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A POSSIBLE P–L RELATION FOR RR-LYRAE VARIABLES IN GLOBULAR CLUSTERS

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Abstract. A study of the mean photographic magnitudes of RR Lyrae variables in galactic globular clusters has shown that the luminosity of RR-ab variables decreases with period. It is found that RR-ab variables, in the cluster ω Centauri, whose distances are greater than 8 arc min from the centre of the cluster, are less luminous than such variables seen in the inner region. The possibility of two transition periods for RR-ab variables, one near $0\overset{d}{.}4$ and the other near $0\overset{d}{.}9$, is suggested to explain the sharp boundaries of the instability gap and of the periods of RR Lyrae variables. It is also shown that the inner RR Lyrae variables in ω Centauri are more massive than the outer ones.

1. Introduction

Mean absolute magnitudes of RR Lyrae variables form the basis for the determination of distances of globular clusters and, hence, to the centre of our Galaxy. In the early studies of globular clusters by Shapley (1930), it was assumed that RR Lyrae stars have a mean photographic absolute magnitude of zero. Later studies on absolute magnitudes of these stars by fitting the Main Sequence or the horizontal branch (HB) of the color-magnitude (CM) diagrams to that of clusters or groups of stars with known absolute magnitudes have shown that the luminosity of RR Lyrae stars differ from cluster to cluster. Large dispersion of magnitudes of RR Lyrae stars is seen within the same cluster. There exists lack of agreement between theoretical predictions and observational results regarding the absolute brightness of RR Lyrae variables. Christy (1966), from theoretical considerations, predicts that RR Lyrae variables of Oosterhoff group I clusters must be fainter than those of group II clusters. But results obtained by Sandage (1969) from Main-Sequence fitting of CM diagrams show just the opposite.

Light curves in two colors were determined for 78 RR Lyrae variables in M3 by Roberts and Sandage (1955) to derive the pulsation characteristics of selected RR Lyrae variables. Results show the sharpness of the red and blue boundaries of the variable gap in the horizontal branch of the H–R diagram of this globular cluster. An interesting result of the investigation was that mean photographic magnitudes show a correlation with period in a sense that RR Lyrae variables of longer period are found fainter than RR Lyrae variables of shorter period. This is contrary to what is seen in the case of Cepheids of longer periods. This also is against the present notion that the photographic magnitude of RR Lyrae variables is independent of their period. The purpose of the present investigation is to see whether what Roberts and Sandage (1955) have observed in M3 exists in other clusters using data available in the *Third Catalogue of Variable Stars*

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by Hogg (1973). Such a relation may be used to revise the distance to RR Lyrae variables in the general field and to the galactic globular clusters.

2. Observations

Helen Sawyer-Hogg's *Third Catalogue of Variable Stars* in globular clusters published by David Dunlop Observatory (1973) comprising 2119 entries, gives periods and photographic magnitudes at maximum and minimum brightness of variables. Twenty-six clusters have more than 20 variables and 5 have more than 100. About 82% of these variables belong to the short period RR Lyrae-type.

Two clusters best suited for the study of the period luminosity relation among RR Lyrae-variables in our Galaxy are M3 with 182 and ω Centauri with 142 objects (Hogg, 1973). Variables in these clusters are well studied and periods and brightnesses accurately determined. Periods of RR-ab variables in M3 cover a range from $0\overset{d}{.}40$ to $0\overset{d}{.}75$ and those in ω Centauri have periods from $0\overset{d}{.}5$ to $0\overset{d}{.}9$. Hence, between these two clusters we get the full range of periods $0\overset{d}{.}40$ to $0\overset{d}{.}9$, what is normally found among RR-ab variables of our Galaxy. M3 is a cluster at high galactic latitude and the reddening is almost zero (Sandage, 1970). Sandage (1970), from a study of the C–M diagram of M3 stars, finds its true distance modulus as 15.03 mag. Scaria and Bappu (1981) from multiband photometry of ω Centauri have shown that this cluster shows no non-uniform reddening. *UBV* photometry of ω Centauri by Dickens and Woolley (1967) gives $E_{B-V} = 0\overset{m}{.}11$ and an apparent visual distance modulus $(m - M)_{V\,\mathrm{app}} = 14.10$ mag. These values give a true distance modulus of 13.77 mag. for ω Centauri.

In Figure 1a we have plotted mean photographic magnitudes $\frac{1}{2}(m_{\max} + m_{\min})$ of variables in M3 against their periods. Roberts and Sandage (1955) have pointed out that brightness determinations of stars in the inner region of M3 are affected by background contribution. Scaria and Bappu (1981) have shown that brightness measurements of stars in M3, with $\gamma \geq 2.5$ arc min, are free from background effects. Hence, variables in M3 are divided into two groups with $\gamma \leq 2.5$ arc min (open circles) and $\gamma > 2.5$ arc min (filled circles) in Figure 1a. Periods of the variables of the inner zone do not show any clear dependence on their luminosity. But it is seen that the luminosity of RR-ab variables of the outer zone show a definite tendency to decrease with period. Roberts and Sandage (1955) have come to a similar result from photographic photometry of a few selected RR Lyrae variables in M3. It is also seen that RR-c stars form a different period-luminosity group.

