Kodaikanal Obs. Bull Ser A (1978) 2, 107-114

AN ANALYSIS OF BABINET COMPENSATOR FRINGES USING JONES CALCULUS

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ABSTRACT

Following Jones Calculus, a comprehensive theory of the Babinet Compensator has been presented The performance of the system governed by the compensator polaroid unit has been evaluated in terms of fringe visibility Criteria have been evolved for the proper choice of the compensator and the polaroid pair for optimising the accuracy of measurements

Key Words Babinet compensator—Jones calculus—polarization

1 Introduction

The Mueller and Jones calculi are two powerful tools available for the analysis of problems involving polarised light and optical devices A detailed account of mathematical formulations, deductions and analysis has been given by Shurcliff (1962) A comparative study (Park, 1948a, 1948b, 1949a, 1949b) of these techniques reveals that

i) By following Jones calculus it is possible to retain information about the absolute phase This facility is not provided by Mueller calculus

ii) The Jones calculus is well suited to the study of interference of coherent beams Mueller calculus on the other hand presents difficulties in dealing with similar problems

iii) The Jones calculus is better suited to the study of problems involving a train of optical devices

iv) Problems involving depolarisation devices can, however, be handled only with the help of Mueller calculus

It, therefore, follows that the choice between the two would depend on the problem that needs a solution However, for analysis of a problem involving polarised light, the Jones calculus provides a simple and systematic approach

The Babinet compensator has been known as a sensitive device for measurement of small phase changes in polarised light It has been shown that, with suitable precautions, the minimum detectable phase change (Jerraid 1949) may be made as low as 2π milliradians Such phase changes may *either* result from variations in the nature of polarised light incident on the compensator (as for example, when the source is located in a region of changing magnetic field) *or* from small changes in the direction of light propagation through the compensator While the former circumstance has been exploited for the study of sunspots (Treanor, 1960, Adam, 1963, Adam, 1967, Adam, 1969, Adam, 1971) the latter has been recently (Saxena and Srivastava, 1973) employed to determine the magnitude of refractive index gradients in transparent media

A high accuracy of measurement can, however, be attained only if careful attention is paid to several aspects of the instrumental combination The basic unit of the relevant optical system, namely the compensator-polaroid unit has, therefore, to be studied in detail for optimising the working conditions

2. Babinet Compensator

The compensator consists of two quartz wedges of equal angles kept with their hypotenuse faces opposite each other so as to form a plane-parallel plate The optic axes in the two wedges are mutually at right angles and also perpendicular to the incident beam When such a plate is placed (Figure 1) between two crossed polaroids the field of view is seen

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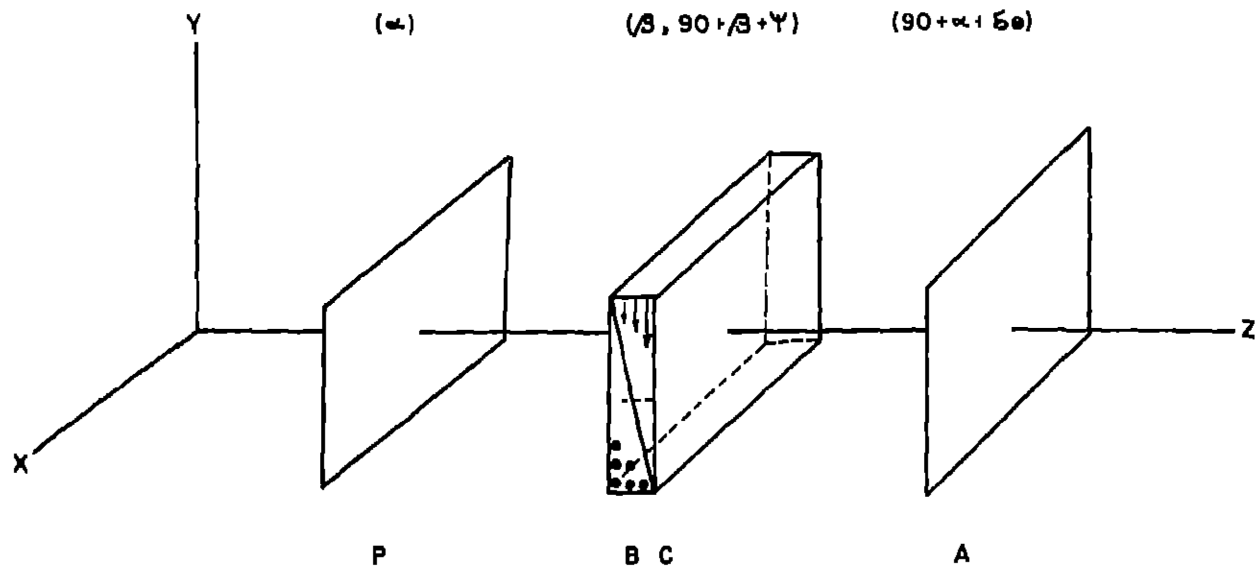

