MAGNETIC FIELD INDUCED METAL-INSULATOR TRANSITION IN A ONE-DIMENSIONAL CHAIN: APPLICATION TO NEUTRON STAR ATMOSPHERES

(Letter to the Editor)

S. CHATTERJEE

Indian Institute of Astrophysics, Bangalore, India

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Abstract. It is shown that, in the non-interacting limit, a one-dimensional metallic system has a transition to an insulating phase, in presence of external magnetic field H, if $\mu_B H > E_B$, where μ_B is the Bohr magneton and E_B is the energy band width of the electronic states. Possible realization of this effect in the atmosphere of neutron stars is pointed out.

For a system of non-interacting electrons, the electrical property of a solid is known to be determined by the level of filling of the energy band states. It is established that a half-filled electron band creates a metallic state for the system such that the mobility of electrons at the Fermi energy is maximum. A completely filled band on the other hand, has its Fermi energy lying at the Brillouin zone edge, where the mobility is zero, thus giving rise to an insulating phase. For a one-dimensional chain of atoms, the half-filled band (metallic) condition results if every atom has a single valence electron, there being two degrees of spin degeneracy for every state in the band. In what follows we show that the system becomes an insulator, if we apply a magnetic field of $H > E_B/\mu_B$, where E_B is the energy band width and μ_B is the Bohr magneton. The importance of this phenomenon in neutron star physics is pointed out.

As we know the application of a magnetic field lifts the spin degeneracy of the electron system, the energy eigenvalues for the wave vectors **k** are then given by

$$E_{\pm}(k) = E_0(k, H) \pm (\tfrac{1}{2})\mu_B H\,, \tag{1}$$

where $E_0(k, H)$ gives the energy eigenvalues in the band as is obtained by considering the electrons to be spinless particles. The energy contribution due to the interaction of the spins with the external magnetic field is given by the last term in (1) with + and − denoting the respective spin states parallel and anti-parallel to the direction of the field.

More specifically, one can write

$$E_0(k, H) = \varepsilon(H) - 2t(a, H)\cos ka\,, \tag{1a}$$

where a is the separation between the nearest ions, $\varepsilon(H)$ is the energy eigenvalue of the atomic states in presence of H, $t(a, H)$ is the tunneling energy for tunneling of the

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electrons between the nearest ionic sites, with $t(a, H)$ being a $+ve$ quantity. Also, it is to be noted that we deal here with a single band, which suffices for the problem under study.

The value of $\varepsilon(H)$ has been given in the review article by Canuto and Ventura (1972) and is not of importance in the context of band filling that we examine below. If the field acts in the direction of the chain, the quantity $t(a, H)$ has H dependence given by

$$t(a, H) = t_0 \exp(-a/l(H)), \tag{1b}$$

where $l(H)$ is the typical length of the atomic orbital along the direction of the field. It can be easily seen that the band width E_B equals $2t(a, H)$.

The question, we ask here, is: at $T = 0$ K, given a chain of N atoms, with nearest neighbour separation a, and each atom donating one electron, till what value of k we have to fill, to accommodate all the electrons? Obviously, for $H = 0$, such a system will posess a half-band configuration, as seen in Figure 1(a), giving rise to a metallic state. For non-zero H, and $\mu_B H < E_B$, as is seen from Figure 1(b), the band filling involves the simple exercise of first filling the states in the $-$ spin band till the energy $\mu_B H$ is reached, and from then onwards, the electrons can be filled in both $-$ and $+$ bands. There are thus more electrons in the $-$ band than in the $+$ one. This results in the well-known Pauli paramagnetism. The important point to be noted is that for $H < E_B/\mu_B$, both the bands are partially filled. Due to this partial filling condition, the metallic state is still retained.

Let us now consider a situation where $H \geq E_B/\mu_B$. As seen in Figure 1(c), the electrons are first to be filled in the $-$ band. Due to the periodic boundary condition, the number of electrons occupying the wave-vectors between k and $k + \mathrm{d}k$ is

$$\mathrm{d}n(k) = (Na/2\pi)\,\mathrm{d}k\,; \tag{2}$$

Na being the length of the chain.

Thus the total number of electrons that can be accommodated in this band is given by

$$n = \int_{k=-\pi/a}^{k=+\pi/a} \mathrm{d}n(k) = (Na/2\pi) \int_{k=-\pi/a}^{k=+\pi/a} \mathrm{d}k = N\,; \tag{3}$$

the spin degeneracy factor being 1 for every state in the band.

Equation (3) shows that for $H \geq E_B/\mu_B$, all the electrons in the system are accommodated in the $-$ band, filling this band completely, while leaving all other bands empty. This condition immediately leads us to conclude, that an insulating phase results as the highest-filled state lies at the Brillouin zone edge where the mobility is zero.

