THE ROLE OF ROTATION IN STELLAR EVOLUTION

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GENERAL CONSIDERATIONS :

In the past two decades, phenomenal advances have been made in our understanding of stellar evolution. Many of these advances, both qualitative and quantitative, can be traced to the introduction of the Henyey method, which is particularly well-suited for machine computations, and the massive use of high-speed computers. The total number of models calculated (and published) during this period can be estimated to be of the order of a few times 10^4. In all but a minute fraction of these calculations, spherical symmetry has been assumed and no attempt made to include the effects of rotation. This minute fraction will constitute the basis for much of our discussion of rotating stars, but before entering into this discussion it is of interest to consider why rotation has been so widely neglected in the other models.

Basically, there are two reasons for ignoring the effects of rotation. First, these effects are inherently non-spherical, and dropping the assumption of spherical symmetry leads to a substantial increase in the complexity of the equations and in the computing effort required to solve them. Second, spherical stellar evolution theory has been so successful in explaining the observations that the assumption of spherical symmetry would seem to be justified. We must remember, however, that this *a posteriori* justification applies only to those stages of evolution for which we have solid observational tests, particularly since theoretical arguments indicate that these are the stages for which rotation will be *least* important.

The classical observational tests (mass-luminosity and mass-radius relationships, H-R diagrams of clusters, etc.) refer either to the main-sequence and early post-main-sequence stages of evolution or to stars of low mass (such as are found in the globular clusters). Low mass stars are generally slow rotators, an observed fact which is related to the formation of planetary systems and magnetic coupling of surface convective zones to solar-type winds. For the more massive ($\gtrsim 2M_\odot$), rapidly rotating stars, calculations show that in order for rotation to grossly alter the overall structure, the interior must be rotating much more rapidly than the surface layers (see, e.g., Sackmann 1970; Bodenheimer 1971). Otherwise, equatorial mass shedding and loss of angular momentum occurs at velocities too small to affect the central regions. For reasons to be discussed in the next section, main-sequence stars should be fairly close to solid-body rotation and the core contraction during the early post-main-sequence stages is of insufficient magnitude to produce any strong differential rotation. Thus, the classical observational tests apply exactly to those cases for which, on theoretical grounds, we would not expect rotation to be important.

The situation changes drastically for the evolution after exhaustion of helium at the center. From the observational standpoint, unambiguous tests either do not exist or have not been successfully met by existing models. From the theoretical standpoint, consider the ratio of the centrifugal force (F_c) due to rotation to the force (F_g) due to gravity. As successive fuels are exhausted and the core contracts, F_c / F_g increases in inverse proportion to the radius of the core, if angular momentum is conserved locally. To see why this is so, write

$$\frac{F_c}{F_g} = \frac{v^2 / r}{GM_r / r^2} = \frac{\omega^2 r}{GM_r/r^2} = \frac{\omega^2 r^3}{GM_r} \qquad (1)$$

where v is the linear velocity, ω the angular velocity, and the other symbols are defined as usual. For conservation of angular momentum, $\omega \propto r^{-2}$, so $F_c / F_g \propto r^{-1}$. In a $7M_\odot$ star evolving from the main-sequence to carbon detonation, the core density increases by a factor of $\backsim 10^9$, implying a decrease in radius of a factor of 10^3. Thus, in order to reach carbon detonation without first encountering critical rotation, the main-sequence counterpart must have $F_c/F_g < 10^{-3}$ in all of the core (the inner $1.4M_\odot$ region). Model calculations (Endal and Sofia 1976a) show that, for a typical surface-rotation velocity of 320 km/s on the main sequence, this condition is not met. Obviously, some angular momentum redistribution will occur but the model calculations referred to above indicate that this only makes the situation worse by moving angular momentum from the very central regions outward to the edge of the helium-exhausted core, where F_c / F_g is already at a maximum. It is interesting that carbon detonation in stars of approximately this mass represents one of the clear-cut failures of non-rotating models in predicting the observations (see Buchler, Mazurek and Truran 1974, for a recent review of this problem). There are several other notable failures of spherical stellar-evolution theory for which rotation may be critical. These will be discussed later.

ANGULAR MOMENTUM REDISTRIBUTION :

Before we can attempt the realistic evolution of a rotating star, it is necessary to consider the mechanisms which can redistribute angular momentum in such a star. At present, these mechanisms are poorly understood and we can only make order-of-magnitude estimates for most of them. In some cases (the role of magnetic fields, for instance) the theoretical picture is so unclear that the best strategy is to ignore them completely and test the resulting models against

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the relevant observations. If these models fail to predict the observations, then there is at least a good possibility that the mechanisms which have been left out are important. This is, of course, the strategy implicitly adopted in leaving rotation completely out of most previous investigations of stellar evolution.

