Kodaikanal Obs. Bull Ser. A (1978) 2, 180-183

ON NEGATIVE INTENSITIES IN DISCRETE SPACE THEORY IN SPHERICAL MEDIUM

A PERAIAH

ABSTRACT

The reasons for the occurrence of negative intensities are discussed A method is described to obtain positive solution of radiative transfer equation in spherical symmetry in discrete space theory by using frequency independent source function

Key Words non-negative fluxes—discrete space theory—radiative transfer

1 Introduction

It is not unusual to encounter negative fluxes in neutron transport problems (see Lathrop 1969). This is perhaps because of the fact that in neutron transport problems the range of magnitudes of cross sections is so large that a fine mesh is not possible In certain cases, the negative intensities become so large that they show up even in angle and frequency integrated fluxes As Lathrop (1969) says, ' there are psychological problems associated with negative fluxes The user who understands the transport equation and not the numerical solution procedure does know that there is no such thing as a negative angle integrated flux and rapidly becomes cynical about the effectiveness of the programme which produces negative numbers' In discrete space theory of radiative transfer in plane parallel medium (Grant and Hunt 1969 a, b) negative intensities can be easily avoided by simply adjusting the step size This is possible because the cross sections in radiative transfer theory applicable in stellar atmospheres are much smaller than those encountered in neutron transport theory However, negative intensities could arise in certain situations in spherically symmetric media which is a purely numerical effect For example, the inequality (3-12) in Peraiah and Grant (1973) cannot be satisfied in vacuum This is, however only a marginal case and for all practical problems this method gives non-negative intensities, provided we choose the step size as small as possible so that the relation (3.11) is satisfied Furthermore, the element Λ^{-}_{11} of Λ^{-} matrix is always negative and large in its absolute value This introduces unphysical negative intensities in the radiation field To avoid these negative intensities, we shall calculate the angle and frequency integrated source function inspite of the fact that they are negative and calculate the solution by using the formal solution. This method need be applied only in situations where negative intensities appear

2 A Brief Outline of Procedure and Results

There are three steps in calculating the solution .

a) Obtain the frequency and angle dependent intensities by solving the transfer equation

$$\mu \frac{\partial I(r,\mu,x)}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I(r,\mu,x)}{\partial \mu} = K_L(r)\,[S(r,\mu,x) - I(r,\mu,x)]$$

as described in Peraiah (1978a or 1978b) Here I (r, μ, x) is the specific intensity of the ray at the radial point r, making an angle $\text{Cos}^{-1}\mu$ with the radius vector with frequency x $(= (\nu-\nu_0)/\Delta\nu_D)$ $\Delta\nu_D$ being the Doppler unit K_L (r) is the absorption coefficient at line centre at r and s (r, μ, x) is the combined source function.

b) Using the intensities obtained in step (a) calculate the angle and frequency integrated source function

c) With the help of the source function obtained in step (b), the solution in spherical symmetry is calculated by using the formal solution

$$I(r, \mu, x) = I(r_1, \mu, x)\, e^{-[\tau(r_2) - \tau(r_1)]/\mu} + \int_{\tau_1}^{\tau_2} S(t)\, e^{-[t - \tau(r_1)]/\mu} \frac{dt}{\mu} \qquad 2$$

