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MOTION OF A PARTICLE IN THE BACKGROUND OF A WHITE HOLE

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ABSTRACT

A study has been made of the motion of a test particle in the background of a white hole particularly from the view point of its visibility to an observer at infinity. It is shown that radial as well as non-radial photons from such a particle can leak through the Schwarzschild radius even when the particle and the white hole boundary have not yet burst through. Implications of the results when applied to the case of a grey hole are also discussed.

Key words : general relativity — white hole — geodesic motion

1. Introduction

The concept of the white hole has been with us for the past fifteen years (Novikov 1935 ; Ne'eman 1965). A white hole is a system in which mass-energy gush out from a highly dense state. To start with, the system is well inside its event horizon, although an observer at infinity can receive photons or slower than light signals emitted from its surface. The event is witnessed as an explosion releasing tremendous amount of energy. The white hole is grey when matter shot out of the singularity lacks sufficient energy to emerge from the horizon into the external universe. In what follows, we study the motion of a particle in the back-ground of a white hole particularly from the view point of its visibility to an observer at infinity.

2. Model of the White Hole

The canonical white hole we shall be concerned with here is a spherical object of uniform density ρ (*t*) and zero pressure in the frame of reference comoving with the outward moving particles (Narlikar and Apparao 1976). The interior of the white hole is assumed to be Friedmannian except for small density fluctuations superposed thereupon. The object emerges from a singular state and subsequently obeys Einstein's field equations. A dust model ensures geodesic motion of all the particles, which can be inferred from the Friedmann metric that describes the white hole interior in terms of the constant, comoving co-ordinates (r, θ, ϕ) of a comoving particle :

$$ds^2 = c^2dt^2 - S^2(t)\left[\frac{dr^2}{1-\alpha r^2} + r^2\,(d\theta^2 + \sin^2\theta\, d\phi^2)\right], \qquad (1a)$$

$$r \leq r_b.$$

Here, t is the proper time of a comoving observer and S (t) the scale factor. At the boundary $r = r_b$, a dust model ensures zero pressure so that the metric matches perfectly with an exterior Schwarzschild metric :

$$ds^2 = c^2dT^2\left(1 - \frac{2GM}{Rc^2}\right) - \frac{dR^2}{\left(1 - \frac{2GM}{Rc^2}\right)} - R^2\,(d\theta^2 + \sin^2\theta\, d\phi^2) \qquad (1b)$$

In cosmology, $r = 0$ refers to any assigned particle, but in the present case, r is defined such that the scale factor $S = 1$ when the expansion of the white hole is complete. Then, $r = 0$ refers to the centre of the object. The scale factor $S(t)$ varies with time according to

$$\frac{dS}{dt} = \pm \left(\frac{8\pi G \rho}{3} S^2 - a c^2 \right)^{\frac{1}{2}}, \tag{2}$$

where + sign refers to explosion, and ρ, the matter density, is time dependent, evolving according to

$$\rho = \frac{\rho_0}{S^3} , \tag{3}$$

ρ_0 being the density corresponding to the epoch when expansion is complete ($S = 1$). The parameter a, a constant, is given by

$$a = \frac{8\pi G \rho_0}{3c^2} \tag{4}$$

Eq. 2 can be integrated to give

$$t = \frac{1}{c a^{\frac{1}{2}}} \left[\sin^{-1} S^{\frac{1}{2}} - S^{\frac{1}{2}} (1-S)^{\frac{1}{2}} \right] \tag{5}$$

The proper time for explosion from $S = 0$ to $S = 1$ is

$$t_0 = \int_0^1 \frac{dS}{(dS/dt)} = \frac{\pi}{2c\, a^{\frac{1}{2}}} \tag{6}$$

The white hole boundary crosses the event horizon at an epoch

$$S(t_s) = a\, r_b^2 = \sin^2 \Sigma \qquad \text{(say)}, \tag{7}$$

from eq 4. The mass of the white hole can be written as

$$M = \frac{a\, r_b^3 c^2}{2G} = \frac{t_0 \sin^3 \Sigma\, c^3}{\pi G} \tag{8}$$

In the following section, we study the radial time-like trajectories of particles and frequency shifts in their radiation, particularly from the instant of their leaving the white hole boundary onwards. For the sake of completeness, we shall briefly study the radial geodesics of a particle of nonzero rest mass in peculiar motion inside the white hole as well, which may be of interest from the cosmic ray physics point of view.

