35

The Engelbert Experience: Pathways from the Past

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It was the best of times, it was the worst of times. For me, that is. This was in the year 1969. I was working at the Institute for Space Studies in New York on a National Research Council Fellowship. I was studying the scattering of gravitational waves by black holes through computer simulation in order to find out whether the black hole left its imprint in some way on the scattered wave. This was done by bombarding the black hole with Gaussian wave packets. When the width of the Gaussian was comparable to or less than the radius of the black hole, there emerged a decaying wave pattern, later to be called the quasinormal mode of the black hole [1]. As is well known, a lot of work has been done on the black hole quasinormal modes since then. Today they are considered to be a means of detecting both black holes and gravitational waves. My original work on the quasinormal modes was terribly exciting—to me. And that is the best-of-times part of the story.

Now for the worst-of-times aspect. Unfortunately, no one at the Institute seemed to be interested in this kind of research. After all, neither black holes nor gravitational radiation had been detected, but only theoretically predicted. Why should anyone spend time investigating the interaction between two unobserved entities perhaps of doubtful existence? The consequence of this attitude, which was not at all uncommon in those days, was rather disconcerting to say the least. My contract was renewed for only three months instead of the normal one year period. As it turned out, however, it was a blessing in disguise, but most effectively disguised at the time, as Winston Churchill might have put it. Frantically I started looking for a job. Jobs were hard to come by, but not negative replies. With some hesitation I thought of writing to Professor Engelbert Schucking, who was working downtown at the New York University. I had a funny feeling that the sort of things I did were not his cup of tea or mug of beer. In reality, Engelbert's cupboard happened to hold a number of mugs filled with a variety of beers. I had listened to Engelbert a few years earlier at the historic Cornell Summer School, but had not spoken to him. Historic, because almost every well-known general relativist of the present era was there. Boyer, Carter, Chandrasekhar, Ehlers, Hawking, Kerr, Misner, Penrose, Robinson, Schild, Schücking, Taub, and so on. Most of the lectures were enormously interesting and some totally incomprehensible. Engelbert gave two or three talks on cosmology. Of all the stuff that seeped into my gray cells, it was only the opening sentence of Engelbert's lectures that was pickled permanently: "An important contribution of the general theory of relativity to cosmology has been to keep out theologians by a straightforward application of tensor analysis." Well, to return to my original story, Engelbert asked me to meet him after about a month. It was a long, dismal month indeed. For, in the meantime, my contract had run out. Unemployed, I had given up my apartment, sold my furniture, and moved in with some friends, dipping into my meager savings. At the end of the month, after a seminar at the New York University, Engelbert told me, "It would be nice if you could come and help us with our research." This was his delicate way of telling me that he was offering me a job! So, I was going to work with Engelbert and his group. The blessing had thrown off its disguise at last.

When I joined Engelbert's group, the physics department occupied a part of the Courant Institute down in the Greenwich Village. It was a modern brownish building which was quite imposing, but a bit intimidating as well. We were to acquire a brand new building of our own soon enough. Although the physicists were surrounded by a number of mathematicians, there was hardly any interaction between the two disciplines. Stranger still, it was difficult to interact with many of the students who had regular jobs during the day and surfaced at the University towards late evening. However, there were some regular graduate students.

One of them was Eli Honig, who was analyzing gyroscopic precession in curved spacetimes to identify the analogue of Thomas precession. Thomas precession arises in flat spacetime when the gyroscope moves along a nongeodesic path such as a circular orbit. However, in curved spacetimes gyroscopes following geodesic worldlines can still precess as a result of the spacetime curvature. Fokker–de Sitter precession in the Schwarzschild spacetime is an example of this. It is customary nowadays to identify automatically extra terms arising on account of nongeodesic motion as representing Thomas precession. Such a procedure was not in the least satisfactory to Engelbert with his penchant for exactitude that characterized his research. As a result, perhaps, none of the work that went in this direction was ever submitted for publication.

