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EQUATION OF STATE OF SUPERNOVA MATTER

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1. Introduction

The equation of state (EOS) of hot, dense matter, with temperature (kT) $\sim$ (1-10) MeV and density (ρ) up to $\sim 10^{14}$ g cm^{-3}, is an important factor that characterizes the gravitational collapse of a star leading to a supernova explosion (Datta & Deo 1983). A major constituent of the pre-supernova collapse matter is finite nuclei. Their presence influences the diffusion of neutrinos produced during the collapse and also determines the maximum density up to which the collapse process will continue. Therefore, a proper understanding of excited nuclei is now regarded as essential in any realistic supernova modelling calculation. This article is aimed at providing a brief review of the EOS of hot, dense matter with particular reference to the treatment of excited nuclei.

2. The Equation of State

The determination of the EOS consists in evaluating the pressure as a function of temperature and density of the system. The contribution to pressure that would come from electrons, neutrinos and photons in the collapsing matter can be treated simply and quite accurately using the standard formulae for non-interacting Fermi and Bose gases (see e.g. Landau & Lifshitz 1958). In contrast, the treatment of hot nuclei, immersed in a (non-degenerate) nucleon (i.e. neutron and proton) gas, is problematic because of uncertainties regarding (a) the effect of temperature on nuclear surface, (b) the effect on the nuclei of neutrons and protons external to the nuclei and (c) the effect of nuclear excited energy states for $kT \gtrsim 5$ MeV.

A detailed calculation by Bethe et al. (1979) indicates that in the stellar collapse beyond $\rho \simeq 10^{12}$ g cm^{-3}, the entropy per baryon (S/A) and the total lepton (i.e. electron plus neutrino) number per baryon (Y_ℓ) will remain nearly constant (~ 1 and 0.35 respectively) because of neutrino-trapping in such matter. So, for such density regimes the relevant EOS will be the one corresponding to the adiabat characterized by (S/A) $\simeq$ 1. It turns out that for ρ up to about 10^{13} g cm^{-3}, a classical, macroscopic approach to derive the EOS is satisfactory, but for ρ beyond that a macroscopic calculation is needed.

3. The Density Regime $\rho \lesssim 10^{13}$ g cm^{-3}

The classical, macroscopic approach to describe this density regime is based on the assumption that the system is equivalent to (classical) Boltzmann gases of nucleons and nuclei in statistical equilibrium (Sato 1975; Arnett 1977; Mazurek et al. 1979; El Eid & Hillebrandt 1980). Given the nucleon-nucleon force, what determines this equilibrium are the neutron and proton chemical potentials (μ_n, μ_p) in the external uniform gas. These quantities are determined iteratively to correspond to the desired values of total ρ and Y_ℓ. The total pressure is the sum of electron and nucleon gas contributions, together with that due to the nuclei which will be given by $\rho_A kT$ Here ρ_A is the number of nuclei (characterized by neutron number N and proton number Z) per unit volume:

$$\rho_A = Q_A \left(\frac{2\pi\hbar}{m_A kT}\right)^{-3/2} e^{(\alpha_A + B_A/kT)} \qquad (1)$$

where Q_A, m_A, B_A are respectively the partition function, mass and binding energy of the nucleus and α_A is the modified *degeneracy parameter*.

The approaches that have so far been adopted to calculate Q_A fall into two main categories (Fowler et al. 1978; Mazurek et al. 1979; Tubbs & Koonin 1979) : (a) modified versions of the simple Fermi-gas model of the nucleus (by introducing density of state cut-offs with some prescription for the continuum state subtractions) and (b) Thomas-Fermi type statistical calculations. A microscopic, Hartree-Fock calculation with Skyrme interaction for Q_A has been performed by Banerjee et al. (1981), who found that for kT up to 10 MeV the nuclei will have a substantially higher internal energy and lower (S/A) than suggested by previous calculations in the literature.

4. The Density Regime $\rho \gtrsim 10^{13}$ g cm^{-3}

Nuclei in matter with $\rho \gtrsim 10^{13}$ g cm^{-3} are neutron-rich, and so are quite different from nuclei studied in the laboratory. Therefore, the methods described earlier are inadequate for their description. The nucleons inside the nuclei and those in the vapour outside must now be treated in some kind of uniform way, taking into account the interactions at short distances. Most of these microscopic methods use the Skyrme force to describe the nuclear interactions, and assume the nuclei to be periodically placed in a lattice.