3. Radial Time-like Trajectories

The motion of a particle can be studied by solving equations of geodesic motion for the interior Friedmann metric (1a) and for the exterior Schwarzschild metric (1b) matching them at the boundary of the white hole, r_b. The Schwarzschild coordinates R and T are related to the comoving coordinates r and t through (Narlikar and Apparao 1975):

$$R = r\,S(t),\ T = \Phi(A)\ ;\ A = \left[\int_{t_b}^{t} \frac{c^2\, dt}{S\,(dS/dt)} + \int_{r}^{r_b} \frac{r\,dr}{1 - a r^2} \right] \tag{9}$$

such that the eq. (1a) is

$$ds^2 = e^{\nu} dT^2 - e^{\lambda} dR^2 - R^2 (d\theta^2 + \sin^2\theta\, d\phi^2) \tag{10}$$

where

$$e^{\nu}=\frac{S^2\,(dS/dt)^2\,(1-\alpha r^2)}{\Phi_1{}^2\,c^4\left(1-\alpha r^2-\dfrac{r^2\,(dS/dt)^2}{c^2}\right)},\quad e^{\lambda}=\frac{1}{\left(1-\alpha r^2-\dfrac{r^2\,(dS/dt)^2}{c^2}\right)},\quad \Phi_1=\frac{d\Phi}{dA} \tag{10b}$$

In what follows, we restrict ourselves to a study of radial geodesics. The equation of motion

$$c^2\,\frac{dt^2}{ds^2}+\frac{S\,(dS/dt)}{(1-\alpha r^2)}\left(\frac{dr}{ds}\right)^2=0, \tag{11}$$

solves, in view of the metric (1a), to give

$$c\,\frac{dt}{ds}=\left(1+\frac{k^2}{S^2}\right)^{\frac{1}{2}}, \tag{12}$$

k being a constant, a measure of energy of the particle with respect to the comoving frame of reference, and $S \geqslant S_1$, where S_1 is the epoch of the commencement of the peculiar motion. Hence, eq. (1a) suggests

$$\frac{dr}{ds}=\pm\frac{k\,(1-\alpha r^2)^{\frac{1}{2}}}{S^2}. \tag{13}$$

The three-velocity of a particle, according to an observer whose instantaneous position coincides with that of the particle at (r, t), is

$$v\,(t)=(-g_{11})^{\frac{1}{2}}\,\frac{dr/ds}{dt/ds}=\pm\frac{kc}{(k^2+S^2)^{\frac{1}{2}}} \tag{14}$$

This shows that v decreases from c at $S=0$ to $kc\,(1+k^2)^{-\frac{1}{2}}$ at $S=1$. For very small k (and $k \ll S_1$ also) $v\,(t) \simeq kc/S$. Solving eq. (14) for k, we have

$$k=\frac{S\,(v/c)}{(1-v^2/c^2)^{\frac{1}{2}}} \tag{15}$$

For k to be > 1, $v\,(1+S^2)^{\frac{1}{2}} > c$. The linear momentum per unit mass is given as

$$P\,(S)=\frac{v}{(1-v^2/c^2)^{\frac{1}{2}}}=\frac{kc}{S}, \tag{16}$$

which decreases with expansion. Eq. (2) and (13) together enable one to study the radial motion of a particle while inside the white hole. As long as the expansion proceeds, eq (13) can be used in the form:

$$\Sigma_1=\int_{r_1}^{r_2}\frac{\alpha^{\frac{1}{2}}dr}{(1-\alpha r^2)^{\frac{1}{2}}}=k\int_{S_1}^{S_2}\frac{dS}{[S\,(1-S)\,(k^2+S^2)]^{\frac{1}{2}}}, \tag{17}$$

while the particle is in motion between the epochs S_1 and S_2. For the sake of convenience, one may choose $r_1=0$ and $r_2=r_b$ so that

$$\Sigma_1=\Sigma=\sin^{-1}\,(\alpha r_b{}^2)^{\frac{1}{2}}=\text{const}$$

The upper limit on the integral on the right hand side of the eq. (17) would then refer to the epoch of emergence, i.e., one of reaching the white hole boundary. By fixing constant values for S_1, Σ and k, one can find S_2 by iteration. If $S_2 \simeq 1$, the particle reaches the white hole surface when the expansion is just complete. It can be seen from the eq. (16) that the particle continues to move along a geodesic even after the expansion is complete once it is set into peculiar motion inside the white hole, unless brought to rest by irregular forces like gravitational scattering etc. Eq. (17) can be rewritten in terms of the transformation

$$S=\sin^2 n,\quad (0 \leqslant n \leqslant \pi/2)$$

$$\Sigma = 2k\int_{n_1}^{n_2}\frac{dn}{(k^2+\sin^4 n)^{\frac{1}{2}}} \tag{17'}$$

We find that the approximations to eq. (17') in the limits $k >> 1$ and $k << S_1$ are

$$\Sigma \simeq \left[2n-\frac{1}{k^2}\left(\frac{3n}{8}-\frac{3}{8}\sin n\cos n-\tfrac{1}{4}\sin^3 n\cos n\right)\right]\Bigg|_{n_1}^{n_2} \tag{18a}$$

and

$$\Sigma \simeq \frac{2k\sin(n_2-n_1)}{\sin n_1 \sin n_2} \tag{18b}$$

respectively. As $k\rightarrow\infty$ ($v\rightarrow c$), $\Sigma\rightarrow 2(n_2-n_1)$, or,

$$n_2 \rightarrow \frac{\Sigma+2n_1}{2} \tag{19}$$

Thus when $n_1 < \frac{\Sigma}{2}$, the particle reaches the boundary of the white hole before the latter crosses the event horizon (at the epoch $S(t_o) = \sin^2\Sigma$). In the extreme case, i.e., $\Sigma = \pi/2$ (expansion complete at the event horizon, the white hole is grey), for n_2 to be $\leqslant \pi/2$, n_1 should be $\leqslant \pi/4$. The sign of equality refers to the case when the particles and the white hole boundary reach the event horizon simultaneously.

For $k << \sin^2 n_1$, i.e., for a slowly moving particle, n_2 would be quite larger than n_1 and eq (18b) suggests

$$\Sigma \simeq 2k\cot n_1;\quad n_2 \simeq \pi/2. \tag{20}$$

For instance, if $\Sigma = \pi/100$ and $k \simeq 0.005$, $n_1 \simeq 18°$.

In case a particle set into peculiar motion at an instant when the white hole expansion is nearing completion, or, a slowly moving particle, reaches the boundary of the white hole at epochs later than $n_2 = \pi/2$, eq. (17) should rather be replaced by

$$\Sigma = \int_{n_1}^{\pi/2}\frac{2k\,dn}{(k^2+\sin^4 n)^{\frac{1}{2}}} + kc\left(\frac{a}{1+k^2}\right)^{\frac{1}{2}}\int_{t_o}^{t'} dt \tag{21}$$

