Stigmatic mounting of holographic plane grating

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SUMMARY. - *A stigmatic mounting of holographic plane grating has been proposed and its properties have been studied theoretically. In this mounting the observation is to be made about a normal direction and different regions of the spectrum are to be brought into the field by altering the angle of incidence by rotation of the grating. The plate holder is to be moved on a straight railing when the grating is rotated to change the region of the spectrum. Design of the grating, for the case of aberrations free spectral images has been given.*

1. - Introduction.

Recent work (1-8) in the production techniques and in the theory of holographic gratings has created new hopes among optical instrument designers. Our study indicated that holographically recorded plane diffraction gratings (*HRPDG*) have self focussing property due to variable spacing of the grooves. This property of *HRPDG* can be usefully exploited for designing mountings similar to concave diffraction grating.

The purpose of this paper is to describe Wadsworth type (concave grating) stigmatic mounting of *HRPDG*. The main advantage of the Wadsworth mounting is that in it the astigmatism of the slit image is very much reduced, giving rise to a considerable increase in the intensity of the spectrum lines. In this arrangement, observation is made about a normal direction and different regions of the spectrum are brought into the field by altering the angle of incidence by rotation of the grating which, however; brings about a change in the position and shape of the focal curve everytime. The plateholder of the photographic plate moves on a parabolic curve as the grating is rotated to change the angle of incidence. But in this case of *HRPDG*, we have found that the plateholder moves on a straight line parallel to the grating normal in the initial position instead of parabolic focal curve.

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2. - Theoretical analysis.

We refer to Fig. 1 to define a rectangular coordinate system XYZ. We assume the origin at the centre of the grating blank 0; the x-axis normal to the grating surface at 0. Let us take $P(o, w, 1)$ as a point on the blank surface, $C(X_c, Y_c, 0)$ and $D(X_D, Y_D, 0)$ the two coherent point sources used for recording interference fringes on the blank. Points $A(X, Y, Z)$ and $B(X', Y', Z')$ are a self-luminous point in the entrance slit, and a point on the diffracted ray from P of wavelength λ in the *mth* order, respectively.

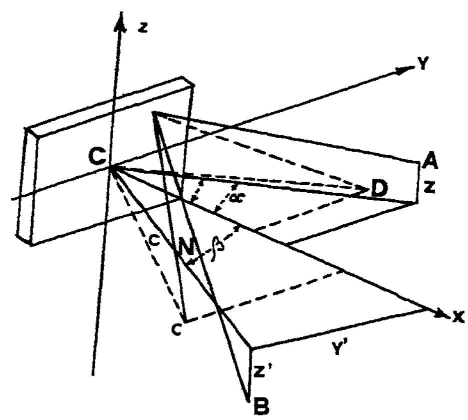

Fig. 1
Representation of coordinate system.

For the ray AP, PB the light path function F is given by (8):

[1] $$F = (AP) + (PB) + \frac{m\lambda}{\sigma_o} \frac{[(CP - DP) - (CO - DO)]}{(\sin\delta - \sin\gamma)}$$

where:

$$\sigma_o = \frac{\lambda_o}{\sin\delta - \sin\gamma} = \text{grating constant at } w = l = 0\,, \quad \delta > \gamma$$

$$(AP)^2 = x^2 + (y - w)^2 + (Z - l)^2$$
$$(PB)^2 = x'^2 + (y - w)^2 + (Z' - l)^2$$
$$(CP)^2 = x_c^2 + (y_c - w)^2 + l^2$$
$$(DP)^2 = x_D^2 + (y_D - w)^2 + l^2$$
$$(CO)^2 = x_c^2 + y_c^2$$
$$(DO)^2 = x_D^2 + y_D^2$$

These relations can be transformed in terms of the cylindrical coordinates of the points A and B; and C and D; which may be written as (ν, α, z) and (ν', β', z'); and $(\nu_C, \gamma, 0)$ and $(\nu_D, \delta, 0)$ respectively. The signs of γ and δ; will be opposite if C and D lie on opposite sides of XZ plane. The same rule for sign holds good for α and β also. Furthermore, their signs should be consistant with the signs of γ and δ. Now using power series expansions for AP, PB; CD, DP, CO and DO in Eq. [1], we get:

[2]
$$\begin{aligned} F = {} & \gamma\left(1+\frac{Z^2}{\gamma^2}\right)^{-\frac{1}{2}} + \gamma'\left(1+\frac{Z'^2}{\gamma'^2}\right)^{-\frac{1}{2}} + w\left[\frac{m\lambda}{\sigma_o} - \sin\alpha\left(1+\frac{Z^2}{\gamma^2}\right)^{-\frac{1}{2}}\right. \\ & \left. -\sin\beta\left(1+\frac{Z'^2}{\gamma'^2}\right)^{-\frac{1}{2}}\right] - l\left[\frac{Z}{\gamma}\left(1+\frac{Z^2}{\gamma^2}\right)^{-\frac{1}{2}} + \frac{Z'}{\gamma'}\left(1+\frac{Z'^2}{\gamma'^2}\right)^{-\frac{1}{2}}\right] + \\ & \frac{w^2}{2}\left[\frac{\cos^2\alpha}{\gamma} + \frac{\cos^2\beta}{\gamma'} - \frac{m\lambda}{\sigma_o Re}\right] + \frac{l^2}{2}\left[\frac{1}{\gamma} + \frac{1}{\gamma'} - \frac{m\lambda}{\sigma_o(\sin\delta - \sin\gamma)}\right. \\ & \left.\left(\frac{1}{\gamma_o} - \frac{1}{\gamma_e}\right)\right] - lw\left[\frac{Z\sin\alpha}{\gamma^2} + \frac{Z'\sin\beta}{\gamma'^2}\right] + \frac{w^3}{2}\left[\frac{\sin\alpha\cos^2\alpha}{\gamma^2} + \frac{\sin\beta\cos^2\beta}{\gamma'^2} -\right. \\ & \left. - \frac{m\lambda}{\sigma_o(\sin\delta - \sin\gamma)}\left(\frac{\sin\delta\cos^2\delta}{\gamma_D^2} - \frac{\sin\gamma\cos^2\gamma}{\gamma_c^2}\right)\right] + \frac{wl^2}{2}\left[\frac{\sin\alpha}{\gamma^2} + \right. \\ & \left. \frac{\sin\beta}{\gamma'^2} - \frac{m\lambda}{\sigma_o(\sin\delta - \sin\gamma)}\left(\frac{\sin\delta}{\gamma_D^2} - \frac{\sin\gamma}{\gamma_c^2}\right)\right] + \frac{l(w^2+l^2)}{2}\left[\frac{Z}{\gamma^3} + \frac{Z'}{\gamma'^3}\right] \\ & - \frac{(u^2+l^2)^2}{8}\left[\frac{1}{\gamma^3} + \frac{1}{\gamma'^3} - \frac{m\lambda}{\sigma_o(\sin\delta - \sin\gamma)}\left(\frac{1}{\gamma_D^3} - \frac{1}{\gamma_c^3}\right)\right] + \frac{w^2}{4} \\ & \left[\frac{(3\sin^2\alpha - 1)Z^2}{\gamma^3} + \frac{Z'^2(3\sin^2\beta - 1)}{\gamma'^3}\right] - \frac{3l^2}{4}\left[\frac{Z^2}{\gamma^3} + \frac{Z'^2}{\gamma'^3}\right] + 0\left(\frac{w^5}{\gamma^4}\right) \end{aligned}$$

$$Re = \frac{\sin\delta - \sin\gamma}{\dfrac{\cos^2\delta}{\gamma_D} - \dfrac{\cos^2\gamma}{\gamma_c}} = \gamma \text{ adins of curvature}$$

By the application of Fermat's principle viz. $\dfrac{\partial F}{\partial w} = 0$ and $\dfrac{\partial F}{\partial l} = 0$ we get the following relations for image formation:

[3]
$$\left(1+\frac{Z^2}{\gamma^2}\right)^{-\frac{1}{2}}\sin\alpha + \left(1+\frac{Z_o'^2}{\gamma_o'^2}\right)^{-\frac{1}{2}}\sin\beta_o = \frac{m\lambda}{\sigma_o}$$

[4]
$$\frac{Z}{\gamma} = -\frac{Z_o'}{\gamma_o'}$$

[5]
$$\frac{\cos^2\alpha}{\gamma} + \frac{\cos^2\beta}{\gamma'} - \frac{m\lambda}{\sigma_o Re} = 0$$

[6]
$$\frac{1}{\gamma} + \frac{1}{\gamma'} - \frac{m\lambda}{\sigma_o(\sin\delta - \sin\gamma)}\left(\frac{1}{\gamma_D} - \frac{1}{\gamma_c}\right) = 0$$

(ν_o', β_o, z_o') are the coordinates of a point, on the principal ray AOB_o. Eqns. [3], [4], [5] and [6] are the grating equation, magnification eqn., horizontal focal curve relation and vertical focal curve relation respectively.