Variables of ω Centauri have been studied in detail by Martin (1938) and Woolley *et al.* (1966). Dickens and Woolley (1967) have estimated the amount of background contribution with distance from the centre in the *B* and *V* bands. Hence, it is possible to estimate the contribution to the mean photographic magnitudes of variables in ω Centauri at any distance from the centre of the cluster. This contribution is about $0\overset{m}{.}12$ in the *B* band for a star at a radial distance of 2 arc min. Dickens and Woolley (1967) have shown that background contribution in ω Centauri is near zero beyond a radial distance of 5 arc min. Mean magnitudes of all RR-ab variables were corrected

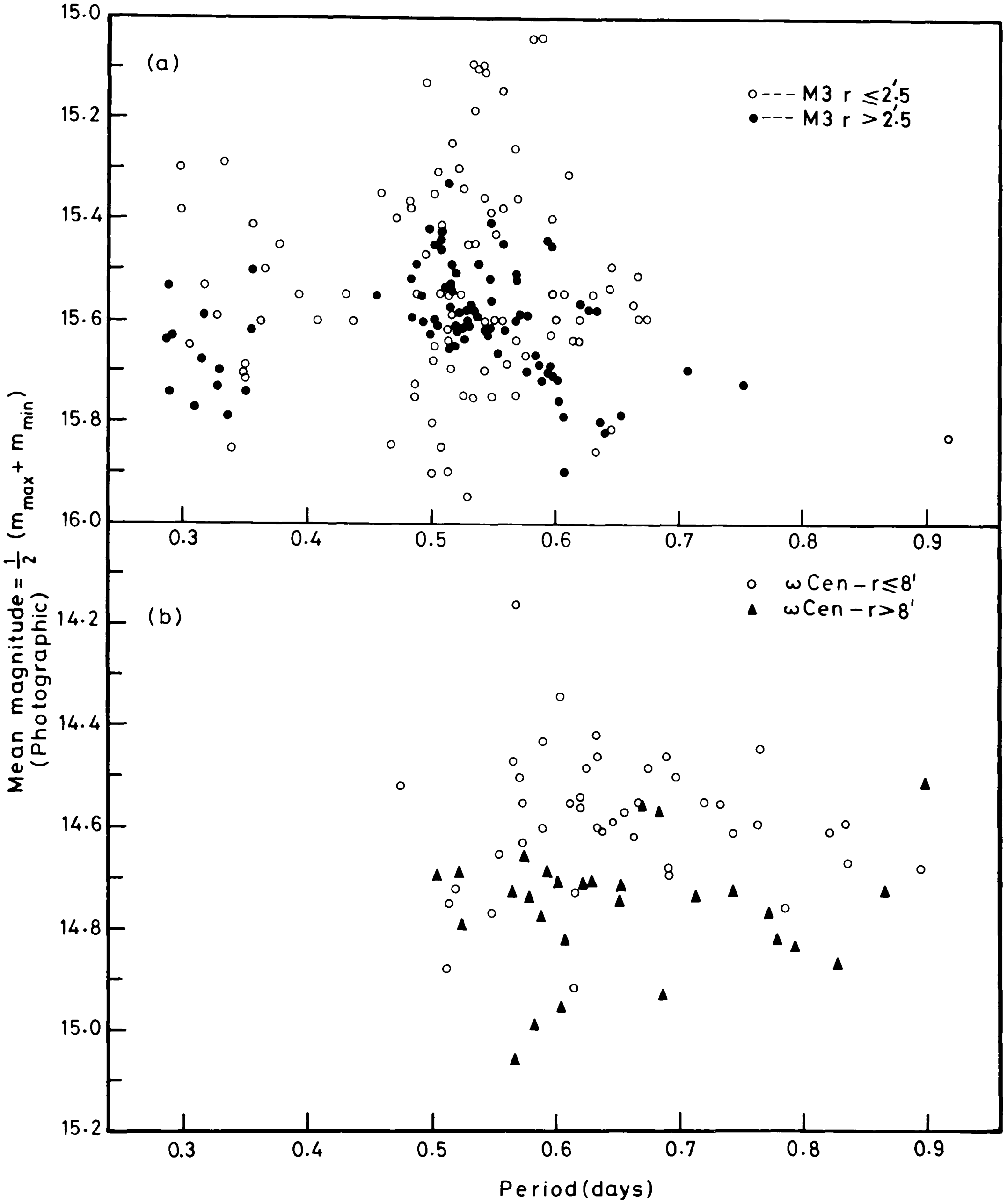

Fig. 1(a). Period/mean photographic magnitude diagram for RR-ab and RR-c variables of M3. Open circles are variables with $\gamma \leq 2.5$ arc min and filled circles are variables with $\gamma > 2.5$ arc min.

Fig. 1(b). Period/mean photographic magnitude diagram for RR-ab variables in ω Centauri. Open circles are variables with $\gamma \leq 8$ arc min and filled triangles are variables with $\gamma > 8$ arc min.

for background contribution using the data provided by Dickens and Woolley (1967).