Fig 1 Schematic arrangement of Babinet Compensator

to be covered with a set of equally spaced dark and bright fringes The spacing between these fringes depends upon the azimuth of the compensator (the angle between the optic axis of the first wedge and the X-axis).

In the case of an ideal set of polarisers and accurate fabrication and mounting of the compensator wedges and other optical elements, the theory of the instrument (Jerrard, 1948, Born and Wolf 1959) is fairly straightforward For a ray entering the compensator at $y = 0$, the lengths of path h_1 and h_2 through the two wedges are equal ($h_1 = h_2 = h$) Hence, the phase difference $2\pi/\lambda\,(n_e - n_o)h_1$ between the ordinary and extraordinary vibrations introduced by the first wedge is balanced by an equal and opposite difference $2\pi/\lambda\,(n_o - n_e)h_2$ due to the second wedge The two vibrations, therefore, emerge from the compensator without any change of phase and combine to give linearly polarised light with its electric vector parallel to the transmission-plane of the polariser The same conditions are fulfilled on either side of the $y = 0$ plane, where

$$\delta = \delta_1 + \delta_2 = \frac{2\pi}{\lambda}(n_e - n_o)\,(h_1 - h_2) = 2n\pi \qquad (1)$$

and n is an integer The corresponding points on the face of the compensator, away from the incident light, therefore, appear dark through a crossed analyser.

In general, the phases δ_1 and δ_2 (numerical values) at points along the planes $y = 0$ will be given by

$$\delta_1 = 2\gamma_1 = 2\pi/\lambda\,(n_e - n_o)\,(h - y\tan A) \qquad (2)$$

$$\delta_2 = 2\gamma_2 = 2\pi/\lambda\,(n_e - n_o)\,(h + y\tan A) \qquad (3)$$

where A is the angle of wedge For certain values of y, the total phase change $\delta = (2n + 1)\pi$ If the Babinet compensator is kept with its axes at 45° to the incident light vector, all those point (at regular intervals of y) will have maximum brightness when seen through the crossed analyser Thus, under these conditions, the fringes would appear to have maximum visibility

3 Performance of the Compensator

Such conditions are, however, rarely available in practice Consequently in any accurate quantitative application the various possible defects have to be taken into account and their effects on the performance of the compensator have to be evaluated Those defects generally fall into one or more of the following categories

i) Defective Polaroids

For a perfect pair of polariods $P_{\parallel}$ and $P_{\perp}$ (transmitted intensities when the polaroids are respectively parallel and crossed) have the values one and zero, In practice such a pair, is difficult to come across.

ii) Misalignment of Polaroids

Even with a perfect pair, the polaroids mayn ot be exactly at 90° when crossed or at zero degrees when parallel. This situation results from inaccurate settings and may be eliminated, at least in theory, by a skilled experimenter.

iii) Fabrication Defects

In addition, the fabrication of the Babinet Compensator may be itself faulty. Such fabrication defects may be the result of

a) Unequal wedge angles

b) Non-Parallelism of the optic axes to basal planes of the wedges

c) Near perpendicularity of the two optic axes

In view of these possibilities, a more elaborate treatment of the Babinet fringes, with all these factors taken into account appears to be worthwhile. It may be, however, mentioned that Jerrard (1949a, 1949b) using the classical treatment, has considered the fabrication defects in some detail. Therefore, following Jones' Calculus (Shurcliff, 1962) a more general treatment, taking all the factors into account, has been presented. It may be added that the effect of defective polaroids on the performance of the compensator is otherwise difficult to handle.