There was, nevertheless, an offshoot of this investigation which became a research project in its own right. As we were discussing precession of spins and magnetic moments, we started looking at the motion of charged particles in homogeneous electromagnetic fields that are, in other words, constant in both space and time [2]. In order to describe the worldlines of these charges, we utilized the Frenet-Serret formalism, which is well known to classical geometers. I had first come across it in Synge's book on general relativity, but had not seen its practical application. In this formalism, a curve is characterized by some scalar parameters that in general vary, say as functions of the arc length, along the curve. In three dimensions these parameters are κ the curvature and τ the torsion. They uniquely and invariantly define the curve. For instance, $\kappa = \tau = 0$ is a straight line,

$\tau = 0$ a planar curve with κ = constant a circle, κ = constant and τ = constant a helix and so on. In a 4-dimensional spacetime we have three such parameters, namely the curvature κ and τ_1, τ_2, the first and second torsions.

All along the curve or the worldline an orthonormal tetrad e_i $(i = 0 - 3)$ is defined which obeys the Frenet-Serret transport law. The vector e_0 is identical to the 4-velocity of the particle. As differentiation along the trajectory is with respect to the invariant arc-length or the proper time, curvature κ turns out to be the magnitude of the 4-acceleration acting along the unit vector e_1. How about the torsions τ_1 and τ_2? Well, they are directly related to gyroscopic precession. A gyroscope, which is by definition Fermi-Walker transported along the particle worldline, precesses with respect to the Frenet-Serret spatial triad at a rate ω_g, which has components τ_1 and τ_2 along e_3 and e_1 respectively. Thus gyroscopic precession can be described in a completely covariant and geometric manner once the torsions and the tetrad have been determined along the worldline of the particle.

In the case of charged particles moving in a constant electromagnetic field given by covariantly constant tensor F_{ab}, all the Frenet-Serret quantities can be determined in terms of F_{ab} and the 4-velocity e_0. We obtained a number of neat results. We could show, for instance, that κ, τ_1, and τ_2 were constants along the worldlines. Two equations connected these parameters to the Lorentz invariants $F_{ab}F^{ab}$ and $F^{ab}\,{}^*F_{ab}$, where ${}^*F_{ab}$ is the dual of F_{ab}:

$$\kappa^2 - \tau_1^2 - \tau_2^2 = \frac{1}{2}F^{ab}F_{ab}$$

and

$$\kappa\tau_2 = -\frac{1}{4}\,{}^*F^{ab}\,F_{ab}\,.$$

The two torsions τ_1 and τ_2, the components of gyroscopic precession, were shown to be proportional to $E \cdot B$ and $E \times B$ respectively, E and B being the electric and magnetic fields in the reference frame comoving with the charged particle.

All this was quite nice. But, then, where do you get constant electromagnetic fields in nature? John Ruskin once wrote something like "The more beautiful the object, the less useful it is." Echoing this opinion, George Ellis, who liked our work, remarked, "Pretty it is, but useful it ain't!" Nevertheless, there was one part of our work which has proved to be not only pretty, but also useful. This is the application of the formalism to particle worldlines following a Killing vector field ξ^a, or Killing trajectories for short. The analogue of the Maxwell tensor is now $F_{ab} = (\xi^c\xi_c)^{-1/2}\xi_{a;b}$. Then all the results of charged particle motion in constant electromagnetic fields follow. The first relation connecting κ, τ_1, and τ_2 as above reduces to the equation that demonstrates the existence of an ergosphere between the stationary limit and the event horizon in stationary spacetimes [3]. On the way to this result, I was able to prove that the gyroscopic precession rate was identical to the vorticity of the Killing congruence after considerable amount of manipulation. I was so excited that I rushed to Engelbert's home to show him my calculations. He was quite happy. He read through my notes, munching on a handful of Pepperidge Farm cookies dipped in lemon tea, exclaiming continually, "Beautiful, beautiful!

Beautiful geometry!" We left these considerations related to Killing trajectories at an abstract theoretical level.

After more than two decades, my colleague B. R. Iyer and I returned to this problem [4]. We developed the formalism further and applied it to a variety of examples like Thomas precession, de Sitter–Fokker precession in the Schwarzschild spacetime, Schiff correction in the Kerr spacetime, and so on. All these results hang together neatly. During the period when there was a lot of excitement about higher dimensional spacetimes, we generalized the Frenet-Serret description to such spaces as well [5].