Figure 2 is a histogram showing the number of RR-ab and RR-c variables at different distances from the centre of ω Centauri. Each distance interval is 2 arc min. The histogram shows that RR-c variables in the cluster have a different distribution than that of RR-ab variables. There is definite increase in the number of RR-c variables relative

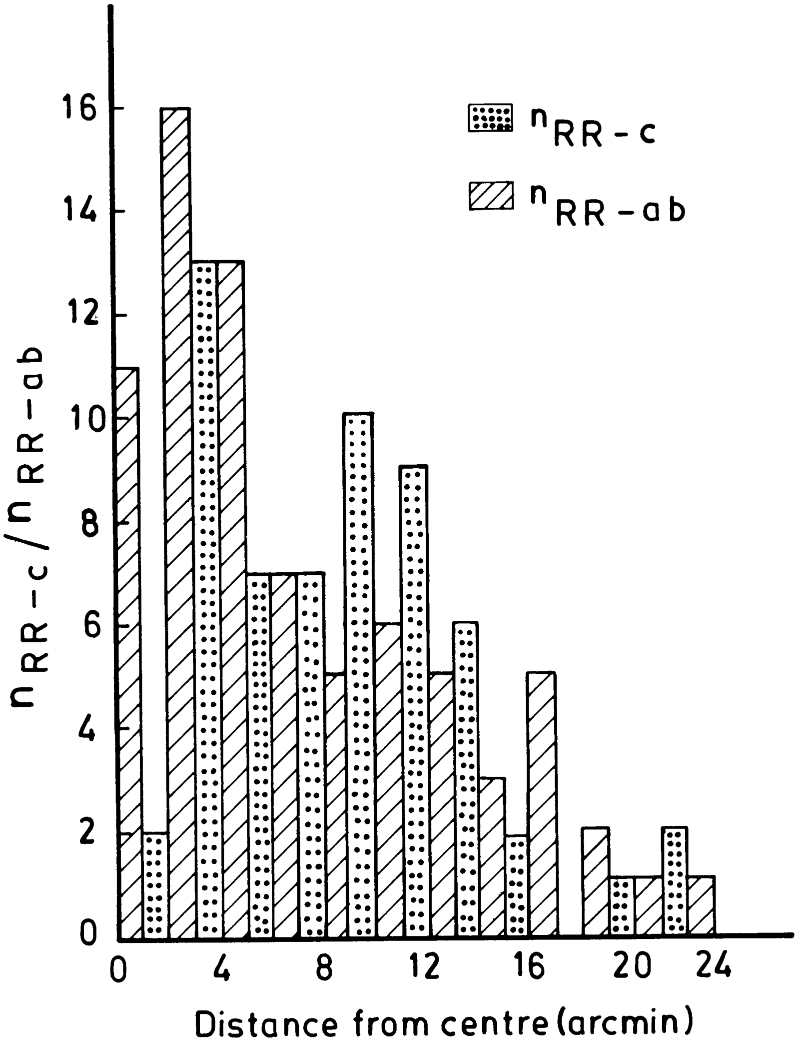

Fig. 2. Histogram showing the change in the number of RR-ab and RR-c variables in ω Centauri with distance from the centre of the cluster.

to the RR-ab variables beyond 8 arc min from the cluster centre. This is not the result of any selection effect. ω Centauri is a cluster of low central concentration (Shapley's class VIII). Horizontal branch stars can be easily identified and measured even at the centre of the cluster (Scaria and Bappu, 1981). The cluster has been thoroughly searched for variables (Martin, 1938; Woolley *et al.*, 1966; Dickens and Saunders, 1965; and Geyer and Szeidl, 1970). Except one variable, all variables in the cluster have amplitudes, larger than 0.30 mag. This one variable with amplitude 0.17 mag. and radial distance 3.5 arc min is located in the inner zone ($\gamma \leq 8'$). Martin (1937) and Oosterhoff (1941) have remarked that RR Lyrae-variables are less concentrated towards the centre than the other stars in the cluster ω Centauri. This effect is actually the result of a larger number density of RR-c variables in the outer region of the cluster. The number ratio n_c/n_{ab} is 0.63 for the inner zone ($\gamma \leq 8'$) and is 1.07 for the outer zone ($\gamma > 8'$). RR-c variables pulsate in the first overtone which is $\frac{3}{4}$ of the fundamental period. The larger number of RR-c stars in the outer zone, indicate that the physical conditions of variables in this region may be more suitable for pulsation in the first overtone rather than in the fundamental tone. The cluster variables are divided into two groups (1) with radial distance $\gamma \leq 8$ arc min and (2) with $\gamma > 8$ arc min. In Figure 1b, mean magnitudes of these variables are plotted against their periods. Open circles are RR-ab variables with $\gamma \leq 8$ arc min and filled triangles represent variables having $\gamma > 8$ arc min. It is seen that ω Centauri variables also follow a P–L relation similar to that of M3. It is also seen in the diagram that variables of the outer zone form a separate group and have a luminosity too small ($\approx 0.2^{\mathrm{m}}$) for their periods. Several variables of the inner zone also occupy the same region in the diagram which is occupied by variables of the outer zone. But it is not possible to tell whether these are variables really close to the centre or variables in the outer region, but seen close to the centre because of their closeness to the line-of-sight of the cluster. The lower luminosity of the outer group of stars seems to be real as most of the variables of the inner zone plotted in the figure have radial distances greater than 4.0 arc min and are far enough from the cluster center where background contribution is negligible. Figure 1b contains only RR-ab variables in ω Centauri as they alone seem to show a P–L relation as in the case of M3.