4 Babinet fringes using Jones Calculus

Let a plane-polarised beam of monochromatic light represented by

$$E_x = E_{x0} \exp i\left(\omega t - \frac{2\pi z}{\lambda} + \delta_x\right) = E_{x0} \exp i\phi_x$$

$$E_y = E_{y0} \exp i\left(\omega t - \frac{2\pi z}{\lambda} + \delta_y\right) = E_{y0} \exp i\phi_y$$

falls successively on a polariser, a Babinet compensator and an analyser with their faces normal to it and further we assume that the transmission axis of the polariser makes an angle α and the optic axis of the first wedge of the compensator makes an angle β with the X-axis.

In the ideal case, the second wedge will be oriented with its optic axis at an angle $(90+\beta)$ to the axis under reference. But, as outlined above, this angle will be taken as $(90+\beta+\psi)$ where ψ can have both positive and negative values. Also, when the polaroids are perfectly crossed the angle between the analyser's transmission axis and the axis of X will be $(90+\alpha)$. The case of imperfect crossing of polaroids can be included by taking this angle as $(90+\alpha+\delta\theta)$ where $\delta\theta$ is the angular misalignment of the polaroids and which may again have negative or positive values.

The resultant emergent intensity can be calculated with the help of the appropriate matrices for different components following Jones Calculus. Thus the chain of matrices for the above system may be written as

$$S\{(90+\alpha+\delta\theta)\}N_2S\{-(90+\alpha+\delta\theta)\}S\{(90+\beta+\psi)\}N'_2 \tag{4}$$

$$S\{-(90+\beta+\psi)\}S(\beta)N_1'S(-\beta)S(\alpha)N_1S(-\alpha)\begin{bmatrix}E_x\\E_y\end{bmatrix}=M\begin{bmatrix}E_x\\E_y\end{bmatrix}$$

where the S matrices are the standard rotator matrix (Shurcliff, 1962). Also,

$$N_1=\begin{bmatrix}P_{1\parallel} & 0\\ 0 & P_{1\perp}\end{bmatrix}, \quad N_2=\begin{bmatrix}P_{2\parallel} & 0\\ 0 & P_{2\perp}\end{bmatrix}$$

$$N_1'=\begin{bmatrix}\exp(i\gamma_1) & 0\\ 0 & \exp(-i\gamma_1)\end{bmatrix}, N_2'=\begin{bmatrix}\exp(i\gamma_2) & 0\\ 0 & \exp(-i\gamma_2)\end{bmatrix}$$

where $P_\parallel$ and $P_\perp$ are respectively the transmissions of the two polaroids indicated by the subscripts 1 and 2 along and perpendicular to their transmission axes. On substituting the values of S and N matrices (Shurcliff, 1962) and appropriate reduction, the M matrix becomes

$$M = A \times B \times C \times D \times E \times F \times G \times H \times I$$

where

$$A=\begin{bmatrix}\cos\eta & \sin\eta\\ \sin\eta & \cos\eta\end{bmatrix} \quad B=\begin{bmatrix}P_{2\parallel} & 0\\ 0 & P_{2\perp}\end{bmatrix}$$

$$C=\begin{bmatrix}-\sin(\alpha+\delta\theta) & -\cos(\alpha+\delta\theta)\\ \cos(\alpha+\delta\theta) & -\sin(\alpha+\delta\theta)\end{bmatrix}$$

$$D=\begin{bmatrix}\exp(i\gamma_1) & 0\\ 0 & \exp(-i\gamma_1)\end{bmatrix} \quad E=\begin{bmatrix}-\sin\psi & \cos\psi\\ -\cos\psi & -\sin\psi\end{bmatrix}$$

$$F=\begin{bmatrix}\exp(i\gamma_2) & 0\\ 0 & \exp(-i\gamma_2)\end{bmatrix}$$

$$G=\begin{bmatrix}\cos(\alpha-\beta) & \sin(\alpha-\beta)\\ \sin(\alpha-\beta) & \cos(\alpha-\beta)\end{bmatrix} \quad H=\begin{bmatrix}P_{1\parallel} & 0\\ 0 & P_{1\perp}\end{bmatrix}$$