The second part here arises by setting $S=1$ in eq. (17) and it is presumed that the white hole remains stable at least for a period t' such that $t_0 < t' \gtrsim t_1$, where t_1 is not arbitrarily large. In the limit $k >> 1$ (which means that $n_1 \simeq n_2 = \pi/2$),

$$\Sigma \rightarrow 2\left(\frac{\pi}{2}-n_1\right)+ca^{\frac{1}{2}}(t'-t_o) \tag{22a}$$

i.e.

$$t' \rightarrow \frac{2t_o}{\pi}(\Sigma+2n_1-\pi/2) \simeq \frac{2t_o}{\pi}\left(\Sigma+\frac{\pi}{2}\right) \tag{22b}$$

For the case of a slowly moving particle ($k << \sin^2 n_1$),

$$t' \simeq \frac{2t_o}{\pi}\left(\Sigma-2k\cot n_1+\frac{\pi}{2}\right) \tag{23}$$

4. The Frequency Shift of Radiation

It has been shown by Narlikar and Apparao (1975) and Narlikar and Kapoor (1978) that leakage of radially and nonradially emitted photons respectively through the event horizon of a white hole is possible even before its surface bursts out of the horizon. Can a radiating particle ejected from the boundary of the white hole also be visible before it comes out of the event horizon ?

For this, let us consider the situation when a particle ejected with a peculiar speed v in the frame of reference of an observer comoving with the outgoing white hole matter instantaneously happens to be at the boundary of the white hole. What is the frequency shift of a photon radiated by the particle in the radial direction according to a distant observer ? For radial emission, we focus our attention on an ejection along the sight line through $R = 0$ toward the observer. Eq. (9) suggests that for such a particle

$$\left.\frac{dR}{ds}\right|_{r=r_b} = \left.\left(\frac{r\,(dS/dt)}{c}\frac{dt}{ds} + S\frac{dr}{ds}\right)\right|_{r=r_b} = \left.\frac{1}{S}\left[(k^2+S^2)^{1/2}\,\frac{r\,(dS/dt)}{c} + k\,(1-\alpha r^2)^{1/2}\right]\right|_{r=r_b} \tag{24}$$

and

$$\left.\frac{dT}{ds}\right|_{r=r_b} = \left.\left(\frac{\partial T}{\partial t}\frac{dt}{ds} + \frac{\partial T}{\partial r}\frac{dr}{ds}\right)\right|_{r=r_b} = \left.\left[\frac{\Phi_1}{S\,(dS/dt)}\left(1+\frac{k^2}{S^2}\right)^{1/2} + \frac{\Phi_1\,kr}{(1-\alpha r^2)\,S^2}\right]\right|_{r=r_b} \tag{25}$$

where $\Phi_1 = \frac{d\phi}{dA}$ and $\frac{dR}{ds} = \frac{r_b\,(dS/dt)}{c}$ refers to the white hole boundary. Now, the frequency shift is given by a formula due to Schroedinger (1950)

$$1+z = \frac{\nu_0}{\nu} = \frac{\vec{u}.\vec{p}\,\big|_{\text{source}}}{\vec{u}.\vec{p}\,\big|_{\text{observer}}} \tag{26}$$

where ν_0 is the frequency at the time of emission, ν that at reception and

$$U^{\alpha}{}_{obs} = (1,0,0,0),\quad U^{\alpha}{}_{source} = \left(\frac{dT}{ds}, \frac{dR}{ds}, 0, 0\right)$$

$$P_{\alpha} = \left(\frac{dT}{d\lambda}, \frac{dR}{d\lambda}, 0, 0\right),\quad c\,\frac{dT}{d\lambda} = \frac{\gamma'}{(1-R_s/R)},\quad \frac{dR}{d\lambda} = \gamma';\quad R_s = \frac{2GM}{c^2} \tag{27}$$

In this equation, λ is an affine parameter and P_{α} refers to the 4-momentum of the photon. A little algebraic manipulation leads one to

$$1+z = \frac{\Delta \sin n_2}{\sin(n_2+\Sigma)},\quad \Delta = \frac{(k^2+\sin^4 n_2)^{1/2} - k}{\sin^2 n_2} \tag{28}$$

