PROFILE FITTING FOR THE STELLAR PHOTOMETRY

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ABSTRACT. The point spread functions used for doing photometry on two-dimensional stellar images are described. Analytical functions used for this purpose are discussed and it is indicated that amongst them modified Lorentzian fits the best to the observed stellar profile. The computer programs presently used in crowded stellar region for accurate photometry are also described.

INTRODUCTION

Stellar photometry is essentially a low resolution spectroscopy. But its utility for astrophysical investigations can be judged from the fact that now a days there are some 75 different photometric systems which are designed to measure a wide variety of features in the stellar energy distribution. Photoelectric photometry (i.e. use of photomultipliers) provides accurate photometric observations (~0.01 mag). To overcome its limitation of observing only a small part of the sky at a time, astronomers use 2-dimensional (D) detectors where a number of program stars or an extended object located within a small region of sky (~ many arc minutes to a few degrees for photographic plates and a few arc minutes for electronic detectors) can be observed at the same time and hence, it is a more efficient way of using telescope time. Also, such obervations are not affected by temporal variations in the instrumental response or in the transmission of earth's atmosphere. One uses microdensitometers to digitize the 2-D images if they are recorded on detectors like photographic and electronographic films/plates. However, direct 2-D digital images can be obtained by using solid state detectors like charge-coupled devices (CCD). Before measuring the stellar magnitudes, these images have to be calibrated. Here only the procedure used for doing stellar photometry on calibrated 2-D digital images are discussed.

METHODS OF STELLAR PHOTOMETRY

It is well known that on 2-D stellar images, fainter the star smaller the diameter i.e. diameter of a stellar image can represent approximately its brightness. However, for doing relatively accurate stellar photometry on 2-D digital images, there are several ways ranging from simply measuring the central density to the complex analysis of the stellar image (see stetson 1986). The main methods may be divided into the following three groups:

1. Where a single one dimensional intensity profile is produced and then compared with the profiles from other images. For example, one can fit 1-D Gaussian profile of the form $B+A \exp[-1/2(x-x_0/R_X)^2]$ and form an index $\mu_G = AR_X$. Here A is the amplitude and R_X is the profile half width.
2. Where the pixel values around entire star image are summed over and some appropriate base values are subtracted. The well known example of this group is to do aperture photometry on 2-D digital images.
3. Where the 2-D empirical or analytical functions are fitted to one star image and compared with that from other stars.

Second type methods suffer from the drawback of conventional photoelectric photometry that the area of sky around the star has to be included with equal weight and a comparison area of sky has to be measured. They introduced noise and possible contamination of field stars and variable background. There is still a gain over photoelectric photometry, however, as considerably smaller measuring apertures for both can be used than are needed for photoelectric photometry, where the aperture must be large enough to contain image dancing due to seeing variations and telescope tracking errors. Smaller apertures allow smaller corrections for the sky, resulting in relatively precise measurements, as well as better separation of program stars from close neighbours. Also, the sky observations are strictly simultaneous with the stellar observations, reducing photometric errors caused by short term variability of the sky. First type method overcome these problems, but at the cost of only using a small area of the image and thus throwing away a lot of information. Third type methods have the advantages of both methods. For this, one should know the 2-D brightness distribution produced in the detector by the image of a star i.e. point spread function (PSF) and it is the topic of next section.

PROFILE OF THE STAR AND COMPUTER PROGRAMS

One expects to get optics diffraction limited star images only with the space telescope if its optical surfaces have no other aberrations. In such idealized situation, diffraction of a point source (i.e. star image) by the primary mirror of a radius R with an axially symmetric obstruction (secondary mirror and assembly) of radius mR and effective focal length of the telescope f, will give rise to the following type of radial intensity distribution $I(\lambda,\gamma)$ at the focus (see Linfoot & Wolf 1953):

$$I(\lambda,\Upsilon) = \left[\frac{J_1(x)}{x} - m \frac{J_1(mx)}{x} \right]^2 / \pi \sigma_2^2(1-m^2);$$

J_1 denotes the Bessel function of the first kind; $x = \Upsilon/\sigma_2$;

$$\sigma_2 = \frac{\lambda f}{2\pi R} \; ; \; \lambda = \text{wavelength} \; ; \; 0 < m < 1$$

and

$$\int_0^{2\pi} \int_0^{\infty} I_2(r) \, r \, dr d\theta = 1$$

For ground based imagery the vagaries of seeing and guiding are such that star images are no more diffraction limited and hence, above type of function cannot represent the star image profile. King (1971) has constructed such profile of a star image from its central peak out to a radius of six degrees. The profile has a central core, an exponential drop, and extended inverse-square aureole. The physical origin of these separate parts of the profile is not clear. Actually, it is determined by various phenomena of atmospheric refraction and both atmospheric and instrumental diffraction and scattering. Atmospheric turbulences causing image motion and blurring are supposed to yield a Gaussian intensity profile at the focus. Instrumental aberrations may arise due to irregular small deviations from the ideal shape of the optical surfaces. All these indicate that an exact derivation of the mathematical function which can represent correctly the PSF of a star image is not an easy task. For simplicity, Gaussian PSF of the following form are used:

$$I(x,y,x_0,y_0) = A \; e^{-(x-x_0/Rx)^2} \; e^{-(y-y_0/Ry)^2} + B$$

In Figure 1a, we have plotted the observed star image profile from a CCD frame i.e. number of counts against radius in pixels (1 pixel = 0.36 arcsec). This figure which is in agreement with Franz (1973) shows that Gaussian distribution cannot represent satisfactorily the observed star image profile. Actually it fails to fit the peak as well as the tail part of the stellar image. Efforts to find a better PSF have been made by several people and are still on. A brief account of some of them are given here. They may be mainly divided into the following two groups:

1. The analytical methods

In this case a mathematical function $I(x,y,x_0,y_0)$ to describe intensity as a function of 2-D distance from the centroid (x_0,y_0) of a star are used and its parameters are adjusted to give the best possible representation of actual star image profiles in a given frame. Some of such functions are given below:

a) In most cases a good description of the PSF on photographic plate is given by following function proposed by Moffat (1969):

$$I(x,y,x_0,y_0) = A \left[1 + \frac{(x-x_0)^2 + (y-y_0)^2}{R^2}\right]^{\beta}$$

Where A = amplitude, x_0 and y_0 are the central coordinates; R and β are the shape parameters. This PSF has been extensively used by Buonanno and coworkers (see Buonanno et al. (1983)) for the stellar photometry.

b) Lorentzian distribution introduced initially by Franz (1973). This function closely approximates the theoretical one expected for a telescope with a central obscuration (i.e. secondary mirror and assembly) looking through turbulent atmospheric elements which have Kolmogorov power spectrum. It has the following form:

$$I(x,y,x_0,y_0) = \frac{A}{1 + \left[\left(\frac{x-x_0}{R_x}\right)^2 + \left(\frac{y-y_0}{R_y}\right)^2 \right]^{p/2}}$$

Where A is amplitude and (x_0,y_0) the peak location of the star image. R_x, R_y are the half power width. P is a variable exponent of the form:

$$P = P_0 \left\{ 1 + \left(\frac{x-x_0}{P_{Rx}}\right)^2 + \left(\frac{y-y_0}{P_{Ry}}\right)^2 \right\}^{1/2}$$

P_0 is the power at the peak location (x_0,y_0) and P_{Rx}, P_{Ry} determines the rate at which P changes as function of (x,y). The parameters P_0, Rx, Ry, P_{Rx}, and P_{Ry} defines the "seeing".

c) Modified Lorentzian is introduced by Penny (cf. Penny & Dickens 1986) to account for the very wide halo around stars. It has the following form:

$$I(x,y,x_0,y_0) = A \left\{ \frac{1}{1+d_1^{P_0(1+d_2)}} + Q \exp(-d_3^{P_3}) \right\} ;$$

Where

$$d_1 = [(x_1/Rx)^2 + (y_1/Ry)^2]^{1/2} ;$$

$$d_2 = [(x_1/P_{Rx})^2 + (y_1/P_{Ry})^2]^{1/2} ;$$

$$x_1 = (x-x_0)\cos\theta + (y-y_0)\sin\theta$$

$$y_1 = -(x-x_0)\sin\theta + (y-y_0)\cos\theta \text{ ; and}$$

$$d_3 = [(x-x_0)^2 + (y-y_0)^2]^{1/2} / r_3$$

Q is small ~ 0.01, r_3 is significantly larger than Rx and Ry. Thus there is a rotated-elliptical Lorentzian with a wide low circular modified Gaussian base.

For the stellar photometry in uncrowded region where background is constant, one has to simply subtract the background and fit the PSF. Gaussian often results in residuals larger than 5% of the peak of the profile while Lorentzian leaves residuals that are rarely larger than one percent of the maximum height of the profile. It is only some tenths of a percent, in the case of modified Lorentzians. It indicates that modified Lorentzian is very close to the true star image profile. This is also noticed in Fig.1 where Lorentzian fits better than Gaussian. If a star image has high S/N ratio, then magnitude estimates from Gaussian, Lorentzian and modified Lorentzian profiles agree fairly well. However, they differ significantly if the star has weak S/N ratio. In such cases modified Lorentzian being closest to true star image will give the best result.

For photometry in crowded regions where background is variable one fits following type of function:

$$I(x,y) = \sum_{i=1}^{N} I(x,y,x_{oi},y_{oi}) + Bx + Cy + D$$

Where (x_{oi},y_{oi}) are the position of ith star. For such type of analysis, it is very much essential that adopted PSF should represent the true star image profiles. Otherwise, residuals of one star image will affect the measurement of the other stellar images.