$$I=\begin{bmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix} \tag{5}$$

where

$$\eta = \{(\beta+\psi)-(\alpha+\delta\theta)\} \tag{6}$$

on simplification, this reduces to a single matrix of the form

$$\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix} \tag{7}$$

5 Ideal case

Considering the case when the polaroids are perfect and there are no fabrication defects in the Babinet compensator (kept at an azimuth zero with respect to the X-axis)

$$P_{1\parallel} = P_{2\parallel} = 1,\ P_{1\perp} = P_{2\perp} = 0,\ \beta = \psi = 0 \text{ and } \delta\theta = 0$$

the matrix elements reduce to

$$a_{11} = [\exp\{i(\gamma_1-\gamma_2)\} - \exp-\{i(\gamma_1-\gamma_2)\}]\cos^2\alpha\sin^2\alpha$$

$$a_{12} = [\exp\{i(\gamma_1-\gamma_2)\} - \exp-\{i(\gamma_1-\gamma_2)\}]\sin^3\alpha\cos\alpha$$

$$a_{21} = [\exp\{i(\gamma_1-\gamma_2)\} - \exp-\{i(\gamma_1-\gamma_2)\}]\cos^3\alpha\sin\alpha$$

$$a_{22} = -[\exp\{i(\gamma_1-\gamma_2)\} - \exp-\{i(\gamma_1-\gamma_2)\}]\cos^2\alpha\sin^2\alpha$$

The intensity of the emergent beam can hence be determined by taking the sum of the products of these matrix elements with their complex conjugates. It can thus be shown that under ideal circumstances the emergent intensity I is given by the standard result

$$I = \sin^2 2\alpha \sin(\gamma_1 - \gamma_2) \qquad (8)$$

6 Case of imperfect polaroids

Consider now the cases of imperfect polaroids, where all the other factors may be taken to be ideal and hence, it may be assumed that $\alpha = 45°$, $\beta = \psi = 0$, and $\delta\theta = 0$. In this situation the matrix elements are given by the expressions

$$a_{11} = \frac{a}{4}\exp\left\{i(\gamma_1-\gamma_2)\right\} + \frac{b}{4}\exp-\left\{i(\gamma_1-\gamma_2)\right\}$$

$$a_{12} = \frac{c}{4}\exp\left\{i(\gamma_1-\gamma_2)\right\} + \frac{d}{4}\exp-\left\{i(\gamma_1-\gamma_2)\right\}$$

$$a_{21} = \frac{d}{4}\exp\left\{i(\gamma_1-\gamma_2)\right\} + \frac{c}{4}\exp-\left\{i(\gamma_1-\gamma_2)\right\}$$

$$a_{22} = \frac{b}{4}\exp\left\{i(\gamma_1-\gamma_2)\right\} + \frac{a}{4}\exp-\left\{i(\gamma_1-\gamma_2)\right\}$$

where

$$a = (P_{1\parallel}P_{2\parallel} + P_{1\parallel}P_{2\perp} + P_{1\perp}P_{2\parallel} + P_{1\perp}P_{2\perp})$$

$$b = -(P_{1\parallel}P_{2\parallel} - P_{1\parallel}P_{2\perp} - P_{1\perp}P_{2\parallel} + P_{1\perp}P_{2\perp})$$

$$c = (P_{1\parallel}P_{2\parallel} + P_{1\parallel}P_{2\perp} - P_{1\perp}P_{2\parallel} - P_{1\perp}P_{2\perp})$$

$$d = (P_{1\parallel}P_{2\parallel} - P_{1\parallel}P_{2\perp} + P_{1\perp}P_{2\parallel} - P_{1\perp}P_{2\perp})$$

Once again, on multiplying these elements with their complex conjugates, adding the products and simplifying, the expression for I is found to be

$$I = \frac{1}{8}\left[a^2 + b^2 + c^2 + d^2 + 2ab\cos 2(\gamma_1-\gamma_2) + 2cd\cos 2(\gamma_1-\gamma_2)\right]$$

Here, it should be mentioned that, on substituting different values of $P_\parallel$ and $P_\perp$ the factors a and c work out to be positive and b and d come out to be negative. Thus the calculated values of I plotted against different values of $(\gamma_1-\gamma_2)$ give the fringe profiles (Figure 2) for different degrees of imperfectness of the polaroids.