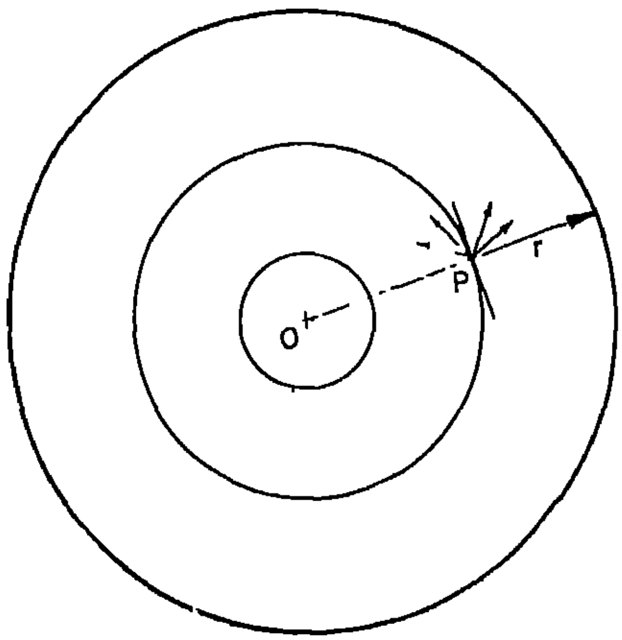

Fig. 1 Diffuse radiation field in spherical symmetry

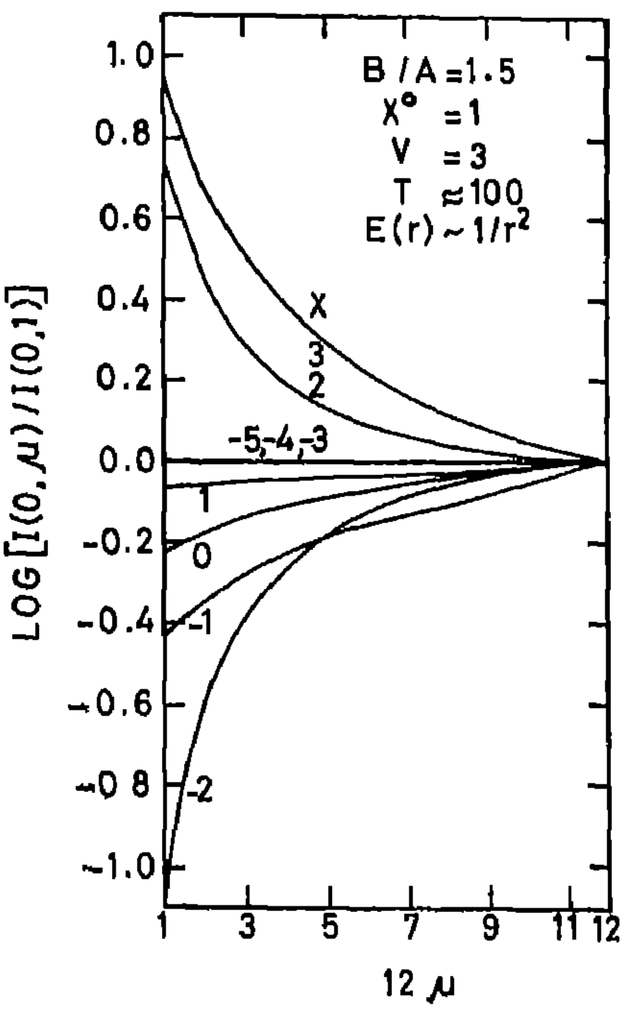

Fig 2 Angular distribution of the emergent radiation field

The optical depth is calculated by using the formula

$$\tau(r, \pm\mu, x) = \Delta r\, K(r, \pm\mu, x) \qquad 3$$

where

$$K(r, \pm\mu, x) = \frac{\pi e^2}{mc} \frac{A}{\Delta\nu_D \sqrt{\pi}} e^{-(x^* \pm \mu v)^2/\delta(r^4)} \qquad 4$$

and

$$\delta(r) \sim \Delta\nu_D(r)/\Delta\nu_D \qquad 5$$

where δ (r) is taken to be unity always

Where v is the velocity of the gas in mean thermal units and A is the abundance Several tests have been, made by comparing the source functions calculated in spherical symmetry by using the descrete space theory with those of Kunasz and Hummer (1974) Agreement has been found to be upto 3 to 4 figures As optical depths increase the agreement is better

To illustrate the method, we have arbitrarily chosen the radial distribution of velocities and densities and calculated the source functions from the steps (a) and (b) The optical depth is calculated at line centre

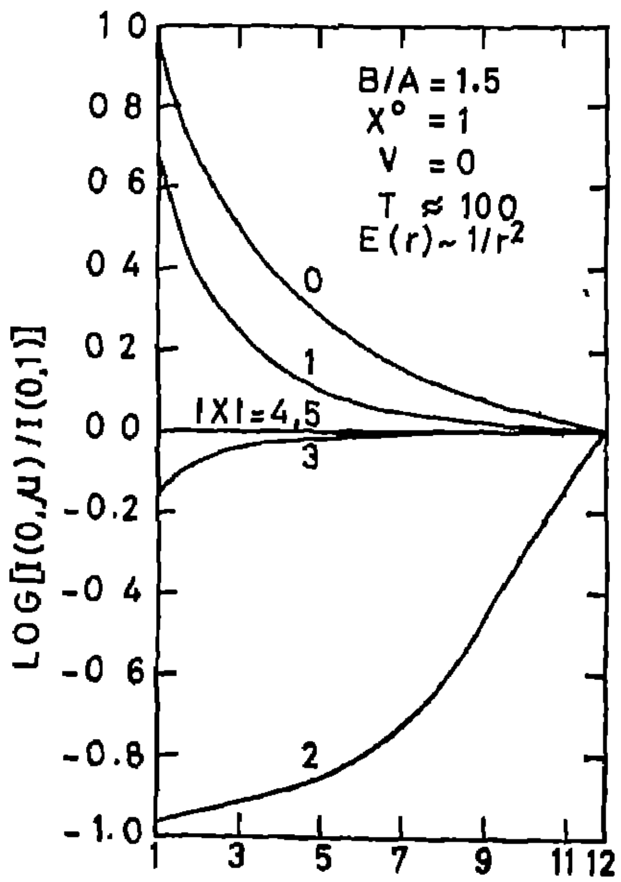

Fig. 3. Angular distribution of the emergent radiation field.