For $k = 0$, eq. (28) refers to the frequency shift of the radially emitted photons from the white hole surface (Narlikar and Apparao 1975). For $k/\sin^2 n_2 << 1$

$$1+z \simeq (1 - k\,\text{cosec}^2 n_2)\,\frac{\sin n_2}{\sin(n_2+\Sigma)}, \tag{29a}$$

while if $k/\sin^2 n_2 >> 1$, $\Delta \simeq \sin^2 n_2/2k$ and the result is

$$1+z \simeq \frac{\sin^3 n_2}{2k \sin(n_2+\Sigma)} \tag{29b}$$

This suggests a highly severe frequency blue shift [$(1+z)^{-1}$] compared to that for radiation from the white hole surface. It is obvious from eq. (28) that at $R_b = R_s = 2GM/c^2$, $(1+z)^{-1}$ is finite and larger than that for the white hole radiation.

Now we consider the ejection of the particle in an arbitrary direction. There would always be photons with a certain value of impact parameter p to be able to reach infinity. The Schwarzschild metric and equations of motion suggest that for a nonradially emitted photon,

$$\frac{dR}{d\lambda} = \gamma' e^{-(\lambda+\nu)/2} F;\ F = F(R,p) = \left(1 - \frac{e^{\nu} p^2}{R^2}\right)^{\frac{1}{2}}$$

$$\frac{d\phi}{d\lambda} = \frac{h}{R^2},\ p = \frac{h}{\gamma'},\ p^{\alpha} = \left(\frac{dT}{d\lambda}, \frac{dR}{d\lambda}, 0, \frac{d\phi}{d\lambda}\right), \tag{30}$$

where h is the orbital angular momentum parameter for the photon; F = 1 for radial and F = 0 for tangentially emitted photons. Then, the frequency shift of a nonradially emitted photon as according to eq. (26) is

$$1+z = \left[\left(1+\frac{k^2}{S^2}\right)^{\frac{1}{2}} \frac{\left\{(1-ar^2)^{\frac{1}{2}} - \frac{r\,(dS/dt)}{c} F\right\}}{(1-R_s/R)} + \frac{k}{S} \frac{\left\{\frac{r\,(dS/dt)}{c} - F\,(1-ar^2)^{\frac{1}{2}}\right\}}{(1-R_s/R)}\right]\Bigg|_{r=r_b} \tag{31}$$

which is less severe than that suggested by eq. (21). For k = 0, this reduces to the result derived by Narlikar and Kapoor (1978). Eq. 31 also is well behaved at $R_b = R_s$. To demonstrate this, we let $R_b \to R_s$, i.e., $F \to 1 - e^{\nu} p^2/2R_s^2$, so that

$$1+z\Big|_{R_b \to R_s} \to \frac{\Delta}{2\cos\Sigma} + \frac{p^2 \cos\Sigma}{2R_s^2}\left(\Delta + \frac{2k}{\sin^2 n_2}\right) \tag{32}$$

which is finite. If $k/\sin^2 n_2 >> 1$,

$$1+z\Big|_{R_b \to R_s} \to \frac{k \cot\Sigma\ \mathrm{cosec}\,\Sigma\ p^2}{R_s^2} \tag{33a}$$