Now from Eq. [5] for $\nu = \infty$, we have:

$$[7] \qquad \gamma' = Re \frac{\cos^2 \beta}{\sin \alpha + \sin \beta}$$

The focal curves according to Eq. [7] at $\alpha = 0°, 30°$, and $80°$ are given in Fig. 2.

Further when $\beta = 0$:

$$[8] \qquad \gamma' = \frac{Re}{\sin \alpha}$$

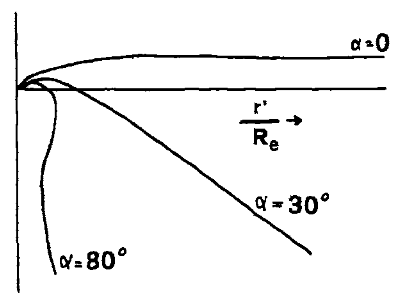

FIG. 2
Focal curves.

Thus Eq. [8] represents a straight line parallel to X axis cutting Y axis (fig. 1) at a distance R_e.

So for the stigmatic mounting arrangement we have, to observe about a normal direction of the grating, and to move the photographic plate-holder on railing parallel to X axis, cutting Y axis at a distance R_e, during the rotation of the grating to change the region of the spectrum. The suggested arrangement is shown in Fig. 3. For a region where $\beta \neq 0$ we have to use Eq. [7].

Now substituting $\nu = \infty$ and ν' given by Eq. [7] into $\frac{\partial F}{\partial w} = 0$ and $\frac{\partial F}{\partial l} = 0$ we get:

$$[9A] \qquad \frac{\partial F}{\partial w} = 0 = \frac{m\lambda}{\sigma_o} - \sin\alpha - \sin\beta \left\{1 + \frac{Z'^2 (\sin\alpha + \sin\beta)^2}{R_e^2 \cos^4\beta}\right\}^{-\frac{1}{2}} +$$

$$\frac{3\, w^2}{2} \left[\frac{\sin\beta\, (\sin\alpha + \sin\beta)^2}{Re \cos^2\beta} - \frac{m\, \lambda}{\sigma_o\, (\sin\delta - \sin\gamma)} \left(\frac{\sin\delta \cos^2\delta}{\gamma_D^2} - \frac{\sin\gamma \cos^2\gamma}{\gamma_c^2}\right)\right]$$

$$-\frac{lZ'}{Re^2}\frac{\sin\beta\,(\sin\alpha+\sin\beta)^2}{\cos^4\beta}+\frac{l^2}{2}\left[\frac{\sin\beta\,(\sin\alpha+\sin\beta)^2}{Re^2\cos^4\beta}-\right.$$

$$\left.\frac{m\,\lambda}{\sigma_o(\sin\delta-\sin\gamma)}\left(\frac{\sin\delta}{\gamma_D^2}\,\frac{\sin\gamma}{\gamma_o^2}\right)\right]+\frac{l\,w\,Z'}{Re^3}\frac{(\sin\alpha+\sin\beta)^3}{\cos^6\beta}-$$

$$\frac{w\,(w^2+l^2)}{2}\left[\frac{(\sin\alpha+\sin\beta)^3}{Re^3\cos^6\beta}-\frac{m\,\lambda}{\sigma_o(\sin\delta-\sin\gamma)}\left(\frac{1}{\gamma_D^3}-\frac{1}{\gamma_o^3}\right)\right]+$$

$$\frac{w\,Z'^2}{2\,Re^3}\frac{(3\sin^2\beta-1)(\sin\alpha+\sin\beta)^3}{\cos^6\beta}+O\left(\frac{w^4}{Re^4}\right)$$