I have worked with a lot of people. My collaborators number some thirty or so. One I never met, since he was a student of one of my colleagues abroad and we worked long distance by mail, that too before the advent of e-mail. Five of these thirty-odd relativists are senior to me, Engelbert being one of them. Usually, the way they guided their graduate students as thesis advisors was to assign a specific problem, indicate the general mode of attack and from time to time discuss the results the student had obtained. But Engelbert's methodology was considerably different from this norm. Often the problem would not be precisely formulated, but only the general area would be identified. He would then relentlessly explore this uncharted terrain until the problem for research emerged. In this process, Engelbert would continually interact with his students. I acted as a sort of multidimensional middleman: coworker, assistant advisor, and liaison officer between Engelbert and his students. Engelbert's reluctance to publish anything in haste—sometimes his disinclination to publish at all—was proverbial. Consequently, some student or the other would ask me to plead with him to have some of the results published. After all, one needed publications to get a job.

He met with each of his students at least once a week for about an hour or two. He would write down his appointments on the blackboard with boxes around them. Often they covered a major part of the blackboard, leaving very little room for writing. If the problem we were working on was interesting enough, these meetings would stretch on, sometimes into the late evening, making Engelbert forget his other engagements. Once Brenda, his companion and our friend, breezed in to whisk Engelbert away to dinner, reminding the absent-minded professor that they had already made reservation at a restaurant in his name and that his name happened to be Engelbert Schucking! Whenever we were engrossed in our discussions, Engelbert did not like to be disturbed by outside agencies. So much so that during these sessions he would not answer the telephone and would let it ring forever. On one such occasion, when he caught me and his graduate student Richard Greene exchanging glances while the telephone rang on, he explained that there was a maniac at the other end and cautioned us not to touch the instrument. How did he know, we wondered.

Richard was fortunate in completing his doctoral research very quickly, culminating in one of the shortest theses I have ever come across. His problem consisted in finding a globally hypersurface orthogonal timelike congruence in the Kerr spacetime and investigating its properties. We were not aware of Bardeen's work defining the locally nonrotating frames (LNRF) [6], which in fact implied the existence of such a congruence. Nevertheless, the global properties of this congruence, especially in a general axially symmetric stationary spacetime, had not been explored. This work was quite close to my heart, since, I felt, the starting point was contained to some extent in my earlier work on the horizon and the stationary limit in the Kerr spacetime [3]. I had shown that although the global timelike Killing vector does not become null on the horizon, the combination $\zeta = \left(\xi_t - \frac{a}{2mr_+}\xi_\phi\right)$ does, where ξ_t and ξ_ϕ are the timelike and axial Killing vectors respectively, m is the mass and a the angular momentum per unit mass, and r_+ the radius of the horizon. In order to obtain the irrotational congruence, one has to multiply ξ_ϕ not by a constant but by a function of coordinates. This combination is in fact given by

$$\chi = \xi_t - \left(\frac{\xi_t \cdot \xi_\phi}{\xi_\phi \cdot \xi_\phi}\right)\xi_\phi,$$

which is no longer a Killing vector field unlike ζ. We studied the global properties of this congruence in an arbitrary stationary spacetime with axial symmetry. We were able to prove some interesting theorems related to χ and also prove that it became null on the horizon provided conditions for orthogonal transitivity were satisfied. The congruence is orthogonal to hypersurfaces defined by t =constant, where t is the global time. These happen to be maximal hypersurfaces. Not only was our paper describing this work [7] accepted very quickly, but also we were rewarded with a glowing referee's report that considered the paper to be "impeccable." Rather a rare occurrence indeed!

The reference frame adapted to χ constitutes the general relativistic analogue of the Newtonian rest frame. It is consequently advantageous to study physical phenomena as seen by these "rest" observers. Abramowicz and coworkers have recently utilized this in defining the analogues of inertial forces [8]. In an axisymmetric stationary spacetime one has in general gravitational force G_a, centrifugal force Z_a, and Coriolis force C_a. Furthermore, they are intimately connected to gyroscopic precession. My young colleague Rajesh Nayak and I have studied this relationship between gyroscopic precession on the one hand and the inertial forces on the other [9]. These relations have striking resemblance to the results of the work on the charged particle dynamics in homogeneous electromagnetic fields. The electric field of the latter case is now replaced by a quantity proportional to the particle acceleration and the magnetic field by a combination of Z_a and C_a which reduces to only the centrifugal force Z_a in static spacetimes. This study in a sense has had its evolutionary origins in the work I did with Engelbert and his students Eli Honig and Richard Greene. Our investigations will, we expect, continue further. After all, the past, present, and future are connected by worldlines frozen in time.