(a) Redistribution in Radiative Regions :—

Even the most elementary considerations will show that angular momentum transport due to the viscosity of the gas and radiation in a star cannot have any noticeable effects. The time scale for such a randomwalk process is

$$\tau \simeq \frac{D^2}{\nu}, \tag{2}$$

where D is the distance over which angular momentum is to be transported and ν is the kinematic (microscopic) viscosity. A typical value for ν is of the order of 10 cm^2/s and ν is invariably less than 10^3 cm^2/s in any (non-crystalline) gas found in stellar interiors. With a typical stellar dimension of one solar radius this gives $\tau > 10^{11}$ years, which is much longer than any stellar-evolution time scale.

(i) MERIDIAN CIRCULATION : Historically meridian circulation has been the most widely considered mechanism for redistributing angular momentum. This type of circulation was first proposed by Eddington (1925, 1929) and Vogt (1925) in order to circumvent the von Zeipel (1924) paradox. von Zeipel showed that, in a rotating star, the rate at which energy is carried away by radiation has a latitude dependence which cannot be duplicated by the local energy - generation rates. To solve this paradox, which follows from the equations of hydrostatic and radiative equilibrium, Eddington and Vogt suggested that a circulation is set up which exactly balances the von Zeipel effect. As a result of a large number of investigations (see the reviews by Mestel 1965 and Kippenhahn 1974), the time scales for meridian circulation are fairly well known. These time scales are of the order of

$$\tau_{MC} \simeq \left(\frac{F_c}{F_g} \right) \tau_{KH} \tag{3}$$

where τ_{KH} is the Kelvin-Helmholtz time scale ($\sim 2 \times 10^7$ yr for the Sun). Exceptions occur in regions of low density and near convective zones, where the time scales may be considerably shorter than indicated by equation (3). A particular important result was obtained by Mestel (1953), who showed that meridian circulation can be choked off by regions of varying mean molecular weight (μ-barriers). In fact, μ-barriers can quench all the angular momentum redistribution mechanisms which we will discuss here. This is the basis for assuming, in the discussion of carbon detonation, that angular momentum will not be transported across the edge of the helium-exhausted core. One problem which has not been solved is that of the angular momentum distribution created by meridian circulation. Circulation-free distributions do not exist (Baker and of Kippenhahn 1959) but the typical Reynolds numbers of the flows are supercritical and the large (macroscopic) viscosities associated with turbulence may tend to enforce solid-body rotation in any stages of evolution which last much longer than τ_{KH}. The main sequence is, of course, the best example of such a stage.

In addition to meridian circulation, there are a number of other mechanisms which can redistribute angular momentum. These mechanisms, which are associated with various instabilities of differential rotation, can be classified as either dynamical or secular, according to the type of analysis used in deriving the stability conditions and according to the time scales involved.

(ii) DYNAMICAL INSTABILITIES : The dynamical-instabilities follow from considering adiabatic perturbations in a rotating (or shearing) fluid. The time scales for growth of such perturbations are of the order of the free-fall time scale ($\sim$1 hour for the Sun). Thus, the angular momentum distribution in a star evolving on a Kelvin-Helmholtz or nuclear time scale should always satisfy the requirements of dynamical stability. For axisymmetric perturbations, the stability criteria are expressed by the vector Solberg-Hoiland condition (Wasiutynski 1946). For non-axisymmetric (swirling)) displacements, the stability condition is usually expressed in terms of the Richardson number :

$$R_i = \frac{g}{c_p} \frac{dS}{d\zeta} \Big/ \left(\frac{dv}{d\zeta}\right)^2 , \tag{4}$$

where ζ is a displacement length, g the component of gravity along ζ , c_p the specific heat at constant pressure, and S the specific entropy. The condition for stability is that R_i be greater than some critical value which lies somewhere between 2 and 1/4 (Yih 1965). The importance of this instability, known as a shear instability, becomes apparent when one considers displacements in the horizontal direction with respect to gravity. In this case, there is no entropy gradient (or gravity component) and any differential rotation is unstable. Thus, as has been emphasized by Zahn (1975), a star should maintain a very close approximation to a spherical distribution of angular velocities throughout its evolution.

