldsymbol{B})\right] \\ + \nu \Big[\boldsymbol{\mathcal{I}}(\tau, x, \mu|\boldsymbol{B}) - \int \boldsymbol{\mathcal{I}}(\tau, x, \mu|\boldsymbol{B}') P(\boldsymbol{B}')\, \mathrm{d}^3\boldsymbol{B}'\Big]. \quad (1)$$

This equation holds for a plane-parallel atmosphere and complete frequency redistribution. The mean conditional source vector $\boldsymbol{\mathcal{S}}(\tau|\boldsymbol{B})$ is defined by

$$\boldsymbol{\mathcal{S}}(\tau|\boldsymbol{B}) = \boldsymbol{\mathcal{G}}(\tau) + \hat{N}(\boldsymbol{B}) \int_{-1}^{+1} \frac{\mathrm{d}\mu'}{2} \int_{-\infty}^{+\infty} \mathrm{d}x' \varphi(x') \hat{\Psi}(\mu') \boldsymbol{\mathcal{I}}(\tau, x', \mu'|\boldsymbol{B}). \quad (2)$$

Here, $\boldsymbol{\mathcal{G}}(\tau)$ is a given primary source term, $\hat{N}(\boldsymbol{B})$ describes the Hanle effect, and $\hat{\Psi}(\mu)$ the resonance scattering. Other notations are standard. The source vector $\boldsymbol{\mathcal{S}}(\tau|\boldsymbol{B})$ satisfies an integral equation which is solved by a Polarized ALI method (Nagendra et al. 1998), extended to random magnetic fields (Frisch et al. 2009). The mean field $\langle\boldsymbol{\mathcal{I}}\rangle(\tau, x, \mu)$ is solution of a standard transfer equation in which the source term is given by the average of $\boldsymbol{\mathcal{S}}(\tau|\boldsymbol{B})$ over the magnetic field PDF.

3 Results

3.1 Dependence on the correlation length

For intermediate values of the line optical thickness T (10-100), some sensitivity to the correlation length can be observed (Fig. 1a). For optically thin lines ($T \ll 1$), and optically thick ones ($T \geq 10^3$), the polarization is essentially independent of the magnetic field correlation length.

The very weak dependence of polarization, on the magnetic field correlation length, indicates that the polarization is created locally, and can thus be estimated by a single scattering approximation. For optically thick lines, Stokes Q may then be approximated by

$$Q(0, x, \mu) \simeq -\frac{3}{2\sqrt{2}}(1-\mu^2) N^2_{00}(\boldsymbol{B}) \bar{J}^2_0 \left(\frac{\mu}{\varphi(x)}\right). \quad (3)$$

The function $\bar{J}^2_0\left(\mu/\varphi(x)\right)$ is a measure of the anisotropy of the radiation field, and $N^2_{00}(\boldsymbol{B})$ is the element of the magnetic kernel coupling Stokes I to Stokes Q (or more precisely I^0_0 to I^2_0). Figure 1b shows the difference between the exact value of Q/I and the single scattering solution, for several choices of the line thickness T and magnetic field. The single scattering approximation appears fairly reliable, in particular for optically thin and thick lines, for both turbulent Hanle and deterministic Hanle scattering problems.

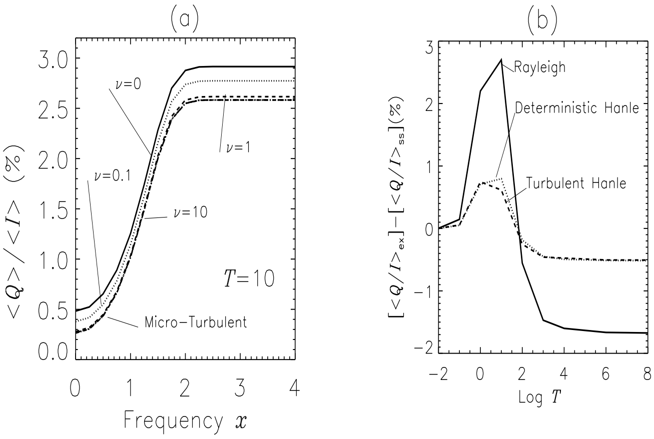

Fig. 1. (a) Correlation length effects on $\langle Q\rangle/\langle I\rangle$, for an exponential field strength distribution (see Table 1), with an isotropic angular distribution. (b) Difference between single scattering (ss) and exact (ex) solution as a function of T, for $\tau = 0$, $x = 0$, and $\mu = 0.05$. The micro-turbulent results (dashed line) are computed using the same model as Panel (a).