Now the problem is whether we can combine both Figures 1a and 1b so that we have a relation covering variables of periods from $0\overset{\mathrm{d}}{.}40$ to $0\overset{\mathrm{d}}{.}9$. The most important observable difference between clusters of Oosterhoff group I and group II, is the abundance of heavy elements. M3 and ω Centauri have similar spectral classes of F7 (Harris, 1976). Fe/H for M3 is -1.6 (Harris and Racine, 1979). Spectroscopic observations of ω Centauri variables by Butler *et al.* (1978) show that, though there is large spread in Fe/H in ω Centauri, the mean Fe/H for the cluster is close to -1.6, similar to M3. The index of metallicity (Kukarkin, 1972) is 5 for both clusters. Thus the available evidences show that, though M3 and ω Centauri belong to two different Oosterhoff groups, they have comparable metallicities and combining Figures 1a and 1b does not look very inappropriate. The true distance modulus of M3 is 15.03 mag. and that of ω Centauri is 13.77 mag. which gives a difference of 1.26 mag. between their distance modulii. We have normalised the mean photographic magnitudes of RR-ab variables of ω Centauri to the distance of M3, by applying the above correction and these normalised values are plotted in Figure 3 along with variables of M3. Filled circles are variables of M3, whose radial distances are greater than 2.5 arc min, open circles are RR-ab variables in ω Centauri with $\gamma \leq 8$ arc min and filled triangles are RR-ab variables in ω Centauri with $\gamma > 8$ arc min. Figure 3 shows a definite continuity in the P–L relation between variables of M3 and ω Centauri.

3. Discussion

Light variation in variables are often studied by photographic methods and, hence, much of the scatter seen in Figure 3 could be, because of errors, due to photographic photometry. These errors, even in the best measurements, could be as high as 0.1 mag. Even though the scatter is high in Figure 3, there is a definite indication of a decrease in

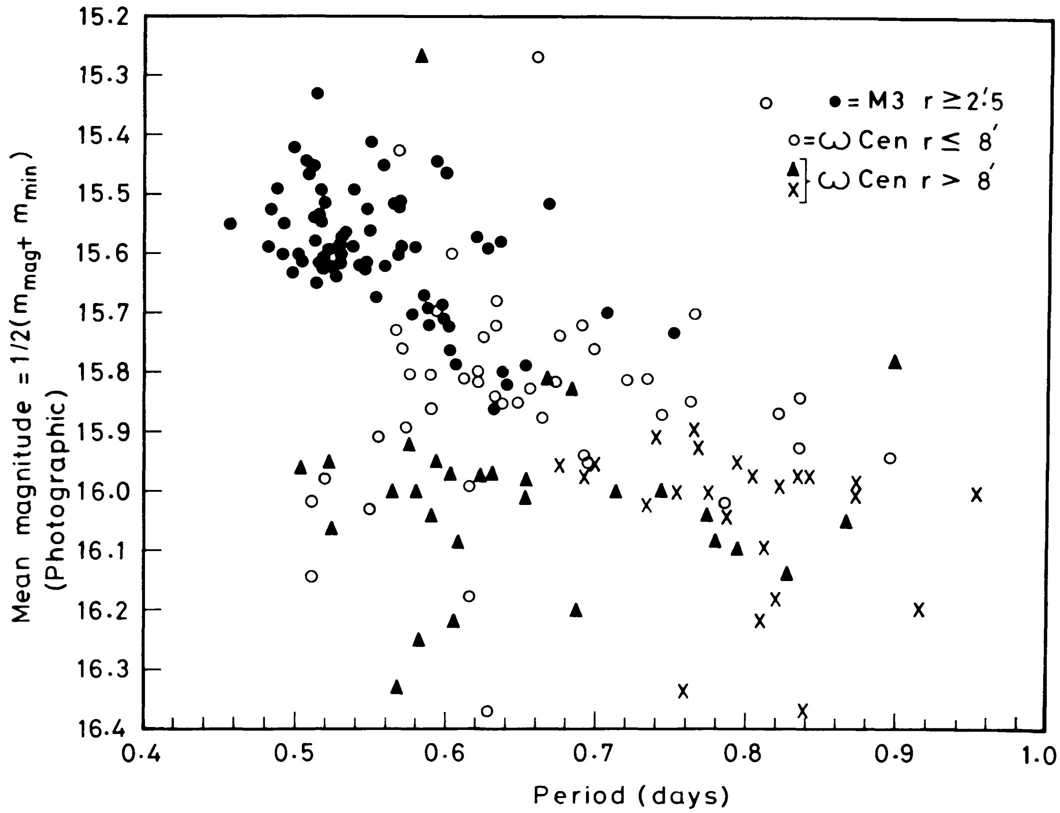

Fig. 3. P–L relation for RR-ab variables in galactic globular clusters.