All my research before joining Engelbert had been entirely in the area of black holes: geometric structure, stability, and quasi-normal modes. As I have mentioned earlier, I was not sure how much interest Engelbert had in this direction. But he was indeed concerned with some of the puzzling questions related to black hole physics.

Those were the days in which research in this field was gathering momentum and finally finding recognition. Some of the ideas, both in theoretical aspects and especially astrophysical implications, were in a fluid state and therefore to some extent confused. Engelbert gave a talk on black holes. Once again, as in the case of his Cornell lectures, one sentence of his has stuck in my memory: "Astronomers often try to make important discoveries by combining the observational techniques of ESP with the theoretical methods of science fiction."

Our work with Richard Greene belonged to black hole physics. I started working with Engelbert on another intriguing problem related to the Kerr metric. Brandon Carter [10] had shown, while studying the geodesics of the Kerr spacetime, that there existed a constant of motion quadratic in the geodesic 4-momentum p^a:

$$Q_{\text{kerr}} = K_{ab} p^a p^b,$$

where K_{ab} is a Killing tensor satisfying the equations

$$K_{ab} = K_{ba}$$

and

$$K_{(ab;c)} = 0,$$

where the parentheses denote symmetrization. We wanted to interpret this quadratic constant Q_{kerr} as something like the square of the angular momentum of the particle tracing the geodesic. The Schwarzschild spacetime, because of its inherent spherical symmetry, admits three noncommuting Killing vector fields ${}_xL$, ${}_yL$, and ${}_zL$. Here

$${}_zL \equiv \xi_\phi = \frac{\partial}{\partial\phi},$$

the axial Killing vector. The scalar products ${}_xL_a p^a$ etc. give the angular momentum components in the x, y, and z directions respectively. If we set $a = 0$ in Q_{kerr} of the Kerr spacetime, thereby going over to the Schawarzschild spacetime, we obtain

$$Q_{\text{sch}} = ({}_xL_a p^a)^2 + ({}_yL_a p^a)^2 + ({}_zL_a p^a)^2,$$

This is indeed the square of the angular momentum. Or equivalently

$$K_{ab} = {}_xL_a\, {}_xL_b + {}_yL_a\, {}_yL_b + {}_zL_a\, {}_zL_b,$$

which is a degenerate Killing tensor. In the Kerr metric, which is axially symmetric, only the Killing vector ${}_zL$ exists in addition to the timelike Killing vector but not the other two Killing vectors ${}_xL$ and ${}_yL$. The question now was whether it is still possible to construct two vector fields ${}_xL$ and ${}_yL$ in the Kerr geometry which are not Killing, but along with ${}_zL$ satisfy the angular momentum commutation relations, namely,

$$[{}_xL, {}_yL] = -{}_zL$$

If so, could one decompose K_{ab} of the Kerr spacetime in terms of these vector fields exactly as in the Schwarzschild spacetime? Could one further relate these fields to some sort of precession of the angular momentum vector?

We tried very hard to answer these questions, working whenever and wherever possible. We even worked in the neighborhood children's park, Engelbert keeping one eye on the calculations and the other on his little son, whom he was baby sitting. At one point another kid squirted water on us, almost washing out our precious but nonfunctional formulae. Engelbert reprimanded the kid for his attempt to sabotage science. The mothers in attendance refused to appreciate the small step we were trying to take, which would mean a giant leap for general relativity. They rebelled against us, insisting that children could do anything they liked in a children's park and suggesting that we had better take our scientific endeavour elsewhere. A shrewd strategist, Engelbert played the role of a retreating relativist with a silent minority of one, namely me, following him.

Anyway, coming back to the Killing tensor, by transforming to the Kerr-Schild coordinates K_{ab} could almost be cast into the required form, but not quite. Abbas Faridi, another graduate student, joined the quest. But no go. However, this goal can be reached not through K_{ab} but through the Killing-Yano tensor F_{ab}, which was obtained later on by Floyd [11]. This antisymmetric tensor is like the "square root" of K_{ab},

$$K_{ab} = F_a{}^c F_{cb}; \quad F_{ab} = -F_{ba}$$

and satisfies the equation

$$F_{a(b;c)} = 0.$$

As a result, the vector $J^a \equiv F^a{}_b p^b$ is parallely propogated along the geodesic.

