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Diurnal variation of horizontal magnetic force at Kodaikanal

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ABSTRACT. The diurnal variation of horizontal force at Kodaikanal on each of the international quiet days during the summer solstices of the years 1949—1951 is subjected to harmonic analysis. The harmonic coefficients of the same period are plotted on a harmonic dial. In the case of a strictly persistent periodicity, affected only by random errors, the probable error figure will be a circle. For the 24-hourly wave the probable error figure is found to be an ellipse having the ratio of the axes, 1·80. The direction of the major axis indicates that the amplitude varies much more than the phase. An attempt is made to find out whether this large variation in amplitude can be accounted for by regular causes like degree of disturbance, variation of sunspot number etc. Sunspot number accounts only to a small extent for this large variation of the amplitude. The variability is mostly fortuitous. It is also noticed that as the value of the first harmonic coefficient decreases, the corresponding decrease in the second harmonic coefficient is less and the second component may even be larger than the first when the latter is very small.

The quiet-day diurnal variation of magnetic elements at a place undergoes irregular or fortuitous changes apart from those depending on seasonal and sunspot-cycle variations. The former are also important, since any theory of diurnal magnetic variation must be able to account for the fortuitous variations as well. These irregular variations at different observatories have been studied by Chapman and Stagg (1929, 1931), Bartels (1932) and others. Kodaikanal magnetograms are particularly suitable for such a study, since here both the diurnal variation in the horizontal force and its variability are large.

2. The 24 hourly values of H-averages for sixty minutes centred at the full hours of Greenwich mean time—are corrected for non-cyclic variation and subjected to the usual harmonic analysis. The harmonic coefficients are further corrected for the use of hourly means so that they represent instantaneous values (Chapman and Bartels, *Geomagnetism*). The phases are expressed in local mean time of Kodaikanal. The harmonic coefficients, thus obtained, are plotted on the harmonic dial. In the case of a strictly persistent periodicity (amplitude and phase both constant), affected only by random errors, the cloud of points in the harmonic dial tends towards a symmetric distribution. The dispersion can be expressed by a single number and a probable error circle drawn about the centroid of the cloud of points. There are cases, however, in which the cloud is considerably elongated in some direction and the probable-error circle is to be replaced by an ellipse. If the harmonic coefficients, a_v, b_v be represented along the x and y axes, σ_x, σ_y be the standard deviations of the x and y co-ordinates and r the correlation coefficient between x and y, the major axis of the probable error ellipse is inclined at an angle θ to the x-axis given by Bartels (1932).

$$\tan 2\theta = \frac{2r\,\sigma_x\,\sigma_y}{\sigma^2_x - \sigma^2_y}$$

The lengths of the major and minor axes are given by

$$P_1, P_2 = 0\cdot8326 \sqrt{(\sigma^2_x + \sigma^2_y) \pm \left\{ (\sigma^2_x - \sigma^2_y)^2 + 4r^2\sigma^2_x\sigma^2_y \right\}^{\frac{1}{2}}}$$

$$= 0\cdot8326\ s_1\ ,\ 0\cdot8326\ s_2 \text{ say.}$$

3. The ellipticity may be either due to its presence in the parent-population itself or due to sampling deviations from a circular parent-population. J. W. Mauchly (1940) defines

$$P\,(\mathrm{Le}) = \left(\frac{2s_1s_2}{s_1^2+s_2^2}\right)^{N-2}$$

where N is the total number of points and s_1 and s_2 have been defined at the end of the last paragraph in terms of the lengths of the major and minor axes of the probable-error ellipse. If P (Le) be quite small compared to unity, the parent-population itself is elliptical. Otherwise, the ellipticity is due to sampling deviations and the parent population is circular.

4. The days chosen are the ten international quiet days in the summer solstices (May to August) of the years 1949, 1950 and 1951. The total number of days selected is 107. The mean values of the different harmonic coefficients for different years or for the different months are found to be not appreciably different from the mean values for all the 107 days. The seasonal variation from May to August or the year to year variation from 1949 to 1951 is small so that systematic variation due to this cause can be neglected. The numerical data regarding the average variation as well as the variability of the harmonic coefficients are given in Table 1.