brightness of RR-ab variables with period. Figure 3 also shows that the group of RR-ab variables found in the outer region of ω Centauri ($\gamma > 8'$) is too faint for their periods. It was mentioned earlier that variables in the outer region of ω Centauri may have preference to pulsate in the first overtone. Probably the RR-ab variables which form this separate group in Figure 3 may also be pulsating in their first overtone. When their periods are converted to fundamental tone by multiplying by the factor $\frac{4}{3}$ and plotted in Figure 3 (crosses), they fall in the straight portion of Figure 3 in places appropriate for their brightness. This shows the possibility of RR-ab stars having two transition regions, one around the period $0\overset{\mathrm{d}}{.}4$ and other around the period $0\overset{\mathrm{d}}{.}9$. A least-squares fit for the straight portion of Figure 3 gives the equation

$$m_{\mathrm{pg}} = 14.54 + 1.91P, \qquad (1)$$

where m_{pg} is the apparent photographic magnitude of RR-ab variables. Since the distance modulus of M3 is 15.03 magnitudes, the absolute photographic magnitude of RR-ab variables is given by the relation

$$M_{\mathrm{pg}} = -0.49 + 1.91P. \qquad (2)$$

The relation is consistent with some of the observations made on the absolute brightness of RR Lyrae variables in globular clusters. Sandage (1969) has shown by Main-Sequence fitting that RR Lyrae variables in globular clusters do not show the same brightness. He finds that RR Lyrae variables of M15 are 0.23 mag. fainter and those of M92, 0.19 mag. fainter than RR Lyrae variables of M3. Mean period of RR-ab variables in M15 is $0^{d}.64$ and in M92 is $0^{d}.62$. These periods are much larger than the mean period of RR-ab variables seen in M3. ‘Period-mean magnitude’ diagrams of RR-ab variables in M15 and M92 were compared with Figure 3 and it was found that RR-ab variables of M15 are 0.20 mag. fainter and those of M92 are 0.15 mag. fainter than RR-ab variables of M3. These values are comparable to the values obtained by Sandage (1969) by Main-Sequence fitting. Since absolute magnitudes of RR Lyrae variables are found to be dependent on their periods, it becomes essential to apply a correction to the distance modulii of those globular clusters whose distances are determined from the mean photographic magnitudes of their RR Lyrae variables and these corrections could be as high as 0.4 mag. in extreme cases. Figure 3 shows that distance determinations of variables from their absolute magnitudes are likely to suffer large errors when their periods are between $0^{d}.50$ and $0^{d}.66$ as this is the region where variables can be pulsating either in their fundamental or first harmonic mode and have a large range in luminosity.

Since RR-ab variables in Oosterhoff group I clusters have shorter periods than RR-ab variables of Oosterhoff group II clusters, it is likely that luminosities of horizontal branch objects in C–M diagrams of group II systems are fainter than those in group I systems. Hence, a part of the contribution to the metallicity parameter ΔV should be coming from difference in the position of the horizontal branch.

Woolley (1967) gives $\langle B \rangle - \langle V \rangle$ colors of 46 RR-ab variables in ω Centauri. These variables have $\gamma > 4.2$ arc min and, hence, their colors need not be corrected for background contribution. In Figure 4 we have plotted the colors of these variables against their periods. Variables with $\gamma \leq 8'$ are plotted with open circles and those with $\gamma > 8'$ are shown with filled circles. It is found that the low luminosity variables have colors comparable to colors of variables of similar period found in the straight line portion of Figure 3. It is also seen that variables of larger period show larger $\langle B \rangle - \langle V \rangle$ colors. It is assumed that RR Lyrae variables are pulsating stars and, hence, must follow the relation $P\sqrt{\rho}$ = constant. When period and color, and, hence, temperature, are the same, a change in luminosity can be only due to a change in mass. Scaria and Bappu (1981) have given evidences of mass segregation in ω Centauri. King (1966) has shown from star counts that there is relative increase in the number density of low luminosity stars towards the outer region of globular clusters and this is explained as result of mass segregation in them. Hence, the presence of low luminosity variables in the outer region of ω Centauri is a direct evidence for mass segregation in ω Centauri and is consistent with the results obtained by Scaria and Bappu (1981) from studies of the distribution of blue horizontal branch stars in the cluster.

