

# ON THE ROTATIONAL TEMPERATURE OF TiO IN SUNSPOTS

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## 1. INTRODUCTION

The rotational temperatures of molecules,  $T(\text{rot})$ , are used to test photospheric and sunspot models (Khlistov 1967; Webber 1971). The molecule TiO is detected in the umbral spectrum only and hence its observed rotational temperature is free from the effects of stray-light contributions from the surroundings (Makita 1968; Wöhl 1970).

Observationally, the following estimates of  $T(\text{rot})$  for the  $\alpha$  system of TiO are given:  $3030 \pm 270$  K (Makita 1968),  $3480 \pm 300$  K (Webber 1971) and  $2980 \pm 200$  K for the (0-0) band and  $3000 \pm 200$  K for the (0-1) band (Sotirovski 1971). Wöhl's (1970) estimates are not included here because an unambiguous rotational temperature has not been obtained. Sotirovski (1971) concludes that due to the larger size of the umbra used (Area =  $250 \times 10^{-6}$  solar hemisphere), he found a lower temperature than that given by Webber (1971). However, the latter author has shown that the rotational temperatures obtained for umbrae of different sizes (Area =  $42 \times 10^{-6}$  and  $100 \times 10^{-6}$  solar hemisphere) are about the same ( $3430 \pm 300$  K and  $3530 \pm 300$  K respectively). Also, Webber (1971) finds a rotational temperature  $3420 \pm 300$  K using the same spectral region as used by Makita (1968). Clearly, Makita's (1968) estimates are significantly lower.

In all these investigations it is not clear whether observationally the coolest parts of umbrae were selected or not. In view of this uncertainty for the  $T(\text{rot})$  observations, we take Webber's (1971) results as the upper limit and as the lower limit the results of Makita (1968) and Sotirovski (1971).

Recently, based on LTE calculations for equivalent widths in Henoux's (1969) sunspot model, Boyer and Sotirovski (1973) gave  $T(\text{rot}) = 3900 \pm 200$  K and  $T(\text{rot}) = 3500 \pm 300$  K for the (0-0) and the (1-2) bands respectively. To resolve the discrepancy between this result and the observations of Sotirovski (1971), departures from LTE in the model calculations and in the molecule formation processes were suggested by them.

Alternatively, however, this discrepancy may be merely a consequence of the sunspot model chosen. Zwaan (1974) presented a more refined and realistic sunspot model. The merits of this model over the other ones, including Henoux's (1969) model, have been discussed in the paper referred to. Here, we present a comparison between the results of an investigation for Zwaan's (1974) and Henoux's (1969) sunspot models to assess the contribution of the choice of models in theoretical LTE estimates of  $T(\text{rot})$  for TiO.

## 2. FORMULATIONS AND CALCULATIONS

The equivalent width of a weak line can be written as  $W = \int_0^{\infty} G(\tau) \frac{k \Delta \lambda}{k_{\lambda 0}} d\tau$  (Gaur et al. 1971). Being less significant the induced emission factor is omitted and also the Doppler broadening factor is replaced by  $\sqrt{\pi}$ . Here  $G(\tau)$  is the weighting factor for pure absorption,  $k \Delta \lambda$  the selective line opacity,  $k_{\lambda 0}$  the continuous opacity at the line centre and  $\tau$  the optical depth in the continuum in the neighbourhood of the lines.

The observations of Makita (1968), Webber (1971) and Sotirovski (1971) refer to sunspot positions close to the centre of the solar disc. For example spot No. 1 used in Sotirovski's (1971) work for the identification of TiO refers to  $\cos \theta$  equal to about 0.9. Moreover, as indicated by saturation function calculations carried out by us, the molecular lines of TiO have a thin-layer formation and consequently no appreciable centre to limb variation of  $T(\text{rot})$  is expected. Therefore, the calculations were carried out for the centre of the solar disc. One can further write

$$W = \int_{-\infty}^{+\infty} \tau G(\tau) \frac{k \Delta \lambda}{k_{\lambda 0}} d(\ln \tau) \quad (2.1)$$

The depth of formation can be defined as the optical depth which, taken in the upper limit of integration, reduces the integral to  $(1/2)W$ . The atomic abundances, the oscillator strengths and the Franck-Condon factors are the same as given by Boyer and Sotirovski (1973). The wavelengths and line identifications are from Sotirovski (1971). The internal partition functions and the dissociation constants needed for the free partial pressure of TiO are tabulated by Tatum (1966). The continuous absorption coefficient is calculated as outlined by Tsuji (1964). The rotational intensity factors,  $S_J$ , for the transition  ${}^3\Delta(a) - {}^3\Delta(a)$  are taken from Kovacs (1966).