The basic idea in such methods is that the nuclei can be considered as small regions of approximately homogeneous nuclear matter, in equilibrium with an external uniform gas of nucleons. The EOS for fixed ρ and T is determined from the total free energy of the system by minimizing it subject to constraints such as total electric charge neutrality and pressure and chemical equilibrium across the nuclear surface.

For electrons and neutrinos, the free energy densities have standard expressions (see Landau & Lifshitz 1958). The free energy density of the lattice cell, where nuclei are placed, can be written, in the simplest picture, as

$$\frac{F}{V} = x\, f\,(\rho_n^i, \rho_p^i, T) + (1-x)\, f\,(\rho_n^o, \rho_p^o, T) \tag{2}$$

In Eq.(2) $f(\rho_n, \rho_p, T)$ is the free energy density of nuclear matter, X is the fraction of the cell volume V that is occupied by the nucleus and the superscripts i and o refer to nucleons inside and outside the nuclei. The r.h.s. of Eq.(2) is minimized with respect to variations in X, ρ_n^i, ρ_p^i, ρ_n^o and ρ_p^o, ensuring that the total density adds up to ρ. Nuclear surface and Coulomb effects, ignored above, will make the computations more involved since the r.h.s. of Eq.(2) will then depend on V (Lamb et al. 1981).

An improvement in obtaining F is the thermal Thomas-Fermi method according to which,

$$F = \int d^3r \;\; f_{nuc}(r) \tag{3}$$

$$f_{nuc}(r) = H\left(\rho_q(r), \tau_q(r), \nabla\rho_q(r)\right) - kT \sum_q S_q \tag{4}$$

where H is the nuclear Hamiltonian density (Rayet et al. 1982) which depends on ρ_q ($q = n, p$), the kinetic energy density τ_q and the density gradient $\nabla\rho_q$. The entropy density S_q is of the form

$$S_q = \frac{5\hbar^2 \tau_q}{6\, m_q^*} - \rho_q \eta_q \tag{5}$$

The quantities ρ_q, τ_q are given in terms of the usual Fermi functions $F_{1/2}$ and $F_{3/2}$:

$$\rho_q = \frac{1}{2\pi^2}\left(2\, m_q^* kT/\hbar^2\right)^{3/2} F_{1/2}(\eta_q) \tag{6}$$

$$\tau_q = \frac{1}{2\pi^2}\left(2\, m_q^* kT/\hbar^2\right)^{5/2} F_{3/2}(\eta_q) \tag{7}$$

which permit the evaluation of the degeneracy parameter η_q. The effective mass $m_q^* \simeq 0.8$ times the nucleon rest mass. The details of this procedure incorporating all the Coulomb interactions can be found in Hartmann et al. (1984). To minimize the total (F/V), usually a Fermi function type expression for ρ_q is chosen. It has now been shown that a fully variational calculation can be performed using the so-called imaginary time-step method (Suraud & Vautherin 1984).

The most detailed calculations of EOS of supernova matter are those based on (spherical) thermal Hartree-Fock method (Bonche & Vautherin 1981; 1982; Hillebrandt et al. 1984). In this method nucleons, put in a Wigner-Seitz lattice cell, are described in terms of a density matrix (D):

$$D = \frac{1}{Z} \exp\left(\sum_i \alpha_i a_i^\dagger a_i \right) \quad (8)$$

where Z is defined by the normalization Tr(D)=1, and α_i and the single-particle states

$$|\phi_i\rangle = a_i^\dagger |0\rangle \quad (9)$$

are variational parameters. The matrix D describes the nucleus as well as the gas of neutrons and protons permeating the lattice. For the Wigner-Seitz cell,

$$F = \mathrm{Tr}(HD) + kT\, \mathrm{Tr}(D \ln D) \quad (10)$$

where the nuclear Hamiltonian H is written in terms of Skyrme interaction formula. The minimization of F is performed with respect to α_i, ϕ_i and the cell radius R_c.

5. Discussion

A comparison of several microscopic EOS models is shown in Fig.1. Model 1 is by Pi et al. (1983) who do not include Coulomb and finite size effects of nuclei. Model 2 is by Bethe et al. (1983) which includes surface and Coulomb effects using the incompressible liquid drop model for nuclei. Model 3 is a thermal Thomas-Fermi calculation by Marcos et al. (1982) and model 4 is a thermal Hartree-Fock calculation by Bonche & Vautherin (1981; 1982).