There are several observational evidences which support the idea that RR Lyrae variables in ω Centauri belong to two different mass groups. From the period frequency-diagram of RR Lyrae variables in ω Centauri, Iben (1971) has shown the presence of two transition periods for the variables, a major transition period at $\log P_{\text{tr}} \approx -0.24$ and a minor one at $\log P_{\text{tr}} \approx -0.32$. Figure 6 of Iben (1971) shows the relation between the

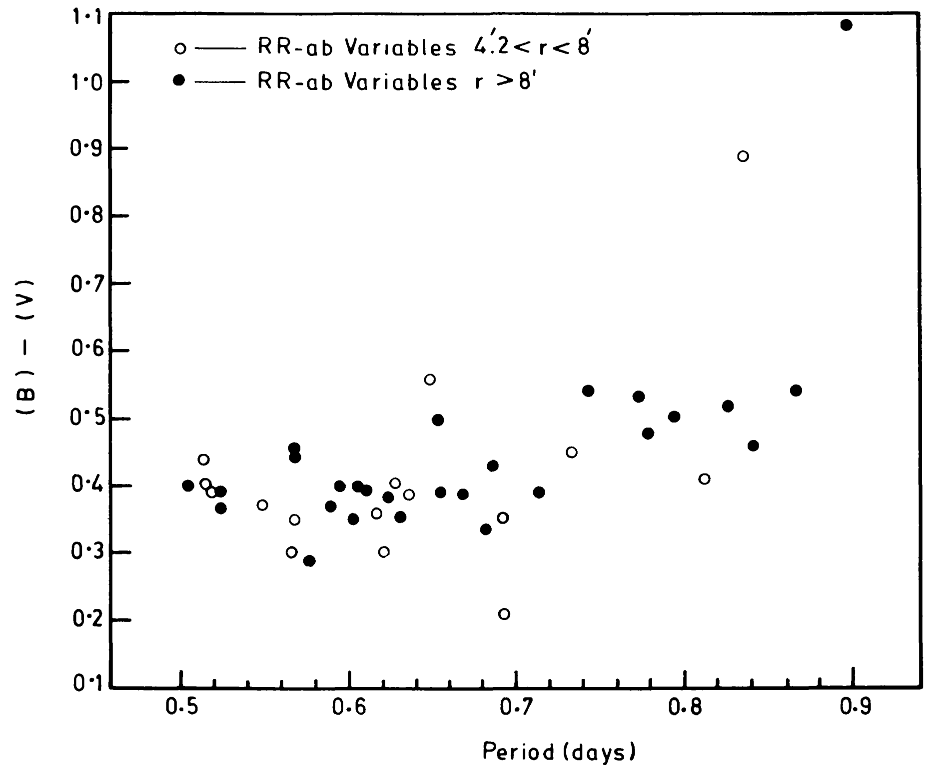

Fig. 4. Period/$\langle B \rangle - \langle V \rangle$ color diagram for RR-ab variables in ω Centauri. Colors are taken from Woolley (1966).

colors and periods of RR Lyrae variables in ω Centauri. This figure shows that the above difference between the transition edges corresponds to a difference of 0.06 mag. in $(B - V)$ color, the one with the larger transition period being the redder of the two. Figure 5 shows the period frequency diagram of RR-c variables in ω Centauri for the inner ($r \leq 8'$) and outer ($r > 8'$) zones. The inner RR-c variables occur in the period range $-0.56 < \log P < -0.36$ and the outer RR-c variables occur in the period range $-0.52 < \log P < -0.26$. ω Centauri is the only globular cluster with the problem of overlap between the RR-ab and RR-c variables in $(B - V)$ color. This problem can be solved if we associate the group of RR-ab variables with the transition edge at $\log P_{\mathrm{tr}} \approx -0.32$, to the RR-c variables of the inner region and the group of RR-ab variables with the transition edge at $\log P_{\mathrm{tr}} \approx -0.24$ to the RR-c variables of the outer region. Under such a classification, the group I (inner) RR Lyrae variables have the RR-ab variables in the period range $-0.32 < \log P < -0.04$ and RR-c variables in the period range $-0.56 < \log P < -0.36$ with the transition edge at $\log P_{\mathrm{tr}} \approx -0.32$ and the blue edge at $\log P_{\mathrm{HBE}} \approx -0.56$, and group II variables have the RR-ab variables in the period range $-0.24 < \log P < -0.04$ and RR-c variables in the period range $-0.52 < \log P < -0.26$, with the transition edge at $\log P_{\mathrm{tr}} \approx -0.24$ and the blue edge at $\log P_{\mathrm{HBE}} \approx -0.52$. The inner group of RR Lyrae variables has a transition edge smaller by 0.08 and the blue edge smaller by 0.04 in $\log P$ than the outer group of RR Lyrae variables. Figure 5 also shows that the inner group of RR-c variables have