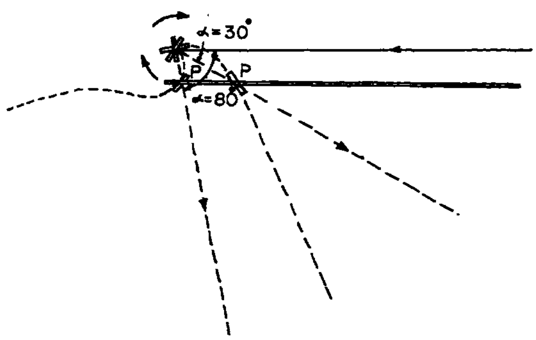

FIG. 3
Suggested Design.

and:

$$[9B]\quad \frac{\partial F}{\partial l}=0=-\frac{Z'(\sin\alpha+\sin\beta)}{Re\cos^2\beta}\left\{1+\frac{Z'^2(\sin\alpha+\sin\beta)^2}{Re\cos^4\beta}\right\}^{-\frac{1}{2}}+l\left[\frac{\sin\alpha+\sin\beta}{Re\cos^2\beta}-\right.$$

$$\left.\frac{m\,\lambda}{\sigma_o(\sin\delta-\sin\gamma)}\left(\frac{1}{\gamma_D}-\frac{1}{\gamma_o}\right)\right]-\frac{w\,Z'\sin\beta\,(\sin\alpha+\sin\beta)^2}{Re^2\cos^4\beta}+wl\left[\frac{\sin\beta\,(\sin\alpha+\sin\beta)^2}{Re^2\cos^4\beta}\right.$$

$$\left.-\frac{m\,\lambda}{\sigma_o(\sin\delta-\sin\gamma)}\left(\frac{\sin\delta}{\gamma_D^2}-\frac{\sin\gamma}{\gamma_o^2}\right)\right]+O\left(\frac{w^3}{Re^3}\right)$$

3. - Aberrations.

(1) *Astigmatism.*

Now let L be the total length of a groove projected on Z-axis, then we get for the length of astigmatic images due to a point source:

$$[Z']_{ast}=L\left[1-\frac{\cos^2\beta}{f}\right]$$

$$[10]\qquad f=\left(\frac{\cos^2\delta}{\gamma_D}-\frac{\cos^2\gamma}{\gamma_c}\right)\Big/\left(\frac{1}{\gamma_D}+\frac{1}{\gamma_c}\right)$$

If we can choose:

[11] $$f = 1$$

during recordin of the interference fringes on the grating blank. Then:

[12] $$[Z']_{ast} = L \sin^2 \beta$$

That is the astigmatism becomes the same as that of a concave diffraction grating in Wadsworth mounting. This condition can be obtained easily by taking $\nu_C = \infty$ and $\delta = 0$. The other parameters γ and ν_D can be selected advantageously. In this case we will be getting zero astigmatism only at one wavelength which is observed normally.

Now if we select $\nu_C \neq \nu_D$ and $\delta = - \gamma$ then, $f = \cos^2 \delta$. In this case we get zero astigmatism at two wavelengths given by $\pm \beta = \delta$. It seems that it will be advantageous to choose $\delta = - \gamma = 5°$ so that we get zero astigmatism at $\beta = \pm 5°$, in the proposed set up. Fig. 4 present *LZ'Iast/L* at different values of *f*.

(2) *Coma.*

The amount of coma is given by:

[13] $$\Delta p_c = \frac{2 R_e \nu^2}{l^2}$$

where:

$$\frac{1}{\nu^2} = \cos \beta \left[1 + \left(2 \tan \beta + \frac{\cos \beta}{\sin \alpha + \sin \beta}\right)^2\right]^{\frac{1}{2}}$$

[14] $$\left[\frac{(\sin \alpha + \sin \beta) \sin \beta}{\cos^4 \beta} - \frac{R_e^2 \left(\frac{\sin \delta}{\gamma_D^2} - \frac{\sin \gamma}{\gamma_c^2}\right)}{(\sin \delta - \sin \gamma)}\right]$$

If we take $\Delta\beta_C$ in Eqn. [13] as the maximum amount of coma permissible for a particular experiment, then the maximum length of the grooves that can be used for obtaining spectral lines with coma less than the allowed magnitude $\Delta\beta_C$ can be given by:

[15] $$L_{max} = 2\,l = 2\,[2\,R_e\,\Delta\,p_c]^{\frac{1}{2}}\,\nu$$

Fig. 5 represents $\frac{1}{\nu^2}$ for two cases of set of recording parameters discussed in the previous section on astigmatism. It is clear that for the set of recording parameters $\nu_C = \infty$, $\delta = 0$ $\Delta\beta_C = 0$ at $\beta = 0$. That is with these recording parameters we get zero astigmatism as well as zero coma at $\beta = 0$.