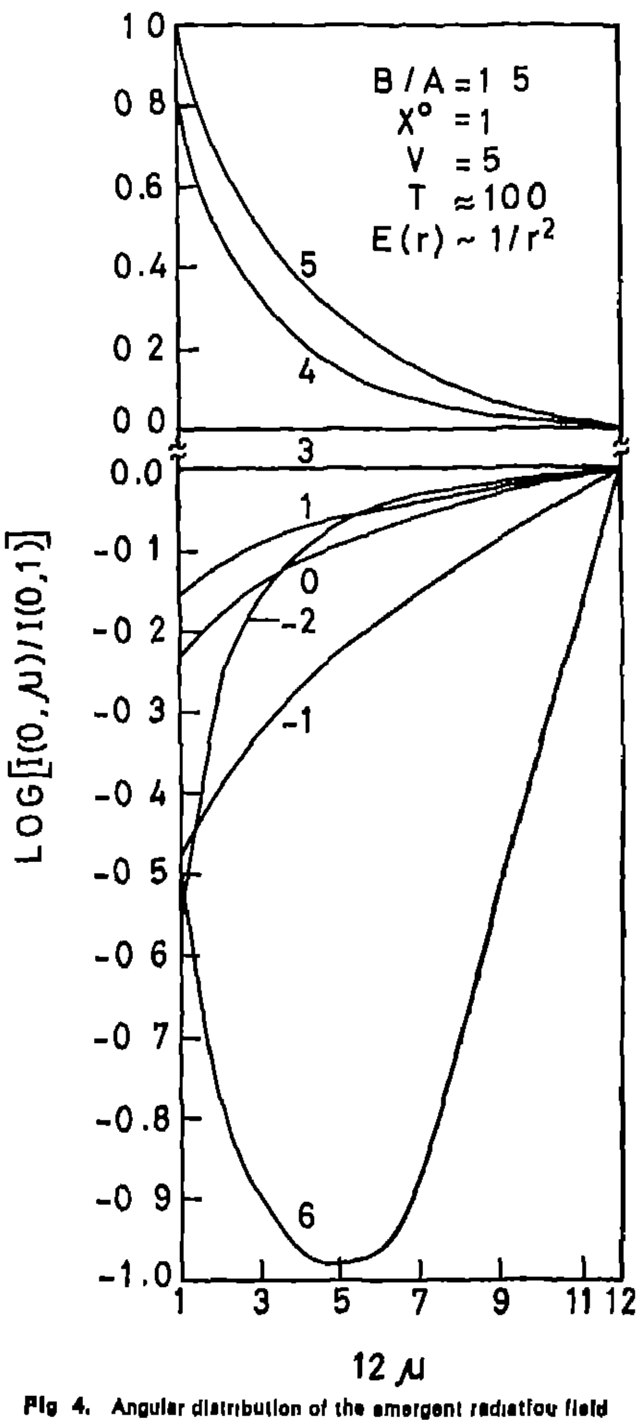

Fig 4. Angular distribution of the emergent radiation field

d the parametres are adjusted in such a way that the total optical depth T at line centre is equal to 100. e geometrical thickness of the atmosphere is taken to be one-half of the stellar radius and the band width ') to be one Doppler unit We have considered equally spaced 12 angle points in $\mu\epsilon$ (0, 1) and a trapezoidal egration is used to evaluate the integral in equation (2). A linear velocity law has been assumed given by

$$v(r) \quad v(A) \;\cdot\; [v(B) - v(A)] \frac{r - A}{B - A} \qquad 6$$

iich represents a medium expanding rapidly outwards Here v (r) is the velocity at radius r A and B are spectively the inner and outer radii of the medium We have set v (A) 0 in all situations, and B/A = 1 5 e density has been assumed to be varying as $1/r^2$ The angular distribution of the emergent radiation for rious cases of velocities are described in Figures 2, 3 and 4 In Figure 2, we have presented the results for (B) = 0 and for different values of x on either side of line centre one can see that for X ± 2, ± 3, there limb darkening whereas the wing frequencies do not show any darkening and again the central frequencies = −1, 0, +1 show limb brightening As the velocity v (B) is increased to 3 and 6 (see Figures 3 and 4), the nb darkening shows up in the red side of the line frequency points This is clear from the fact that the matter oving radially outwards produces red emission and blue absorption, which means that more radiation is ansferred at μ 1

The procedure described above is useful and necessary in a co moving frame calculation (see eraiah 1979)

M S received on May 25th 1979

eferences

rant I P , Hunt, G F , 1969a *Proc R Soc London Ser A* 313, 183
rant I P , Hunt, G E , 1969b, *Proc R Soc London Ser A* 313, 199
unasz, P B , Hummer, D G , 1974, *Mon Not R astr. Soc* 160, 67
athrop, K D , 1969, *J Comput Phys* 4, 475
eraiah, A 1978a, *Kodaikanal Obs Bull Ser A*, 2, 115
eraiah, A , 1978b *Astrophys Space Sci* 58, 189
eraiah, A , 1979 To be communicated
eraiah, A , Grant I P , 1973, *J Inst Math Appl* 12, 75