With this new information in hand, I opened up the original problem again after returning to India for good and worked on it with Joseph Samuel [12]. The close resemblance of F_{ab} to that of the flat spacetime could be demonstrated by transforming to the Kerr-Schild coordinates (x^0, x, y, z), in which the metric takes on the familiar form,

$$g_{ab} = \eta_{ab} + 2H(x, y, z)l_a l_b.$$

In these coordinates the spatial part of F_{ab} has exactly the same form as in the flat spacetime:

$$F_{ab} = \epsilon_{abcd} x^c t^d,$$

where t^d is the timelike Killing vector and x^c stands for either the Cartesian coordinates or the Kerr-Schild coordinates (x, y, z). We can therefore define the angular momentum operators of the Kerr spacetime, in the same way as in the flat spacetime leading to the same commutation relations. The surface on which the angular momentum operators act is given by

$$\frac{1}{2} F_{ab} F^{ab} = x^2 + y^2 + z^2 = \text{constant}$$

which is the "sphere" in the Kerr spacetime. Furthermore, to first order in the Kerr parameter a, we have the equations for the precession of the angular momentum about the z-axis:

$$ {}_x\dot{L} = \omega\, {}_yL; \qquad {}_y\dot{L} = -\omega\, {}_xL; \qquad {}_z\dot{L} = 0, $$

where

$$ \dot{L} \equiv (L^a p_a)_{;b}\, p^b $$

and

$$ \omega = \frac{ma}{r}\, l^a p_a. $$

All this and some more considerations worked out quite well. In 1985 I visited Engelbert at the New York University and gave a seminar on the above findings. He was very happy. But, unfortunately, our paper ran into the usual problems with the referees. In the meantime I discovered, to my utter amazement, our old friend Abbas Faridi had not only come up with results similar to ours, but had also published them very recently [13]. This almost convinced me of the existence of telepathy. We did not pursue our efforts to publish our paper further. Questions such as the possibility of obtaining precession equations for the angular momentum without linear approximation in a as well as further investigation of the properties of the angular momentum and possible applications to astrophysical situations still remain an unexplored territory.

One of the memorable events that occurred during my tenure at the New York University was Professor S. Chandrasekhar's visit to the physics department. Chandra gave a beautiful talk on rotating fluids. I had met Chandra at the University of Maryland three years earlier. This was the time when Chandra was just getting interested in black holes. I had given him the correct equations governing perturbations of the Schwarzschild metric. Now we discussed the work I had done on scattering of gravitational radiation by the black hole, in particular the quasinormal modes. In the evening Engelbert gave a party. At that time Engelbert lived in the East Village, which was known for its Bohemian life as well as a nonnegligible crime rate. Engelbert's apartment was predominantly furnished with crates retrieved from supermarkets and topped with cushions to sit on. Normally I occupied orange crates, leaving the apple crates to senior people. Chandra, impeccably dressed in a gray suit as always, did not sit down and left early so as not to miss his customary bedtime. The party, however, continued. Somewhat late in the evening, the bell rang and Engelbert opened the door. In rushed an excited young couple. Behind them stood two burly gentlemen. Engelbert invited them in, waving his arms expansively in a gesture of welcome. To Engelbert's great disappointment, instead of accepting his invitation, the two men took to their heels. These two gentlemen happened to be muggers who had been chasing the young couple! The party went on till the wee hours in the morning.

Around five or so, Engelbert was awakened from his deep, tired slumber by the ringing of the telephone. It was Chandra calling. There was no hot water in his hotel and he could not shave. The pipes had frozen and it would be at least a couple of hours before he could have running hot water. This delay in shaving would throw Chandra's schedule into unthinkable confusion, hamper his calculations and ruin his day. So Engelbert had to arrange with Larry Spruch, another professor who lived close to Chandra's hotel, to have hot water fetched to Chandra so that he could shave in time.