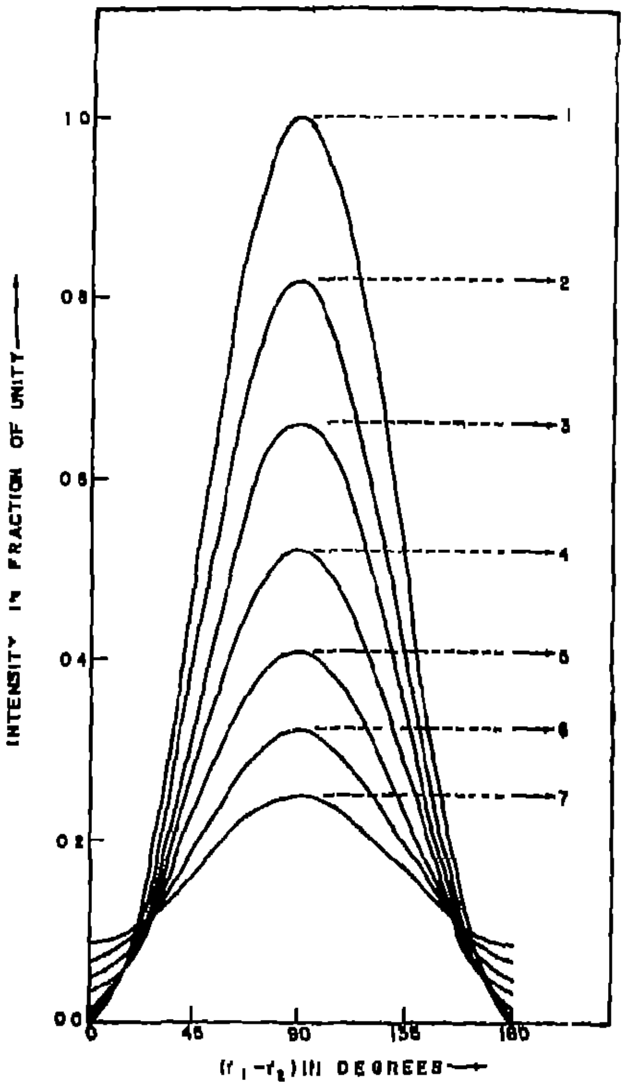

Fig 2 Fringe profiles for different degrees of imperfectness of the polaroids
1 Perfect polaroids, 2 to 7 Imperfectness 5% to 30%

7 Case of imperfect crossing of polaroids

When the polaroids are not perfectly crossed but other alignments are ideal it may be assumed that

$\beta = \psi = 0$ and hence $\eta = -(\alpha + \delta\theta)$

Also, for perfect polaroids

$$P_{1\parallel} = P_{2\parallel} = 1 \text{ and } P_{1\perp} = P_{2\perp} = 0$$

The values of the matrix elements can be again worked out by making these substitutions and the intensity of the emergent beam found out as before. It can be thus shown that

$$I = \cos^2\alpha \sin^2\alpha - 2\sin\alpha \cos\alpha \sin(\alpha+\delta\theta)\cos(\alpha+\delta\theta)$$

$$\cos^2(\gamma_1-\gamma_2) + \cos^4\alpha \sin^2(\alpha+\delta\theta) + \sin^4\alpha \cos^2(\alpha+\delta\theta) \quad (10)$$

With $\alpha = 45°$, Equation (10) reduces to

$$I = \tfrac{1}{2}[1 - \cos 2\delta\theta \cos 2(\gamma_1 - \gamma_2)] \quad (11)$$

This relation represents the effect of an imperfect crossing on fringe profile (Figure 3).