The rotational temperature is obtained from the slope of the line

$$\log (W_J/S_J) = \text{Const.} - 0.62473 B_v'' J(J+1) / T(\text{rot}) \quad (2)$$

where  $B_v''$ , the rotational constant and  $J$ , the appropriate rotational quantum number (Webber 1971) are used. The adjustment of  $J$  as  $K \pm 1/2$  changes the  $T(\text{rot})$  value by about 50 K. The molecular constants used are from Phillips' (1973) data.

### 3. RESULTS AND DISCUSSION

At the depths of formation  $\tau_2$ , tabulated by Boyer and Sotirovski (1973), the difference between the model temperatures for the (0-0) and the (1-2) bands seems smaller than that between the theoretical estimates of  $T(\text{rot})$  for these bands by the authors. So, the rotational temperatures for the (0-0), (0-1) and the (1-2) bands were calculated, using the computed equivalent widths tabulated by Sotirovski (1971). However, it is not clear to us whether Sotirovski (1971) used the  $S_J$  values for the transition  ${}^3\Delta(a) - {}^3\Delta(a)$  or  ${}^3\pi(\text{int.}) - {}^3\pi(\text{int.})$  from

TABLE 1  
Rotational temperatures of the  $\alpha$  system of TiO molecules for two sunspot models.

Model	Band	T(rot)		
		For ${}^3\pi(a) - {}^3\pi(a)$ Transition	For ${}^3\Delta(a) - {}^3\Delta(a)$ Transition	Boyer & Sotirovski (1973)
HENOUX (1969)	(0-0)	$3900 \pm 70$	$3890 \pm 70$	$3900 \pm 200$
	(0-1)	$3890 \pm 70$	$3875 \pm 70$	—
	(1-2)	$3700 \pm 140$	$3700 \pm 140$	$3500 \pm 300$
ZWAAN (1974)	(0-0)	$3220 \pm 50$	$3220 \pm 50$	—
	(1-2)	$3290 \pm 30$	$3290 \pm 30$	—

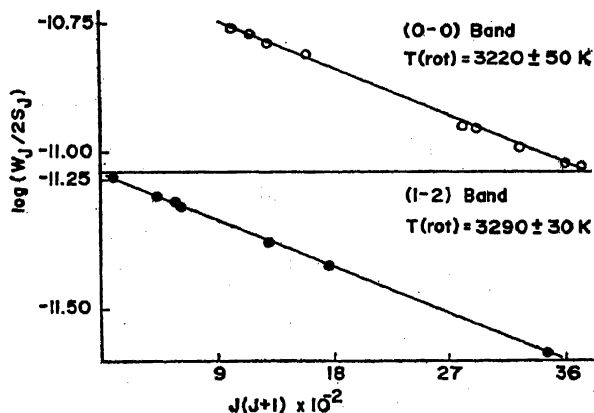


Fig. 1 Rotational temperatures for the (0-0) and the (1-2) bands of the  $\alpha$  system of the TiO molecules in Zwaan's (1974) sunspot model.

the tables of Kovacs (1966). We used the  $S_J$  factors for both of these transitions separately and also that for  ${}^3\Pi(a) - {}^3\Pi(a)$  (Schadee 1964). The  $S_J$  for the  ${}^3\Pi(\text{int.}) - {}^3\Pi(\text{int.})$  and  ${}^3\Pi(a) - {}^3\Pi(a)$  turned out to be the same except for a normalising factor of 3, the spin multiplicity. The results of  $T(\text{rot})$  calculations are summarised in Table 1. It may be inferred from this table that Hanoux's (1969) model gives larger rotational temperatures, inconsistent with observations.

$T(\text{rot}) = 3220 \pm 50$  K and  $T(\text{rot}) = 3290 \pm 30$  K are obtained for the (0-0) and the (1-2) bands respectively (cf. fig.1) in our calculations for Zwaan's (1974) sunspot model. This is midway between the observed range of temperatures i.e., 3000 K to 3480 K. We obtain slightly larger equivalent widths than those computed by Sotirovski (1971) for the (0-0) band, whereas the weaker (1-2) band remains practically unaffected.

Calculations similar to Pande (1968) and Sarychev (1971) show that the various vibrational and the rotational states of TiO are populated through collisions, under the physical conditions prevalent in the umbral atmosphere as represented by Henoux's (1969) and Zwaan's (1974) sunspot models. In the absence of any dipole moment measurements (as known to us) for TiO, it was taken as large as 1 Debye and the radiative life time of the rotational states was obtained to be of the order of few tenths of a second, close to the estimates made in other cases by Hinkle and Lambert (1975). Therefore, an equality between the vibrational, rotational, Kinetic and the model based temperatures is expected under LTE. Further, for several molecules in the photosphere Withbroe (1968) observationally found evidence for an approximate equality of the excitation temperatures of solar molecular lines and the local electron temperature in the region of line formation.

Summing up, for TiO rotational temperatures, consistent with observations can be obtained through a change to a more refined and realistic model atmosphere, without invoking non-LTE.

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