THE DETERMINATION OF A SPECTROSCOPIC BINARY ORBIT

MAHIPAL SINGH

Indian Institute of Astrophysics, Bangalore, India

(Received 21 June, 1983)

Abstract. An efficient analytical method for determining the orbit of spectroscopic binary is presented. The advantages of the method are: (1) its accuracy, (2) its rapidity, (3) the possibility of making adjustments in the elements before the final presentation. This method seems to have very definite advantages in comparison to the other methods. It gives a standard set of velocity curves also, for a set of (e, ω) values with amplitude ranging between $+1$ K and -1 K. These curves in tabulated, or graphical form can be used in routine fashion for the determination of preliminary elements preparatory to a least-squares correction. Application to the velocity curve of o And and δ Ori is given.

1. Introduction

The determination of the elements of a spectroscopic binary system is usually facilitated by the fact that observations have been made over a time long enough to find the period with sufficient accuracy and with enough care in the distribution of them to make known fairly precisely the extreme range in the variation of the radial velocity. With these data it is very simple matter to determine from the velocity curve the other orbital elements. A large number of practical methods have been proposed. The method which gives the best results in one case may not yield in another. Under unfavourable circumstances any method by itself is liable to give untrustworthy results. The best plan would be to determine the approximate elements in several independent ways. The degree of agreement between the results so obtained will be a valuable guide to the judgement.

Wilsing (1893) developed an analytical method applicable to very small eccentricity. Russell (1902) extended this method so that it may be generally available. In this method the observed radial velocity is developed into a trignometric series (Fourier series) and the elements are found by comparing this series with the corresponding analytical expression (Fourier series) for the velocity. In this method the time consumed is considerably longer than the geometrical methods of Lehmann-Filhés (1894), Schwarzschild (1900), and Zurhellen (1907a, b). But is should be very useful in special cases. One such case would occur when a star had a period of about a year, and the maximum or minimum velocity fell in the interval when it was near the Sun. This phase being unobservable, the geometrical methods will give results, which might be seriously in error. Another case in which this method is useful is that of a star attended by two dark companions with commensurable periods. In this case the resultant velocity curve may have several unequal maxima and the geometrical methods fail altogether. This method, however enables us to separate the resultant motion into the two-component orbital motions, except when the perturbations are large.

Astrophysics and Space Science **100** (1984) 13–30. 0004–640X/84/1001–0013$02.70.

In the present paper we present a method for determining the orbits of spectroscopic binaries. The approach is similar to that of Wilsing (1983) and Russell (1902), as for as the expansion in trignometric series is concerned, but differs in certain aspects. In this method instead of expanding the radial velocity V, we expand $(V - V_m)K$, where $V_m = \frac{1}{2}(V_{\max} + V_{\min}) = V_0 + Ke \cos \omega$.

2. Theory of the Method

The conditions of the problem are shown in Figure 1. The XY-plane is taken as the plane tangent to the celestial sphere at the center of motion, the Z-axis being the line-of-sight in which the velocities are measured and perpendicular to XY-plane. The orientation of the X- and Y-axes in space remain unknown.

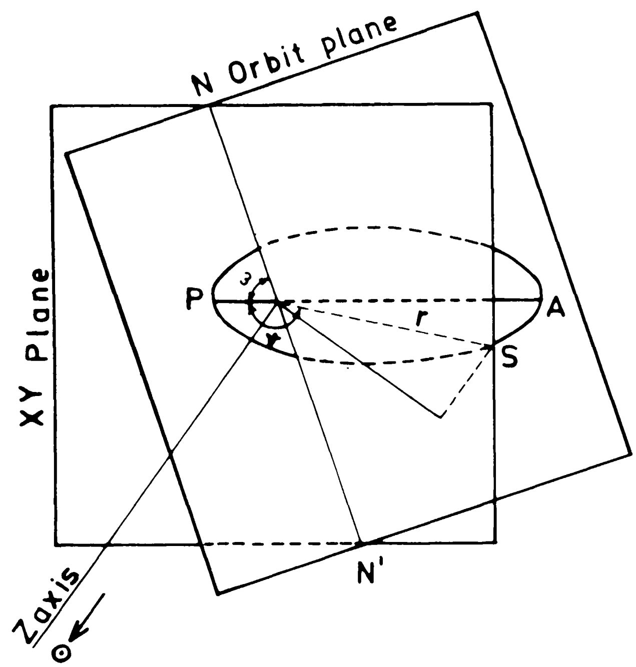

Fig. 1. Schematic diagram of the orbit, and axes of coordinates.