In order that a null ray have a blue shift, the corresponding impact parameter can be found out by the following considerations. We multiply eq. (31) by a factor

$$\left[\left(1+\frac{k^2}{S^2}\right)^{\frac{1}{2}}(1-ar^2)^{\frac{1}{2}} + \frac{k}{S}\frac{r\,(dS/dt)}{c} + F\left\{\left(1+\frac{k^2}{S^2}\right)^{\frac{1}{2}}\frac{r\,(dS/dt)}{c} + (1-ar^2)^{\frac{1}{2}}\frac{k}{S}\right\}\right]\Bigg|_{r=r_b} \tag{33b}$$

in the numerator and denominator. The factor $(1-R_s/R_b)$ cancels out. The result is

$$1+z = \frac{1+\left[\frac{p}{R}\left\{\left(1+\frac{k^2}{S^2}\right)^{\frac{1}{2}}\frac{r\,(dS/dt)}{c} + (1-ar^2)^{\frac{1}{2}}\frac{k}{S}\right]^2}{\left[\left(1+\frac{k^2}{S^2}\right)^{\frac{1}{2}}(1-ar^2)^{\frac{1}{2}} + \frac{k}{S}\frac{r\,(dS/dt)}{c} + F\left\{\left(1+\frac{k^2}{S^2}\right)^{\frac{1}{2}}\frac{r\,(dS/dt)}{c} + (1-ar^2)^{\frac{1}{2}}\frac{k}{S}\right\}\right]}\Bigg|_{r=r_b} \tag{33c}$$

Now we let $R_b \to R_s$. Then $(1-ar_b^2)^{\frac{1}{2}} \to r_b\,(dS/dt)/c$ and $F \to 1$; consequently,

$$1+z = \frac{1+\left[\frac{p}{R_b}(1-ar_b^2)^{\frac{1}{2}}\left\{\left(1+\frac{k^2}{S_2^2}\right)^{\frac{1}{2}} + \frac{k}{S_2}\right\}\right]^2}{2\,(1-ar_b^2)^{\frac{1}{2}}\left[\left(1+\frac{k^2}{S_2^2}\right)^{\frac{1}{2}} + \frac{k}{S_2}\right]} \tag{33d}$$

For $S << 1$, $1 - ar_b^2 \simeq ar_b^2/S$. Hence, requiring $(1+z)^{-1} \geq 1$ we are led to

$$p^2 \lesssim \frac{S_2}{k} r_b^2 S_2^2 \left(\frac{k}{S_2} >> 1\right) \tag{34a}$$

This is considerably smaller than that for a nonradial photon from the white hole surface ($k=0$). For $k << S_2$, a similar consideration leads us to

$$p^2 \lesssim \left(1-\frac{2k}{S_2}\right) r_b^2 S_2^2 \tag{34b}$$

This is about the same as that for nonradial photons from the white hole surface ($p \lesssim r_b S$). Eqs. (34) apply only to epochs when the particle just leaves the surface of the white hole. In the exterior of the white hole, Schwarzchild geometry governs the propagation of photons. Photons emitted at an angle

$$\delta < \delta_0 = \sin^{-1}\left[\frac{3.3^{1/2} R_s}{R_l}\left(1-\frac{R_s}{R_l}\right)^{1/2}\right] \tag{35}$$

when the particle is at $R = R_l$ from the center of the white hole, are captured by the latter. Here $\delta = \pi$ refers to a radial photon going into the white hole and $\delta = 0$ for one going outwards. Photons emitted at the capture angle $\delta = \delta_0$ i.e., ones with an impact parameter $= 3.3^{1/2} R_s/2$, contribute to the ring of emission formed round the white hole by the nonradially emitted photons from the latter's surface. The ring persists till the white hole boundary occupies the region $R = 3GM/c^2$.

In the exterior of the white hole, the Schwarzschild metric suggests the frequency shift of radiation from the particle to be

$$1+z \simeq \left(\gamma+\frac{dR}{ds}\cos\theta\right)^{-1}, \quad \frac{dR}{ds} = \left[\gamma^2-\left(1-\frac{R_s}{R}\right)\right]^{1/2} \tag{36}$$

for ejection at an angle θ with respect to the line of sight through $R=0$. Here γ is the energy of the particle per unit rest mass as measured at infinity which can be evaluated with reference to the position of the particle when instantaneously coincident with the white hole boundary. Comparing the last equation with eq. (24), we have

$$\gamma(\eta_2) = \frac{1}{\sin^2\eta_2}\left[(k^2+\sin^4\eta_2)^{1/2}\cos\Sigma + k\sin\Sigma\cot\eta_2\right] \tag{37a}$$