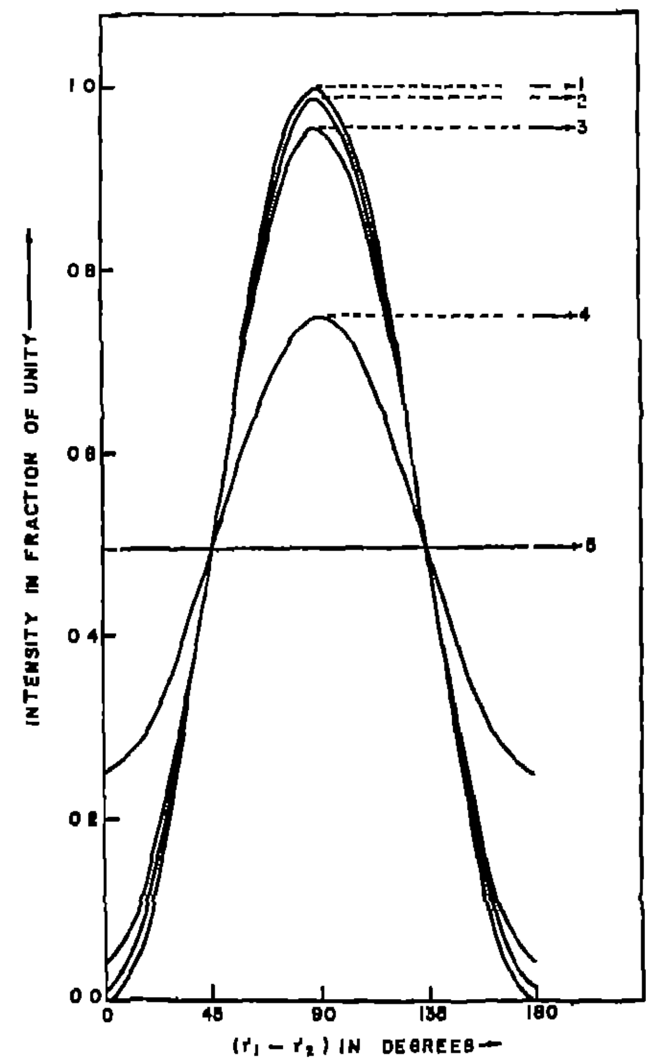

Fig 3 Fringe profile for different errors in cross
1 $\delta\theta = 0°$ 2 $\delta\theta = 6°$ 3 $\delta\theta = 12°$ 4 $\delta\theta = 30°$
5 $\delta\theta = 45°$

8 Visibility of Babinet fringes

It will be seen from equation (8) that, whatever be the values of $(\gamma_1 - \gamma_2)$, the intensity I of the field of view undergoes variations on changing the values of α. Differentiating this expression with respect to α, we find

$$\frac{dI}{d\alpha} = 2\sin 4\alpha \sin^2(\gamma_1-\gamma_2)$$

which becomes zero for $\alpha = 0, \pi/4, \pi/2$ etc. The second differential

$$\frac{d^2I}{d\alpha^2} = 8\cos 4\alpha \sin^2(\gamma_1-\gamma_2)$$

will thus be seen to have positive values for $\alpha = 0$ and $\pi/2$ and a negative value for $\alpha = \pi/4$. Under ideal circumstances, therefore, the field of view will have a uniformly dark illumination when the transmission axis of the polariser and also the analyser is either parallel or perpendicular to the optic axes of the compensator wedges (or *vice versa*). But when the transmission axes happen to be at 45° to the optic axes of the compensator, the intensity of the field attains a maximum value. Under this condition, however, because of the $\sin(\gamma_1 - \gamma_2)$ term the field of view is interspersed with a set of a dark fringes. The visibility of fringes, defined by

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \quad (12)$$

will thus have a maximum value (of one). Accordingly, the Babinet fringes will be most useful when $\alpha = \pi/4$.

With imperfect polaroids, the expressions for I_{max} and I_{min} equation (9) become

$$I_{max} = \frac{1}{8}[a^2 + b^2 + c^2 + d^2 - 2ab - 2cd]$$

for $(\gamma_1 - \gamma_2) = (2m+1)\pi/2$ where m = 0, 1, 2, 3,

and

$$I_{min} = \frac{1}{8}[a^2 + b^2 + c^2 + d^2 + 2ab + 2cd]$$

for $(\gamma_1 - \gamma_2) = 2m\pi/2$ where m = 0, 1, 2, 3,

Thus, the visibility in this case, (Sec 5) is given by

$$V = -\frac{2(ab + cd)}{a^2 + b^2 + c^2 + d^2} \quad (13)$$

The values of V calculated from equation (13), for different amounts of imperfectness in the polaroids, have been given in Table 1 Their plot (Figure 4) shows that the visibility of fringes does not significantly alter for a small (less than 10%) degree of imperfectness of the polaroids, which correspond to values of 0 90 (for $P_{\parallel}$ s) and 0 10 (for $P_{\perp}$ s)