5. The ellipticity is very pronounced for the first harmonic. The other ellipses, especially the third and the fourth have small ellipticities. P(Le) is quite small for the 24-hourly cloud, but is not so for the others. The standard deviation, M, decreases with the order of the harmonic, but the proportional scattering is greatest for the fourth harmonic coefficient. A similar study of variation of declination at Huancayo has been made by Bartels (1932). The dispersion for the first harmonic coefficient is much less at Kodaikanal (c/M=3·1 as compared to 1·7 at Huancayo), but the ellipticity (1·80) is greater than that for Huancayo (1·63). The value of P(Le) at Huancayo (10^{-5}), though quite small, is comparatively larger. The 24-hourly cloud at Kodaikanal has a marked and significant ellipticity.

6. The harmonic dial for the 24-hourly wave is shown in Fig. 1 and the probable-error ellipse is also drawn. The direction of the major axis indicates that the amplitude of the wave varies much more than the phase. The amplitude has varied from 18·0 γ to 64·3 γ . Let us consider how far these variations are regular and how far they are fortuitous.

7. On disturbed days, the range is usually larger than on quiet days. Since international quiet days have been chosen for this analysis, the disturbance daily variation on these days is small. It has been found that up to a magnetic character figure of 1·1, the variability of S (solar daily variation) with character figure is very little (Bartels 1932). Still, these ten quiet days in each month can be further subdivided into five international quiet days and five additional quiet days. The latter are relatively more disturbed. The probable-error ellipses for the two sets of days are determined and the data, for the first harmonic coefficient, along with those for all the 107 days are given in Table 2. It is clear from an inspection of the table that disturbance is not the cause for this large variability.

8. The variation considered does not represent truly the solar daily variation, S_q alone. It is, in fact, $S_q + L$. However, even at Huancayo, where the lunar daily variation is abnormally high, it is only about 8 per cent of the S_q variation and has been neglected by Bartels (1932) who has, for a similar study, taken the S_q+L variation. Moreover, the lunar variation, being semi-diurnal in type, is more likely to affect the second rather than the first harmonic coefficient.

9. It is well-known that the amplitude of the S_q variation is greater in years of sunspot maximum than of sunspot minimum. The daily sunspot number during these 107 days has varied from 15 to 218 with an arithmetic mean of 100 and a standard deviation of 29. The correlation coefficient, +0·66 between the daily Zurich sunspot number, R_z, and the amplitude of the first harmonic coefficient, c_1, is not high enough to account fully for the very large variability of the diurnal range of H at Kodaikanal, exhibited by the ellipticity of the probable-error figure. For example, on 30 August 1951, the Zurich