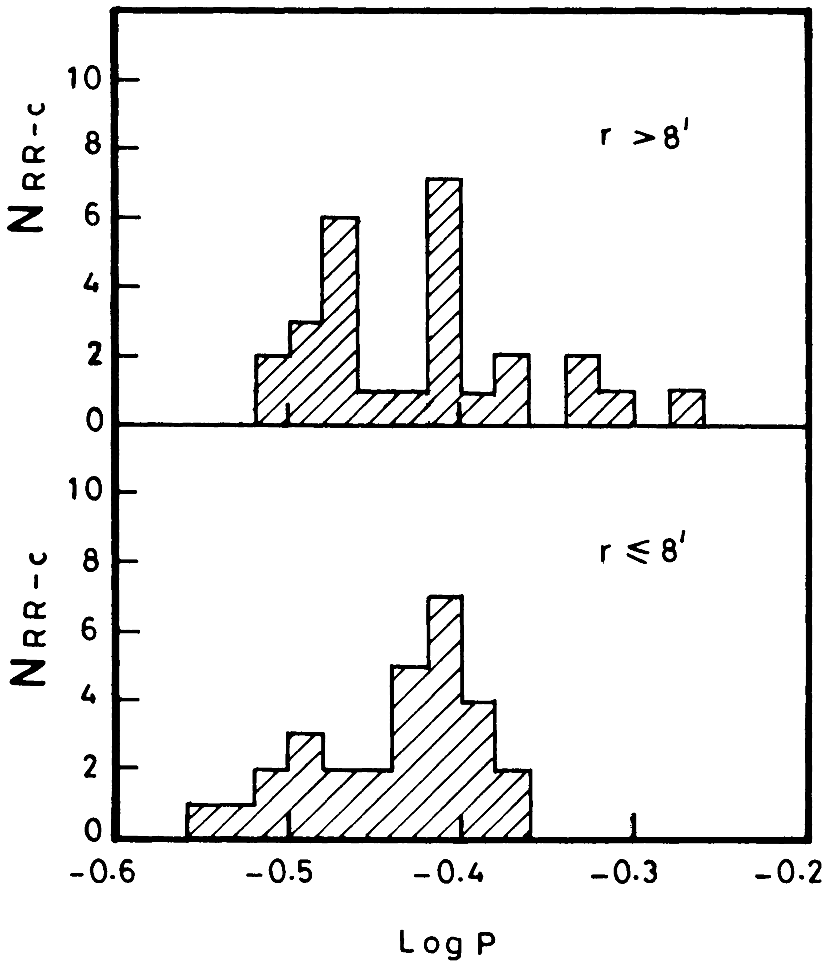

Fig. 5. Period-frequency diagram for RR-c variables in ω Centauri for the inner ($\gamma \leq 8'$) and outer ($\gamma > 8'$) zones.

a smaller period range than the RR-c variables of the outer group. Iben (1971) argues that the presence of two transition periods and two blue edges in ω Centauri is because of two mass groups among the RR Lyrae variables. He points out that transition and blue edges shift to shorter periods, i.e., larger effective temperatures, with increasing mass of the variables. Theory also predicts that the difference in color between blue edges will be smaller than the difference in color between the transition edges. When the above results of Iben (1971) are compared with the transition and blue edges of the inner and outer zones in ω Centauri, as postulated in this paper, the inner group of RR Lyrae variables appears to be of larger mass than the outer group. The difference between the transition edges of the inner and outer group of variables is 0.08 in logarithmic scale. Dickens (1971) gives the equation for the mean absolute magnitude of RR Lyrae stars $M_v = 0.46–4.17 \log P_{\mathrm{tr}}$. It is seen that for a difference of 0.08 in $\log P_{\mathrm{tr}}$, the difference in luminosity is ≈ 0.30 mag. The difference in brightness between the inner and outer variables in Figure 3 is of this order. The only problem is that, theory predicts the inner variables to be fainter than the outer ones, whereas our results show just the opposite. Sandage (1969) points out a similar problem regarding the brightness of RR Lyrae variables of M3 when compared to similar variables in M15 and M92. Christy's (1966) theory predicts the M3 variables to be 0.35 mag. fainter than M92 variables, but by Main-Sequence fitting, Sandage (1969) found them to be 0.35 brighter. Sandage's result is consistent with the result obtained here. The inner region of ω Centauri behaves like a Oosterhoff group I cluster and the outer region like a Oosterhoff group II cluster, though the difference between the inner and outer regions is not as large as one finds between M3 (Oosterhoff group I) and M92 (Oosterhoff group II).

There are evidences which show similar mass differences among the non-variable stars of the horizontal branch in ω Centauri. Dickens and Woolley (1967) have given the H–R diagram of ω Centauri for the inner and outer zones separately. The inner zone is bounded by radii 3.5′ and 8′ and the outer zone is bounded by radii 8′ and 22′. They have used only reliable values for the plot. When the H–R diagrams for the inner and outer zones are compared, it is found that the HB stars of the inner region are ≈ 0.20 mag. redder than the HB stars of the outer region. In the absence of a better explanation, Dickens and Woolley attributed this to background contribution to measurements of the $B - V$ color. This explanation is doubtful because all program stars have radial distances larger than 3.5 arc min and the estimated background contribution in the $B - V$ color is even at a distance of 2 arc min only one third of the above value. According to theories of stellar evolution redder stars along the horizontal branch are more massive than the bluer ones and, hence, the redder inner horizontal branch stars are more massive than the outer ones. There is probably no increase in helium abundance, Y towards the centre of ω Centauri, because, if there were any increase, it would have made the HB bluer in the inner region, rather than redder.