This magnetic field induced metal to insulator transition can be realised if the spin term $\mu_B H$ is greater than $E_B = 2t(a, H)$. The former, as is seen, is linear in H while the latter is weakly dependent on H. This is because, the variation of $l(H)$ with H is very weak (Canuto and Ventura, 1972). For example, for $H = 0$, $l = a_0$ (the radius of the first Bohr orbit), while for $H = 10^{12}$ G, we have $l \sim a_0/2$. This indicates that for $a \sim 10^{-9}$ cm

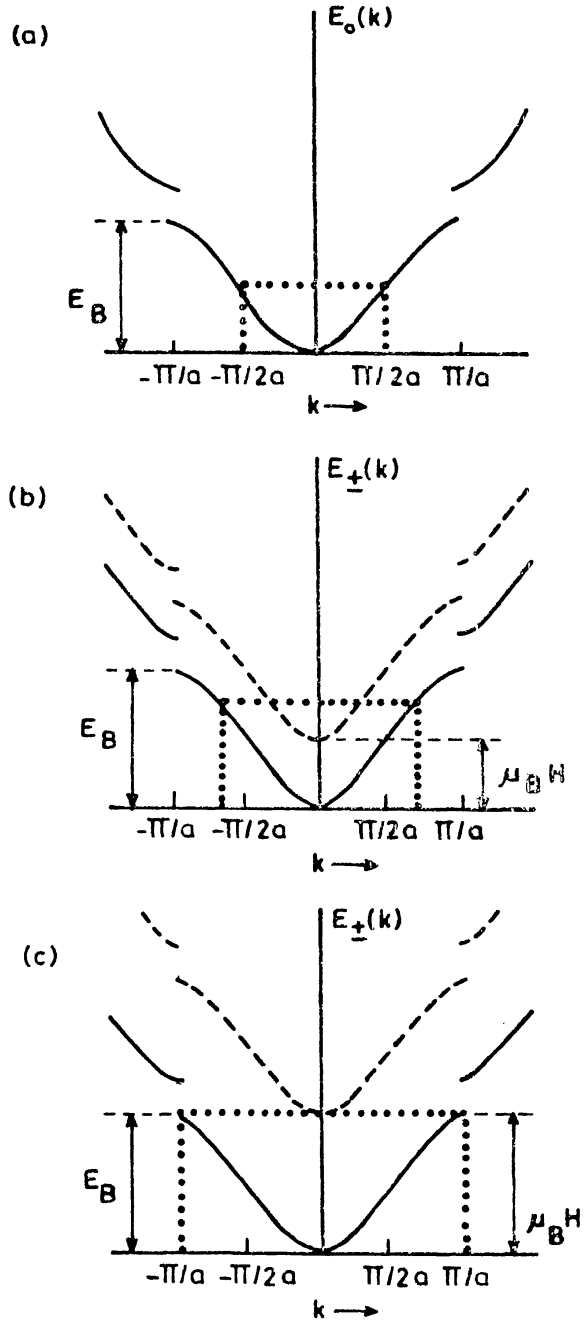

Fig. 1. Schematic band structure and band-filling diagram. (a) $H = 0$ case, (b) $H < E_B/\mu_B$ case, (c) $H = E_B/\mu_B$. ——— denotes $E_-(k)$, ---- denotes $E_+(k)$, ... designates the level of band filling.

(i.e., ion density of 10^{27} particles per c.c.) and $H \sim 10^{12}$ G, we have $t(a, H) \sim t(a, 0)e^{1/2}$, i.e., $t(a, H)$ in this case is nearly of the same order as $t(a, 0)$. This shows that the spin term rises faster than the band width as H increases and can, hence, overtake E_B if H be sufficiently large.

This particular result is of considerable interest in the physics of neutron star atmospheres. In neutron stars one can obtain magnetic fields $H \geq 10^{12}$ G. This makes

the electron motion highly anisotropic. The electrons are constrained to move in cylindrical orbitals aound ion sites, with the axis of the cylinder lying along the direction of the magnetic field. The strong orbital overlap between electronic states, along the field direction is expected to give rise to a one-dimensional electron band structure, referred to as a magnetic polymer, which has metallic conduction along the direction of the field (Ruderman, 1972). Our result, however, shows that this assumption is invalid if the condition $H \geq E_B/\mu_B$ is satisfied, which in the context of neutron star signifies $\rho^2 \leq (2\pi)^{-4} (eH/c\hbar)^3$, ρ being the number of electrons per unit volume. For a typical $H \geq 10^{12}$ G, this insulating phase will occur for $\rho \leq 10^{27}$ cm^{-3}. This requires many of the properties of the neutron stars to be re-examined. For example, the insulating phase will have a low thermal conductivity and alter the cooling rate of neutron stars. Other candidates for this effect are electrons in the narrow impurity bands of semi-conductors.

In conclusion, we state that the result derived here is a manifestation of two fundamental results of quantum physics: namely (1) Pauli's exclusion principle and (2) the vanishing $\partial E(k)/\partial k$ at the Brillouin zone edge. These two phenomena, operate with the lifting of spin degeneracy in presence of magnetic field to create the insulating phase for very high magnetic fields. The present result shows that for high fields the electron system can become insulating because of the constraints of band filling, even if the atoms donate one electron per atom, i.e., even though the system is expected to be metallic for $H = 0$. An electron system in presence of magnetic field may exhibit other types of transitions to the insulating phase like the Wigner crystal phase or the Peierls's insulating phase (Peierls, 1955) which will be discussed in a future communication.

References

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