If $k/S_2 >> 1$,

$$\gamma(\eta_2) \simeq \frac{k}{\sin^2\eta_2}\left[\frac{\sin(\eta_2+\Sigma)}{\sin\eta_2}\right] >> 1 \tag{37b}$$

Eq. (36) would then suggest

$$(1+z)^{-1} \simeq \gamma(1+\cos\theta) = \text{const} \tag{38}$$

and, from Liouville's theorem, it can be seen that intensity enhancement takes place when

$$\cos\theta > \left(\frac{1}{\gamma}-1\right) \tag{39}$$

If the particle is ejected at an epoch $S_2 \simeq 1$ or after the expansion of the white hole is complete,

$$\gamma\left(\frac{\pi}{2}\right) = (1+k^2)^{1/2}\cos\Sigma \tag{40}$$

Hence, in general,

$$\gamma(\eta_2) > \gamma(\pi/2) \tag{41}$$

5. The Case of a Grey Hole

In the case of a grey hole, the expansion is complete at $R_b = R_s$, so that

$$a r_b^2 = \sin^2\Sigma = 1 \tag{42}$$

The frequency shift for a photon is given as

$$1+z = \frac{1+\frac{p^2}{R^2}\left[\frac{1-S}{S}\right]}{F\left[\frac{1-S}{S}\right]^{1/2}} \tag{43}$$

A radial photon from the grey hole surface, for instance, will be blueshifted when $r_b\,(dS/dt) \gtrsim 1$ or $n \lesssim \pi/4$.

The radial photons from a particle in peculiar motion suffer a frequency shift amounting to

$$1+z = \Delta \tan n_2$$

as according to eq. (28). For this particle,

$$\left.\frac{dR}{ds}\right|_{r=r_b} = \frac{r\,(dS/dt)}{c}\left(1+\frac{k^2}{S^2}\right)^{\frac{1}{2}}\Bigg|_{r=r_b} \tag{44}$$

Therefore, if the particle reaches the grey hole boundary just when the expansion is complete, $\frac{dR}{ds} \to 0$ and $(1+z) \to \infty$, i.e., neither the particle nor the photons it emits can leak through the Schwarzschild barrier. The photons and the particle leak through only when $S_2 < 1$, since then the eqs. (33c) and (44) suggest that $(1+z)$ and dR/ds are finite.

Our studies have been based on an ideal white hole model. However, it is a crude approximation to the white hole explosions that in general would be anisotropic, the expansion may be nonhomogeneous and the exterior spacetime may quite possibly be non-empty. Can matter ejection, in the form of high energy particles and lumps of gaseous masses, from a white hole be possible? The possibility at least in principle exists. Although our results apply to test particles, they should as well be applicable to lumps of masses negligible compared to that of the white hole in the first approximation. Considering a simple geometry, the speed of separation between the particle and the white hole can be shown to be superluminal for $\gamma \gtrsim 1$ (see, *e.g.* Rose 1973). As k is positive definite, the frequency shift in the radiation from the white hole and the ejected particle would be different. In principle, the velocity of ejection of a particle from the white hole surface should be $0 \lesssim v \lesssim c$; although the condition $\gamma \gtrsim 1$ is easily met for ejection during the expansion phase of the white hole, our analysis would not be adequate for large ejecta masses. When ejection is not violent and the ejectum carries an insignificant fraction of the white hole mass, its emergence from the white hole boundary would appear as a flare phenomenon. An answer to the question whether a white hole can develop into a multi-component object with its components (of comparable luminosity) separating from each other at appreciable speeds and with different frequency shifts can, however, come from a many-body treatment of the problem only.

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