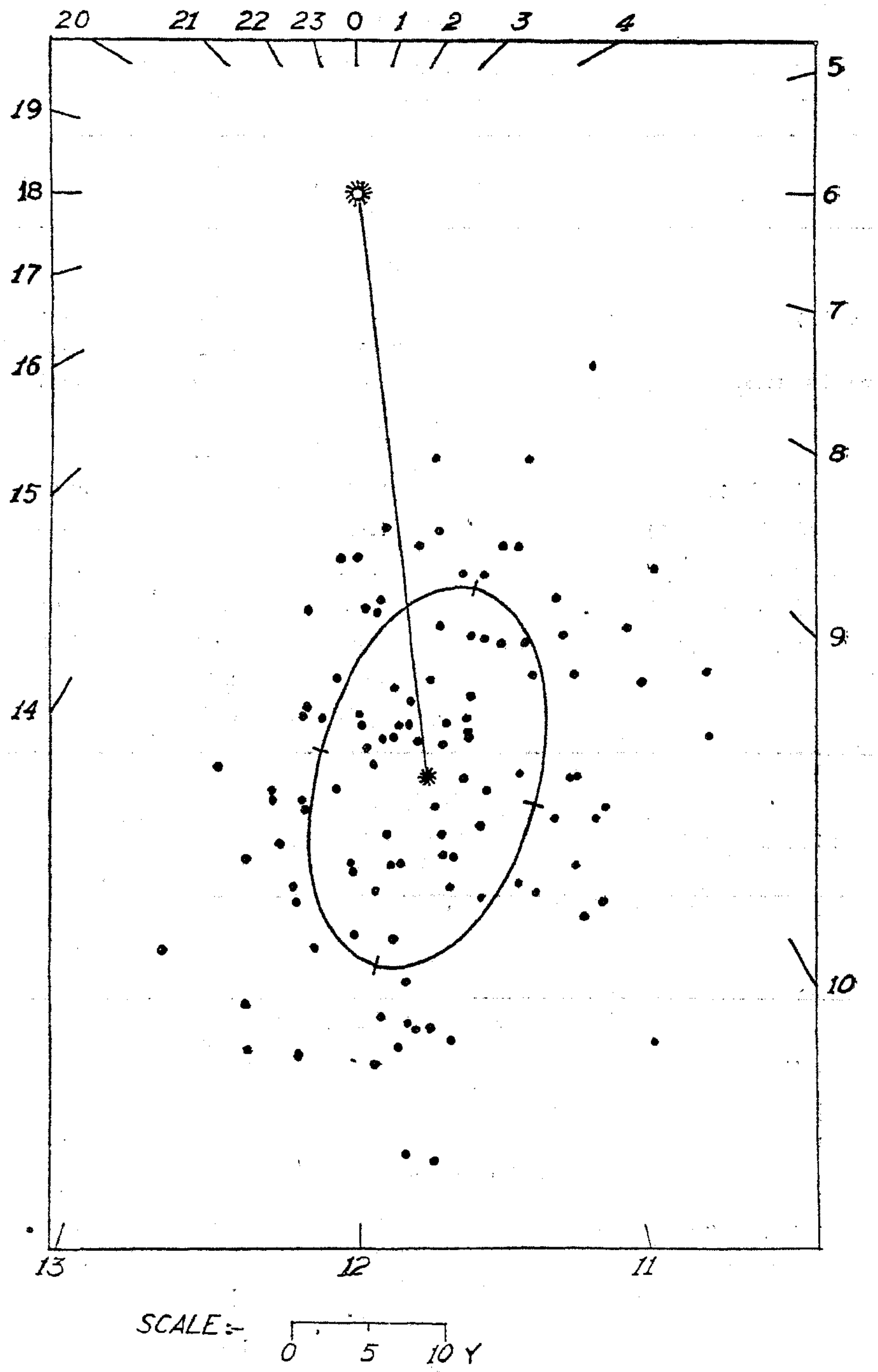

Fig. 1. Harmonic dial for the 24-hourly sine-wave in the diurnal variation of the horizontal force at Kodaikanal Observatory on 107 international quiet days in the summer solstices of the years 1949—1951

Each dot marks one day ; the length of the vector from the centre gives the amplitude and its direction, as read from the dial gives the local mean time of the maximum of the sine-wave. The average vector and the probable ellipse are drawn

TABLE 1

Harmonic analysis of diurnal variation of *H* at Kodaikanal and its variability derived from 107 international quiet days in the northern summer during the years 1949, 1950 and 1951

	Period (*hr*) 24	12	8	6
	Average diurnal variation			
Amplitude, *c* in γ	38·8	22·5	9·5	1·7
Phase	276°·5	108°·2	308°·3	150°·3
Local time first max.	11·6*h*	11·4*h*	3·1*h*	5·0*h*
	Variability			
Standard deviation, *M* (γ)	12·5	8·2	6·2	3·7
Major axis probable ellipse (γ)	12·84	7·46	5·20	3·85
Minor axis probable ellipse (γ)	7·14	6·05	5·07	3·45
Direction of major axis	13·0*h*	1·4*h*	1·0*h*	0·2*h*
Ratio of the axes (ellipticity)	1·80	1·23	1·03	1·11
Ratio, *c*/*M*	3·1	2·7	1·5	0·5
P (Le)	$3{\cdot}6\times10^{-8}$	$9{\cdot}8\times10^{-2}$	0·98	0·53

TABLE 2

24-hourly wave for (a) five quiet days, (b) five additional quiet days and (c) ten quiet days in the summer solstices of 1949, 1950 and 1951