All the EOS in Fig.1 correspond to Y_l = 0.35 and S/A = 1. The axes in this figure are logarithms of pressure in MeV fm^{-3} and density in nucleons per fm^3. The density regime here is subnuclear : from 0.02 to about 0.08 nucleons per fm^3. Nuclear matter density is 0.17 nucleons per fm^3 or $\sim 2.8 \times 10^{14}$ g cm^{-3}.

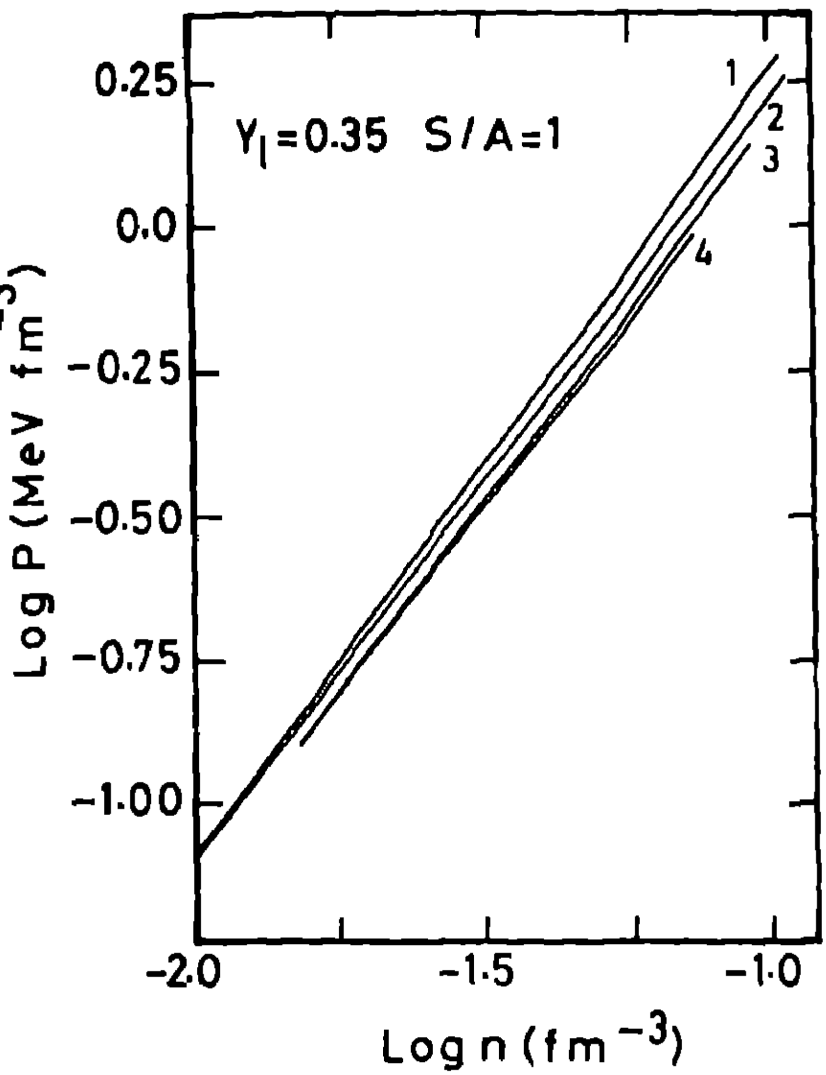

Figure 1. Equation of State of (pre-supernova) hot, dense matter according to different models 1-4 (see text for reference).

Fig.1 shows that the inclusion of nuclear surface and Coulomb effects makes the EOS softer, implying a trend towards a smaller value for the adiabatic index $\gamma = d\ln P/d\ln\rho$. It may be recalled that collapse will continue till $\gamma > 4/3$. If nuclei were absent, $\gamma \sim 5/3$ and collapse would halt much before reaching nuclear densities ($\sim 10^{14}$ g cm^{-3}). Fig.1 indicates that collapse will continue till $\rho \sim 10^{14}$ g cm^{-3}. This comes about because of the presence of excited nuclei which have large values of the partition function so that they serve as efficient store-houses of energy and do not undergo photodisintegration, thus keeping the EOS soft.

In conclusion, we mention a few points which merit detailed investigation before any consensus is reached on the EOS of supernova matter in its final stages before core bounce:

(1) is the Skyrme force the best one to obtain nuclear level densities?

(2) although temperature dependence of the effective interaction can be neglected for kT up to about 3 MeV (Buchler & Datta 1979), is this valid for kT up to $\sim$ 10 MeV?

(3) departure from sphericity in the nuclear structure at high and T, and how it will influence the nuclear free energy.

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