(iii) SECULAR INSTABILITIES : The conditions for dynamical stability define the balancing point between the destabilizing effects of a velocity gradient and the stabilizing effect of an entropy gradient. This latter gradient may be thought of as being composed of two components : a gradient in ρ/μ (or temperature) and a gradient in the mean molecular weight, μ(ρ is the density). Because of the small Prandtl numbers ($P_r = \nu/\varkappa$, where $\varkappa$ is the thermal diffusivity) found in stellar interiors*, the cooling time for a perturbation is much shorter than the time scale for angular momentum diffusion. Thus, it is valid to consider a slow (non-adiabatic) perturbation which maintains both pressure and radiative equilibrium with its surroundings but retains its original angular momentum. The effect of this is to substantially reduce the stabilizing effects of a gradient in ρ/μ and this leads to the secular

* The Prandtl number may be considered as the ratio of the momentum and heat transport rates. A typical value for stellar interiors is $P_r = 10^{-6}$.

instabilities. For an axisymmetric perturbation, we have the well-known Goldreich-Schubert instability (Goldreich and Schubert 1967; Fricke 1968). We will not discuss this any detail since it has been widely discussed in the literature (cf. James and Kahn 1971). The secular analog of the shear instability was first considered by Townsend (1958) in order to explain the turbulence observed high in the earth's atmosphere. An attempt by Zahn (1974) to apply Townsend's theory to stellar interiors leads to the stability condition

$$R_i \gtrsim 1/R_{crit}\, P_r , \qquad (5)$$

where R_{crit} is the critical Reynolds number ($\sim 10^3$). For typical stellar interior conditions, this raises the critical Richardson number to $\sim 10^3$ to 10^4. The time scale for the Goldreich-Schubert instability is of the order of the time scale for meridian circulation, whereas the secular shear time scale may be considerably shorter (though not as short as the dynamical time scales).

(b) Redistribution in Convective Regions :—

Thermal convection will redistribute angular momentum on a very short time scale. This much is known with great certainty. Unfortunately, we do not know what type of angular momentum distribution it will lead to. The two possibilities seem to be solid-body rotation and something approaching constant specific angular momentum throughout the convective region (Tayler 1973). The latter case would imply infinite angular velocities on the axis of rotation, but this can be avoided by considering quenching of certain modes of convection by rotation. In order to decide which possibility is correct, we need a better understanding of the (possibly anisotropic) turbulent viscosity and of turbulent convection in general.

EFFECTS OF ROTATION ON STELLAR STRUCTURE AND EVOLUTION :

In this section, we will try to give a general view of the effect of rotation on main-sequence and post-main-sequence stars. We will not consider pre-main-sequence evolution because the problems encountered there are of a very different nature. Also, we will not describe the methods used in the calculations. An excellent summary and comparison of the various methods is already available (Papaloizou and Whelan 1973).

(a) Main-sequence structure :—

The effects of non-critical rotation on the structure of main-sequence stars can be understood by thinking of rotation as reducing the effective mass of the star. This is because the effective gravity is reduced below the value which would be computed by considering only the GM/r^2 term. Thus a rotating star has a lower luminosity, a lower central temperature, and a higher central density than its non-rotating counterpart. The increase in central density is a second-order effect due to the decreased importance of radiation pressure. These effects have been explored in detail by Sackman (1970), who finds that, for solid-body rotation and critical surface velocities, the maximum decerease in the luminosity is about 25 percent (the effect is greatest for low mass stars which have higher surface gravities and, thus, spin faster at critical rotation). For the other parameters, the changes are smaller and can be thought of as being due to, at most, a 4 percent decrease in the effective mass. On other hand, for differential rotation, the luminosity decrease can easily be 80 percent (cf. Bodenheimer 1971), though we would not expect to find such strong differential rotation on the main sequence.

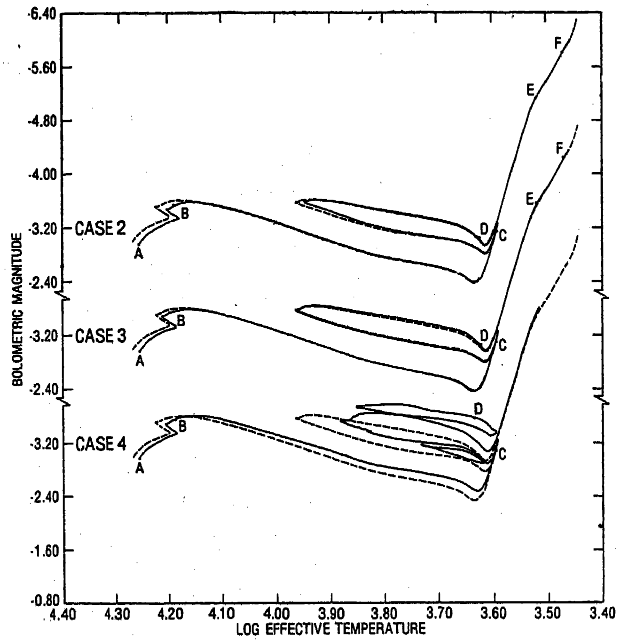

Fig. 1 Evolution in the H–R diagram of rotating (solid lines) and non-rotating (dashed line) $7M_\odot$ stars. In CASE 2, solid-body rotation was enforced on all chemically homogeneous regions (both radiative and convective). This corresponds to assuming that μ-barriers can decouple two regions of differing chemical composition. In CASE 3, no redistribution of angular momentum was allowed. CASE 4 is the same as CASE 2 except that constant angular momentum per unit mass was used in convective regions. The tracks are displaced vertically to avoid confusion. From Endal and Sofia (1976a).