	Five quiet days	Five additional quiet days	Ten quiet days
	Average diurnal variation		
Amplitude, *c* in γ	39·9	37·6	38·8
Phase	274°·6	278°·7	276°·5
Local time first max.	11·7*h*	11·4*h*	11·6*h*
	Variability		
Standard deviation, *M* (γ)	12·3	12·2	12·5
Major axis probable ellipse (γ)	12·99	12·45	12·84
Minor axis probable ellipse (γ)	6·44	7·25	7·14
Direction of major axis	12·5*h*	13·3*h*	13·0*h*
Ratio of the axes (ellipticity)	2·02	1·72	1·80
Ratio *c*/*M*	3·2	3·1	3·1
P (Le)	$9{\cdot}2\times10^{-6}$	$9{\cdot}3\times10^{-4}$	$3{\cdot}6\times10^{-8}$

sunspot number was only 15 and c_1=49·3 γ, whereas on 21 May 1951, R_z was 180 and yet c_1 was only 23·7 γ. From out of the 107 days, 58 days were chosen so that the values of R_z for these days lie between 71 and 124. The numerical data pertaining to the probable error ellipse for these 58 days are given in Table 3.

TABLE 3

24-hourly wave for the 58 international quiet days with their Zurich sunspot number lying between 71 and 124

Average diurnal variation	
Amplitude, c in γ	37·6
Phase	275°·4
Local time first max.	11·6*h*
Variability	
Standard deviation, M (γ)	10·5
Major axis probable error ellipse (γ)	10·4
Minor axis probable error (γ)	6·68
Direction of major axis	13·1*h*
Ratio of the axes (ellipticity)	1·6
Ratio c/M	3·6
P (Le)	$4{\cdot}5\times10^{-}$

10. Though the lengths of the major and minor axes are somewhat less, the ratio of the axes has not decreased appreciably. The direction of the major axis also remains unaltered. This clearly demonstrates that the variation of the sunspot number cannot fully account for the very large variability of the diurnal variation of H at Kodaikanal.

11. It has been found by Chapman and Stagg (1931) that if a quiet day is followed by another, the ranges on the two days are very nearly equal. Bartels (1932) has verified this with his harmonic analysis of D-variation at Huancayo, where he finds that for successive quiet days, the values of the corresponding harmonic coefficients are same. Though this is also generally true of Kodaikanal, exceptions are not uncommon. 20, 21 and 22 May 1951 were all included in the five international quiet days for the month. Still, the daily ranges—difference between the maximum and minimum in the hourly values on these days—were 133 γ, 96 γ, and 141 γ respectively; the values of the first two harmonic coefficients were 50·0 γ and 31·5 γ; 23·7 γ and 18·9 γ and 54·4 γ and 31·2 γ respectively. It is clear that both the magnitude and type of variation of the 21st were different from those of the preceding and succeeding days. 20 and 21 July 1951 were two other successive quiet days when the ranges were 86 γ and 128 γ; the first and the second harmonic coefficients on these days were 18·6 γ and 43·0 γ and 17·3 γ and 31·5 γ respectively. These further illustrate the large fortuitous variability of the horizontal force at Kodaikanal.

12. Another interesting feature was observed in the course of this analysis. Bartels (1932), in his study of the D-variation at Huancayo, finds that days with small or large amplitude in one wave are found to have correspondingly small or large amplitudes in the other waves. The mean value of c_1/c_2 for all the 107 days is 1·74. If we divide the 107 days into two nearly equal parts with 53 and 54 days respectively with the former having comparatively large values of c_1 and the latter smaller values, the mean values of c_1/c_2 in the two cases are found to be 1·82 and 1·67 respectively. With smaller c_1, the decrease in the second harmonic coefficient is proportionately less. Of the 107 days considered, on 20 July 1951, c_1= 18·6 γ and c_2=17·3 γ. It is possible that if c_1 decreases still further c_2 may become even greater than c_1. Such a case has been noticed on 29 July 1952, when c_1=15·0 γ and c_2=18·1 γ.

13. My sincere thanks are due to Dr. A. K. Das, Director of the Kodaikanal Observatory and to Mr. B. N. Bhargava, Officer-in-charge of the magnetic and ionospheric section for allowing me to use the above data and for helpful criticism.

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