Table 1 Visibility of Babinet fringes for imperfect polaroids

No	Values of $P_{1\parallel}$ & $P_{2\parallel}$	Values of $P_{1\perp}$ & $P_{2\perp}$	e_n Imperfectness	Visibility
1	1 00	0 00	0	1 000
2	0 90	0 10	10	0 950
3	0 80	0 20	20	0 770
4	0 70	0 30	30	0 476
5	0 60	0 40	40	0 190
6	0 50	0 50	50	0 000

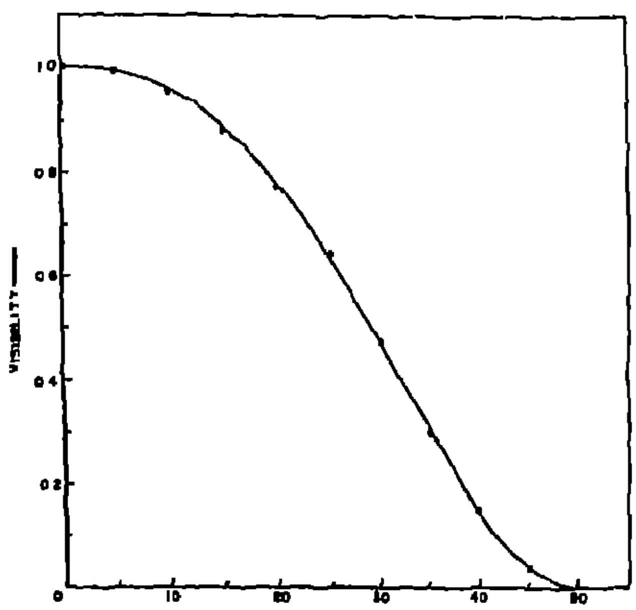

Fig 4 Visibility of Babinet fringes for imperfect polaroids

It may also be noted from equation (9) that the Babinet fringes are still obtained at phase intervals of 2π because the factors a, b, c and d are constant for a pair of polaroids, but their visibility undergoes a change

In the case of imperfect cross (Sec 6) the visibility can be similarly obtained with the help of equation (11)

Thus, with the values of I_{max} and I_{min} given by the expressions

$$I_{max} = \frac{1}{2}(1 + \cos 2\delta\theta)$$

and

$$I_{min} = \frac{1}{2}(1 - \cos 2\delta\theta)$$

the visibility is given by

$$V = \cos 2\delta\theta$$

Table 2 Visibility of Babinet fringes for imperfect cross

No	Misalignment in degrees	Visibility
1	0	1 000
2	10	0 939
3	20	0 766
4	30	0 500
5	40	0 174

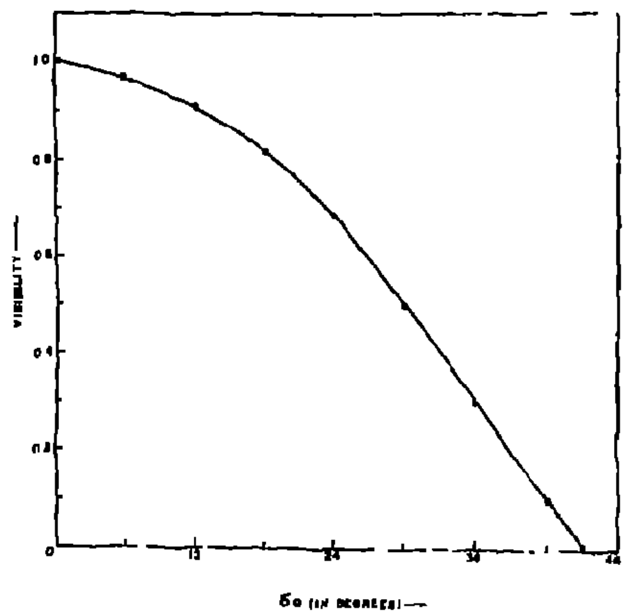

Fig 5 Visibility of Babinet fringes for imperfect cross

The visibility in this case is seen to be directly proportional to cosine of twice the error in the crossing of polaroids For the calculated values (Table 2) the nature of visibility curve is given by Figure (5) Once again as expected, the spacing between the fringes the same as that obtained under ideal circumstances It may be noted that their visibility is not seriously impaired by an error of a few degrees in setting the polaroids in crossed position