It is dusk. Hemmed in on all sides by tall buildings, one never sees the sunset in New York. The sun just dissolves in the smog, leaving behind a gray twilight. I finish my light evening meal in the little German restaurant Zum Zum. Zum Zum Zum Zum .... written around the restaurant. Bratwurst with fried onion washed down with a mug of beer followed by strudel and tea. I walk down to the Avenue of the Americas and hang around the Washington Square for a while. The dying strains of a distant saxophone wafts across the square while the tired girl in the faded overalls gathers up the unsold posters. As I near the newly constructed physics building, a musty old figure suddenly materializes from the side alley. I am scared. One is always scared walking the mean streets of New York. But I have never been mugged or murdered in New York. Only happy memories. "Brother, can you spare a quarter for a glass of beer?" the man stammers. I am about to fish out two quarters out of my pocket—one for beer and one for honesty. No one admits that the handout is for a drink; it is for food, medicine or to pay the vet for fixing the cat. The man is frightened. More than I am. "Officer, please don't shoot me. I've done nothing," he pleads. I assure him that I am not a cop and that I do not carry a gun. I give him the two quarters and the alley swallows him up.

The physics building is ablaze with light. Dracula's castle on the Walpurgis Night. Evening students chattering away as they swarm in and out of class rooms. Once a week I gave informal lectures in general relativity to some of the students working with Engelbert and some who intended to do so in the near future. After the lecture, we would retire to one of the neighborhood basement bars to discuss everything under the sun except physics. Engelbert taught an extremely popular course in astronomy meant for nonscience majors. It was also televised as part of the educational program called the Sunrise Semester under the title "Astronomy and Astrology: the Heavenly Twins." In his course he taught the students how to cast horoscopes. Maybe some of them are rolling in money as professional astrologers, who knows! He set hilariously worded problems in astronomy. Something like this: "Henry Kissinger is kidnapped by a bunch of gigantic CIA agents disguised as a gaggle of Croatian dwarfs. He is left to languish in a vertical elevator shaft of an abandoned building without a roof. By consulting the ephemeris concealed in his wrist watch, which is also a two-way wireless unit, he recognizes the star above him as Sirius. What time is it in New York?" Once a week some of us met to discuss current problems in relativistic astrophysics, especially those related to the areas of gravitational collapse, neutron star models, quasars, and so on. Dressed in a lemon yellow shirt and blue jeans, one long leg crossed over the other, Engelbert would sit amidst the graduate students gathered around him and lead the discussions. Once in a while, one of us would present some classic paper. I remember discussing in detail Hermann Bondi's paper establishing the upper limit on gravitational redshift produced by a spherical mass distribution. It was a very nice paper. I presented it in all its mathematical complexity and the students left one by one, unable to stomach the calculations. Only Engelbert, his student Bill Wallace, and I remained till the bitter end.

Bill, among other things, was working on the apparent superluminal velocities displayed by some quasars like 3C 279. The model we concentrated on involved a component of the quasar moving almost directly towards the observer with relativistic speed. In such a situation, the motion of the component projected normal to the line of sight onto the celestial sphere is endowed with apparent superluminal velocity. We made detailed computations on the spectral characteristics of the quasar, magnetic fields associated with it and so on. The paper was published in *Astronomical Journal* [14]. Engelbert called our model the "three penny model." Let me quote from the introduction, which Engelbert wrote, in order to explain the reason for choosing this name:

> In our analysis one component is taken as the parent quasar while the other visible component would be a radio source ejected at a relativistic speed in our general direction. Presumably this component would be only one out of many ejects which were emitted isotropically or equatorially by the quasar but remain invisible because the relativistic intensity shifts make them inconspicuous. This model, which presumably goes back to Rees, might be described as the 'three penny model', after the Brecht and Weill opera in which the chorus sings:
>
> Therefore, some are in darkness;
> Some are in the light, and these
> You may see, but all those others
> In the darkness no one sees.

I have not seen any other technical paper that includes such a literary quotation. Engelbert carried out some very elegant calculations in the velocity space of the quasar component in motion. These were never published. They may still be languishing, buried deep in the pile of my ancient notes gathering dust. Perhaps some day I should pull them out and publish them as a joint paper.

For three years I worked as Engelbert's research associate. It was for me a period of education. I learned a lot, not just from the problems we worked on and the physics that went into them. More than that, I learned a great deal from his knowledge, insight, and thoroughness. Working in an area of physics which has become increasingly competitive, he has never wavered from his pursuit of excellence and from his fairness to other researchers. Beyond the confines of science, he is a model of warm friendship and gentleness. His wonderful sense of humour never fails to delight his friends and listeners. The experience I gained in those three years of association with Engelbert has made pathways from the past meandering into the future passing through that strange interlude called the present.

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