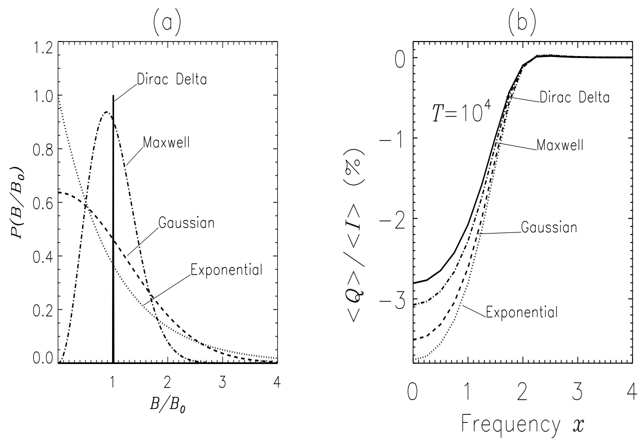

Fig. 2. (a) Various $P(B/B_0)$ as a function of (B/B_0). (b) The corresponding emergent $\langle Q\rangle/\langle I\rangle$ profiles for $\mu = 0.05$.

3.2 Dependence on the magnetic field vector PDF

The ratio $\langle Q\rangle/\langle I\rangle$ is calculated in the micro-turbulent limit for PDFs of the form

$$P(\boldsymbol{B})\mathrm{d}^3\boldsymbol{B} = P_{\mathrm{S}}(B)P_{\mathrm{A}}(\theta_{\mathrm{B}},\chi_{\mathrm{B}})\,\mathrm{d}B\,\mathrm{d}\theta_{\mathrm{B}}\,\frac{\mathrm{d}\chi_{\mathrm{B}}}{4\pi}. \quad (4)$$

Here, B is the random field strength and $(\theta_{\mathrm{B}},\chi_{\mathrm{B}})$ are the polar angles of the field. The surface value of $\langle Q\rangle/\langle I\rangle$ is shown in Fig. 2b for an isotropic mag-

Table 1. Field strength PDFs with mean field B_0 used in this paper[a]

$P_{\rm D}(B) = \delta(B - B_0)$	$P_{\rm M}(B) = \frac{32}{\pi^2 B_0}\,(B/B_0)^2 \exp\left[-\frac{4}{\pi}\,(B/B_0)^2\right]$
$P_{\rm G}(B) = \frac{2}{\pi B_0}\exp\left[-\frac{1}{\pi}\,(B/B_0)^2\right]$	$P_{\rm E}(B) = \frac{1}{B_0}\exp(-B/B_0)$

[a] D : Dirac delta, G : Gaussian, M : Maxwell, E : exponential

netic field ($P_{\rm A}(\theta_{\rm B}, \chi_{\rm B}) = \sin\theta_{\rm B}$), and different choices of the field strength distribution $P_{\rm S}(B)$ (see Table 1 and Fig. 2a). They were inspired from PDFs proposed in the literature (see e.g., Vögler et al. 2005; Stenflo & Holzreuter 2003; Sánchez Almeida 2007; Sampoorna et al. 2008). We observe that the polarization strongly depends on the choice of $P_{\rm S}(B)$. It grows when the probability of having very weak fields increases. A similar conclusion is suggested in Trujillo Bueno et al. (2004).

4 Conclusions

The choice of the magnetic field strength distribution strongly affects the mean magnetic field strength deduced from the Hanle effect analysis. The sensitivity to the correlation length is in general weak, and in most cases of astrophysical interest, micro-turbulence can be safely assumed. The sensitivity to the correlation length is related to the number of scatterings events that are needed to create the emergent polarization. In the presence of random magnetic fields, the single scattering approximation is applicable, especially for lines with small and large optical depths. For the Rayleigh scattering, contributions from higher orders of scattering become important, and the single scattering approximation can not be used, except for optically thin lines.

References

Frisch, H. 2006, A&A, 446, 403
Frisch, H. 2007, A&A, 476, 665
Frisch, H., Anusha, L. S., Sampoorna, M., Nagendra, K. N. 2009, A&A, accepted
Nagendra, K. N., Frisch, H., Faurobert-Scholl, M. 1998, A&A, 332, 610
Sampoorna, M., Nagendra, K. N., Frisch, H., Stenflo, J. O. 2008, A&A, 485, 275
Sánchez Almeida, J. 2007, ApJ, 657, 1150
Stenflo, J. O. 1982, Solar Phys., 80, 209
Stenflo, J. O., Holzreuter, R. 2003, Astron. Nachrichten, 324, 397
Trujillo Bueno, J., Shchukina, N., Asensio Ramos, A. 2004, Nature, 430, 326
Vögler, A., Shelyag, S., Schüssler, M., et al. 2005, A&A, 429, 335