(b) Post-Main-Sequence Evolution :—

We have already indicated what the effects of rotation might be on the carbon detonation scenario. Another possible means of testing rotating models is in the H-R diagram. Figure 1 shows the evolution of a rotating $7M_\odot$ star from the main sequence (A to B), through helium burning (C to D), and on to formation of thin double-shell energy sources (E to F). The solid lines correspond to rotating models with different assumptions about angular momentum redistribution (for more details, see the figure caption) and the dashed lines to non-rotating models. The calculations were performed for stars with typical (observed) rotation velocities (in the main-sequence models) and illustrate that it would be very difficult to test rotating models using the H-R diagram. Except for CASE 4, where constant specific angular momentum in convective zones was assumed, the differences between rotating and non-rotating models are very slight. This is because the effects of differential rotation are strongest in the core, which spins up as it contracts. Unfortunately, detailed calculations have not yet been carried to the point where observational tests in terms of nucleosynthesis in the core or of carbon detonation, for instance, are possible.

FURTHER WORK NEEDED:

The angular momentum redistribution mechanisms are poorly understood and a great deal of work remains to be done here. In the case of meridian circulation, the time scales are fairly well known but the angular momentum distribution produced by the circulation is still unknown. For the various instabilities, we may safely assume that the resulting angular momentum distribution tends towards one which meets the stability criteria. Such criteria do not exist for meridian circulation. An investigation by Sakurai (1972) indicates that the effect of circulation in the Sun *may* be to spin up the core by moving angular momentum toward the center. The results, however, are incompatible with the recent solar oblateness measurements (Hill and Stebbins 1975).

For the dynamical instabilities, the time scales are certainly so short that they may be assumed instantaneous. The secular instabilities, on the other hand, probably have time scales comparable to the time scales for post-main-sequence evolution. In this case, accurate estimates of the time scales are needed and they should be based on nonlinear analysis because the linear analysis is generally misleading. For example, the linear estimates of the time scale for the Goldreich-Schubert instability indicated something on the order of one rotation period (Goldreich and Schubert 1967), whereas even the simplest nonlinear considerations raises this time scale to the order of the Kelvin-Helmholtz time scale (Kippenhahn 1969). A picture of angular momentum redistribution due to magneto-hydrodynamic effects would be nice but this will have to await some basic advances in MHD theory.

In terms of modelling the evolution of rotating stars, a great deal can be done even with our present poor understanding of angular momentum redistribution. All the previous studies have assumed asymptotic angular momentum redistribution laws and have ignored the time-dependent aspects of redistribution. They have also ignored the fact that the known angular momentum redistribution mechanisms all involve chemical homogenization, as well. Efforts are now underway (Endal and Sofia 1976b) to include the time-dependence and chemical mixing but there is still plenty of room for more investigations.

In looking for problems to tackle with models of rotating stars, a good guide is to look at the areas where non-rotating models have failed or have run into severe problems. Some of these areas are :

(1) the carbon detonation scenario - non-rotating models indicate that intermediate-mass stars ignite carbon in an explosive manner which leads to complete disruption. This is completely at variance with numerous observational results.

(2) surface composition anomalies - many stars show atmospheric abundances which must have originated from nuclear reactions within the stellar interiors. The difficulty lies in bringing the elements to the surface. While the helium shell flashes were once thought to be the answer, detailed calculations indicate that the resulting mixing is only effective for a small range of masses. The mixing associated with angular momentum redistribution might help. This might also have an impact on the problem of the solar surface lithium abundance (cf. Durney 1976).

(3) the final stages of stellar collapse - this cannot be considered an area of failure for non-rotating models. because detailed calculations are not yet available. However, the arguments presented in the first section with respect to carbon detonation are even more compelling here because the increase in density is so much greater (see Sofia 1971).

(4) solar neutrinos - arguments presented earlier would indicate that the Sun does not have a rapidly rotating core but the situation with regard to the missing solar neutrinos is so desperate that this possibility should be re-examined. Rotation is certainly capable of lowering the neutrino flux to the present observed upper limits (Demarque, Mengel and Sweigart 1973; Bahcall and Davis 1976) and, as long as the rapid rotation is confined to a small enough region, the oblateness limits can also be satisfied. This solution certainly does less harm to present physical conceptions than most of the solutions which have been proposed (and have failed).

The above list is not exhaustive, but it should keep researchers busy for quite some time.

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