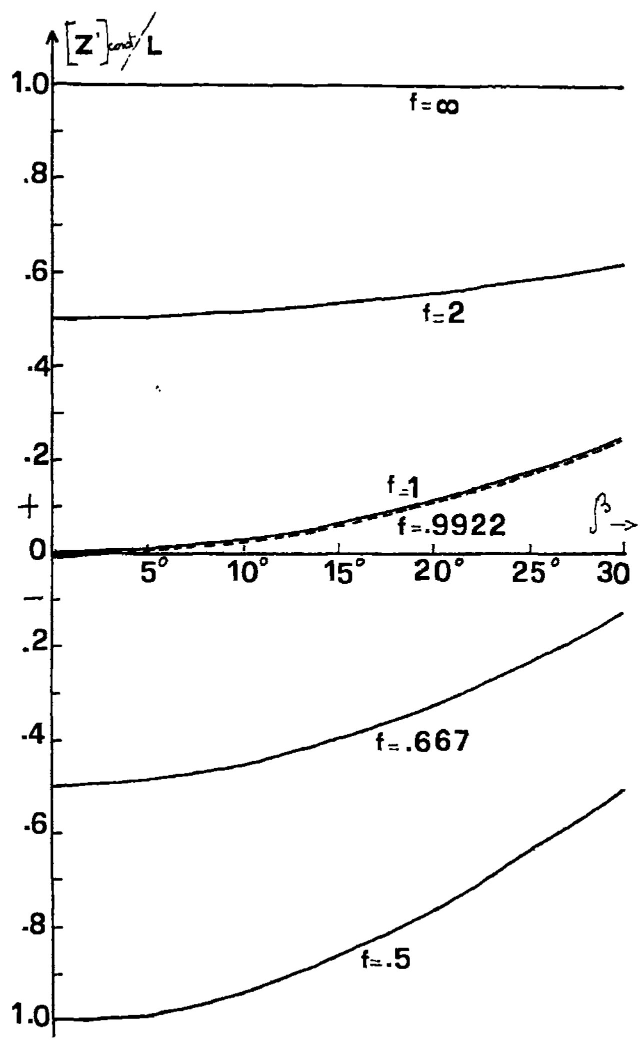

FIG. 4

LZ'Iast/L at different values of *f*.

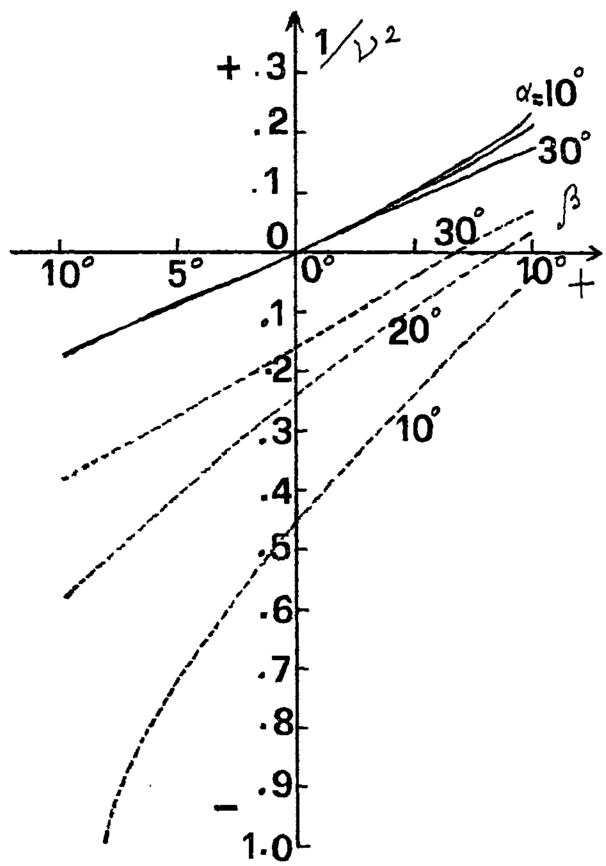

FIG. 5

Plot of $\Delta\beta_C/(l^2/2R_e) = \frac{1}{\nu^2}$. Solid curve for $\nu_C = \infty$, $\delta = 0$, $\nu_D = \gamma$ finite dotted curve for $\delta = -\gamma = 5°$, $\nu_C = 2\nu_D$.