Computer programes based on analytical PSF, for doing stellar photometry on 2-D digital images have been developed and used by several astronomers (cf.Peterson et al. 1978; Buonanno et al. 1983; Penny & Dickens 1986; and references therein). The analytical PSF integrates numerically over the area of each pixel in a stellar image and hence, the adverse effects of finite pixel size in an undersampled image are minimized. However, they are relatively time consuming and imperfectly formed stellar images require either large number of parameters for their description as in the modified Lorentzian PSF or else one has to live with relatively poor measurements.

2. The empirical Methods

The computer programs based on the empirical PSF are RICHFLD and VISTA (See Stetson 1986; and references therein). The empirical PSF is estimated using bivariate interpolation to estimate intensity values at fractional-pixel positions within an observed stellar profile. This PSF is then numerically interpolated and scaled to match the observed data for each program star. This method is much faster than the analytical one but is poorly suited to undersampled data because in such cases assumptions used in the interpolation are not able to provide interpolations correct to a few percent and the most severe failure of it is near the centre of the profile where most of the photometric information resides (Stetson 1986).

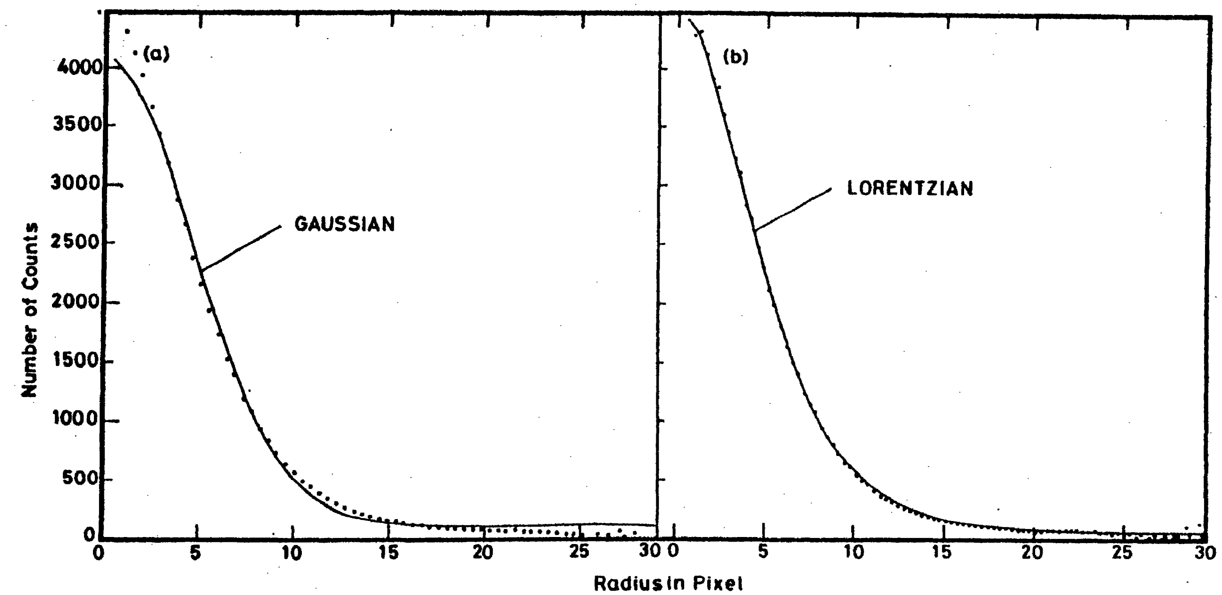

Figure 1. Plot of pixel counts of a stellar image from a CCD frame against its radius in pixels (1 pixel = 0.36 arc sec). (A) and (B) show respectively the best fitted Gaussian and Lorentzian to the observed stellar image profile.

DAOPHOT computer program by Stetson (1986) uses a model for the PSF which attempts to overcome the limitations of both the analytical and empirical methods and exploit the best aspects of them. The principles on which this program works have been described extensively by Stetson (1986). It defines the PSF on the basis of a 2-D look up table containing brightness values actually observed within the profiles of bright stars and hence, it is an empirical PSF. To overcome the limitations of empirical methods i.e. to improve the accuracy of interpolation, an analytical Gaussian PSF as a first approximation to the actual stellar profile is used and empirical corrections are made for the observed residuals of the actual stellar profile from the best fitted Gaussian. This residual has much smaller amplitude, and hence the interpolation procedures can provide the desired accurate interpolation.

If one is not interested in very accurate photometry of either faint stars or located in crowded region, any one of the above methods is suitable. However, for accurate photometry one has to use either modified Lorentzian type analytical functions or DAOPHOT type programs.

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