Green (1980) has measured a parameter r, which is essentially a measure of the height to which the red giant branch extends above the horizontal branch. It is found that for distances less than 10 kpc, r increases linearly with distance. Green suggests that a likely explanation for this phenomenon lies in an increase in the helium abundance towards the galactic centre. Globular clusters of Oosterhoff group I are, in general, more metal rich than Oosterhoff group II clusters. From the P–L relation seen in Figure 3, we find that the luminosity of RR Lyrae variables and, hence, the luminosity of the horizontal branch stars, are brighter in Oosterhoff group I clusters than in Oosterhoff group II clusters. Globular clusters closer to the galactic centre are more metal rich than clusters farther away (Morgan, 1959). Hence, the luminosity of the horizontal branch in clusters closer to the galactic centre could be brighter and this could be one of the possible reasons for the small r values obtained by Green for clusters close to the galactic centre. It is likely that the mass-loss mechanism is less efficient in metal rich globular clusters and, hence, their horizontal branch stars are more massive than the horizontal branch stars in metal poor globular clusters.

Iben (1971) has shown the difficulty in reconciling the present observational data on variables in ω Centauri with the existing theories of stellar evolution and pulsation. He suggests to observe the stars in ω Centauri again. *UBV* measurements of all blue horizontal branch stars in ω Centauri are in progress. A wide giant branch for ω Centauri is not very surprising, because the inner and outer regions of the cluster show very different mass, chemical abundance, and dynamical properties.

ω Centauri is a rotating globular cluster and flattened in shape (Scaria and Bappu, 1981). It is possible that some of the horizontal branch stars could be having an excess angular momentum and might have developed fast rotating cores. Rotating cores result in larger core-mass and bluer color (Castellani, 1981). The horizontal branch stars in the inner region of ω Centauri are redder than the outer ones and, hence, it is unlikely that core rotation plays any role among the HB stars of the inner region.

M3 and ω Centauri belong to two different Oosterhoff groups. But their metallicity is the same and both together share a period-luminosity relation as given by Equation (1). Bimodal distribution of mass has been used to explain the horizontal branches of several clusters like NGC 2808, M4, etc. (Harris, 1974; Norris, 1981). The importance of mass as a candidate for second parameter is to be investigated with the above results in mind.

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References

Butler, D., Dickens, R. J., and Epps, E.: 1978, *Astrophys. J.* **225**, 148.

Castellani, V.: 1981, in D. Sugimoto, R. Q. Lamb, and D. N. Schramm (eds.), 'Fundamental Problems in the Theory of Stellar Evolution', *IAU Symp.* **93**, 275.

Christy, R. F.: 1966, *Astrophys. J.* **144**, 108.

Dickens, R. J.: 1970, *Astrophys. J. Suppl.* **22**, 187.

Dickens, R. J. and Saunders, J.: 1965, *Roy. Observ. Bull.* **128**.

Dickens, R. J. and Woolley, R.: 1967, *Roy. Observ. Bull.* **128**.

Geyer, E. H. and Szeidl, B.: 1970, *Astrophysics* **4**, 40.

Green, E. M.: 1980, in J. E. Hesser (ed.), 'Star Clusters', *IAU Symp.* **85**, 441.

Harris, W. E.: 1974, *Astrophys. J.* **192**, L161.

Harris, W. E.: 1976, *Astron. J.* **81**, 1095.

Harris, W. E. and Raccine, R.: 1979, *Ann. Rev. Astron. Astrophys.* **17**, 241.

Hogg, H. S.: 1973, *DDO Publ.* **3**, 6.

Iben, I., Jr.: 1971, *Publ. Astron. Soc. Pacific* **83**, 697.

King, L. R.: 1966, *Astron. J.* **71**, 276.

Kukarkin, B. V.: 1972, in J. D. Fernie (ed.), 'Variable Stars in Globular Clusters and Related Systems', *IAU Colloq.* **21**, 8.

Martin, Chr.: 1937, 'Fotografische fotometric van veranderlijke sterren in ω Centauri', Thesis, Leiden.

Martin, Chr.: 1938, *Ann. Sterrew. Leiden* **17**, 2.

Morgan, W. W.: 1959, *Astron. J.* **64**, 432.
Norris, J.: 1981, *Astrophys. J.* **248**, 177.
Oosterhoff, P. Th.: 1941, *Ann. Sterrew. Leiden* **17**, 4.
Roberts, M. and Sandage, A.: 1955, *Astron. J.* **60**, 185.
Sandage, A.: 1969, *Astrophys. J.* **157**, 515.
Sandage, A.: 1970, *Astrophys. J.* **162**, 841.
Scaria, K. K. and Bappu, M. K. V.: 1981, *J. Astrophys. Astron.* **2**, 215.
Shapley, H.: 1930, *Star Clusters,* Cambridge Univ. Press, Cambridge.
Stobie, R. S.: 1971, *Astrophys. J.* **168**, 381.
Woolley, R.: 1966, *Roy. Observ. Ann.* **2**.