9 Fabrication defects

Considering fabrication defects of the compensator, Jerrard (1949a,b) has shown that

a) When the wedge angles are not the same, the two wedges joined together do not form a parallel plate Even the normally incident light, therefore, emerges at a non-zero angle Accordingly, the position of the central dark fringe is shifted from the symmetrical and the spacing of the fringes alters This spacing, however, remains uniform over the field of view

b) When the optic axes are not parallel to the basal planes (but are still mutually perpendicular), the light falling normally on the compensator will be travelling at an angle slightly less (or greater) than $\pi/2$ Under these circumstances, the birefringence ($n_e - n_o$) of the wedges has to be replaced by ($n_e - n_o$) $\sin^2\theta$ for the purpose of calculating the phase changes introduced by each wedge

Thus

$$\delta_1 = \frac{2\pi h_1}{\lambda} (n_e - n_o) \sin^2\theta$$

and

$$\delta_2 = \frac{2\pi h_2}{\lambda} (n_e - n_o) \sin^2\theta$$

so that

$$\delta = \frac{2\pi}{\lambda} (h_1 - h_2) (n_e - n_o) \sin^2\theta$$

It can hence be seen that the dark fringes obtained for $\delta = \pm 2m\pi$ are still formed at regular intervals although the new fringe spacing is slightly larger than that in the ideal case $\theta = \pi/2$

Jerrard has shown that such a defect, if present in the compensator, gives rise to a set of fringes across the field of view if the azimuth of the wedges makes an angle of zero or $\pi/2$ with the transmission axis of the polariser (and hence $\pi/2$ or zero with the axis of the analyser) This test can be, therefore, utilized to select or fabricate a compensator suitable for use in studies mentioned in Sec 2

c) If the two wedges are cut in such a way that when mounted together, the axes are not exactly perpendicular to each other but make an angle ($\pi/2 + \psi$), then for small values of ψ, the intensity of the light emerging from the analyser is given by

$$I = a^2 \sin^2 2\beta + b^2 \sin^2 2(\beta + \psi) - 2ab\,(AB + \cos 2\psi) \sin^2 \beta \sin 2(\beta + \psi)$$

where

$$a = \sin\gamma_1 \quad b = \sin\gamma_2 \quad A = \cos\gamma_1 \quad B = \cos\gamma_2$$

and the intensity of light emerging from the polarizer is taken to be unity

It was shown that when the azimuth of the first wedge is at an angle of zero or $\pi/2$ to the transmission axis of the polarizer, a series of fringes is obtained across the field of view, instead of complete extinction This test can be used once again for choosing a suitable compensator

It may be, nevertheless mentioned that the effects on the non-parallelity of the compensator faces (case a) and the non-perpendicularity of the optic axes (case c) are less apparent when the compensator is placed at an azimuth of 45°

10. Conclusions

Thus, Jerrard's findings, along with the results of the present analysis provide guide lines that cover selection, fabrication and the use of the compensator The main points may be summarized as follows

1 The compensator should be examined for the presence of fabrication defects. These are readily revealed, by the appearance of fringes when it is mounted at an azimuth of zero or $\pi/2$ with respect to the crossed polaroids It should then be arranged at an azimuth of $\pi/4$ with respect to the polaroids, for optimum sensitivity

2 The polaroid pair should be carefully chosen so as to satisfy the criterion that the values of $P_{\parallel}$ and $P_{\perp}$ (for the unit) should be of the order of 0 90 and 0 10 for the range of wavelength used in the investigation

3 The accurate setting of the polaroids (in the crossed position) is, however, not so critical An angular misalignment of a few degrees is not expected to materially effect the performance of the system

Acknowledgment

The authors are thankful to Dr M K V Bappu for his continued interest and encouragement

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