(3) *The Cubic Terms.*

The aberrations arise from the contribution of the higher order terms given in Eq. [9] of the optical path function. Now we shall find out the limiting value of grating width so that the aberration introduced by cubic terms is $\leq \frac{\lambda}{4}$. This is given by:

$$[16] \qquad \frac{W^3}{2}\left[\frac{\sin\beta(\sin\alpha+\sin\beta)^2}{R_e^2\cos^2\beta} - \frac{m\lambda}{\sigma_o(\sin\delta-\sin\gamma)}\left(\frac{\sin\delta\cos^2\delta}{\gamma_D^2} - \frac{\sin\gamma\cos^2\gamma}{\gamma_C^2}\right)\right] \leq \frac{\lambda}{4}$$

Thus the optimum value of W is given by:

$$W_{opt} = \left(\frac{\sigma_o}{2m}\right)^{\frac{1}{3}} \left|\frac{R_e}{\frac{\sin\beta(\sin\alpha+\sin\beta)}{\cos^2\beta} - B}\right|^{1/3}$$

$$[17] \qquad B = R_e^2\left(\frac{\sin\delta\cos^2\delta}{\gamma_D^2} - \frac{\sin\gamma\cos^2\gamma}{\gamma_C^2}\right) \Big/ \sin\delta - \sin\gamma)$$

Value of $W_{opt}/\left(\frac{\sigma_o R_e^2}{2m}\right)^{1/3}$ are plotted in Fig. 6 for two sets of recording parameters viz. $\nu_C = \infty$, $\delta = 0$ (solid curve) and $\delta = -\gamma = 5^\circ$, $\nu_C = 2\gamma_D$ (dotted curve).

4. - Description of the mounting.

The proposed arrangement based on these studies is shown in Fig. 3. The grating, G, is to be rotated in the clockwise direction to get the different regions of the spectrum into the field. The photographic plate P, is to be moved on the straight railing accordingly. The inclination of the photographic plate to the railing is given by $\alpha + \tan^{-1}(\sin\alpha)$ which is to be adjusted every time along with the curvature of the plate holder. The distance of the railing from the normal to the grating at $\alpha = 0$ is given by R_e.

5. - Results and discussion.

From this analyses it appears that the *HRPDG* can be used advantageously in Wadsworth type of mounting with the modification suggested in the proposed design. In the two examples of sets of recording parameters, the set $\nu_C = \infty$, $\delta = 0$ appears to be better.

6. - Acknowledgment.

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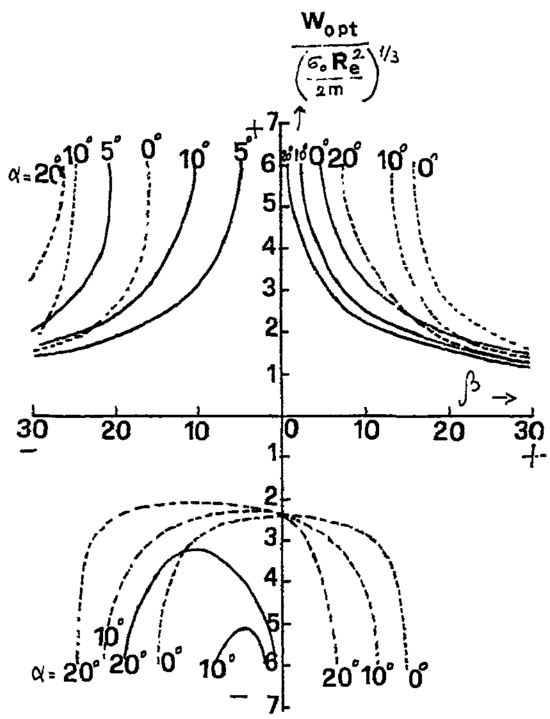

FIG. 6

Plot of $W_{opt}/\left(\frac{\sigma_0 R_e^2}{2^m}\right)^{1/3}$. Solid curve for $\nu_C = \infty$, $\delta = 0$, ν_D & γ finite dotted curve for $\delta = -\gamma = 5°$, $\nu_C = 2\nu_D$.

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