The following notation will be used:

P = period;
n = mean daily motion;
Ω = ascending node on XY-plane;
a = semi-major axis of true ellipse;
K = semi amplitude of the velocity curve;
ω = angular distance of periastran from ascending node;
u = $v + \omega$ = argument of the latitude;
r = radius vector of the star;
t = time;

T = the time of periastron passage;
v = true anomaly;
M = mean anomaly;
E = eccentric anomaly;
e = eccentricity = $\sin\phi$, ϕ = eccentric angle;
V_0 = velocity of center of mass of the system;
V = radial velocity of the bright star;
Z = the projection of r on the line-of-sight.

Then we must have

$$V = V_0 + \frac{\mathrm{d}Z}{\mathrm{d}t} \tag{1}$$

$$= V_0 + K[e\cos\omega + \cos(v+\omega)]. \tag{2}$$

The mean velocity V_m is defined by

$$V_m = \tfrac{1}{2}(V_{\max} + V_{\min}) = V_0 + Ke\cos\omega. \tag{3}$$

Therefore

$$= -e\cos\omega + \cos\omega\cos^2\phi \sum_{p=1} \{J_{p-1}(pe) + J_{p+1}(pe)\}\cos pM -$$

$$-\sin\omega\cos\phi \sum_{p=1} \{J_{p-1}(pe) - J_{p+1}(pe)\}\sin pM, \tag{5}$$

$$= A_0 + \sum_{p=1} (A_p\cos pM - B_p\sin pM), \tag{6}$$

where $J_P(pe)$ is the Bessel function of order p and argument pe,

$$\cos^2\phi = (1-e^2), \tag{7}$$

$$A_0 = -e\cos\omega, \tag{8}$$

$$A_p = \cos\omega\cos^2\phi\, C_p, \tag{9}$$

$$B_p = \sin\omega\cos\phi\, S_p. \tag{10}$$

Now from (7), (8), (9), and (10) one can find out e and ω. That is

$$A_1^2 + B_1^2 = \cos^4\phi\cos^2\omega\; C_1^2 + \cos^2\phi\sin^2\omega\; S_1^2, \tag{11}$$

$$C_1 = 1 - \frac{e^2}{8} + \ldots, \tag{12}$$

$$S_1 = 1 - \tfrac{3}{8}e^2 + \ldots. \tag{13}$$

The accuracy of the values of e and ω depends upon the number of terms retained in the Bessel functions

$$J_p(x) = \frac{x^p}{2^p p!}\left\{1 - \frac{x^2}{2(2p+2)} + \frac{x^4}{2.4\times(2p+2)(2p+4)} - \ldots\right\}, \tag{14}$$

$$e^2 = \frac{(A_1^2+B_1^2)+0.5A_0^2-1}{0.125A_0^2-(A_1^2+B_1^2)-0.75}. \tag{15}$$

As a first approximation and next approximation

$$\begin{aligned} e^2 = {} & -\{0.125A_0^2-(A_0^2+B_1^2)-0.75\} \pm \\ & \pm[\{0.125A_0^2-(A_1^2+B_1^2)-0.75\}^2-(1-0.5A_0^2-A_1^2-B_1^2)\times \\ & \times\{\tfrac{9}{16}-(A_0^2/16)-4(A_1^2+B_1^2)\}]^{-1/2}/(\tfrac{9}{32})-(A_0^2/32)-2(A_1^2+B_1^2). \end{aligned} \tag{16}$$

Further accurate value of e can be obtained by matrix iteration method for solving a polynomial in powers of e as represented by (11).

In order to find A_0, A_1, and B_1, let us divide the period into any even number $p = 2N$ of equal parts, beginning at the epoch to. Let v_0, $v_1, \ldots, v_{2N-1}$ be the corresponding values of $(V - V_m)/K$ (v_0 corresponding to t_0), then

$$A_0 = \frac{1}{2N}(v_0+v_1+\ldots+v_{2N-1}), \tag{17}$$

$$A_1 = \frac{1}{N}\left(v_0+v_1\cos\frac{\pi}{N}+v_2\cos\frac{2\pi}{N}+\ldots+v_{2N-1}\cos\frac{(2N-1)\pi}{N}\right), \tag{18a}$$

$$B_1 = \frac{1}{N}\left(v_0+v_1\sin\frac{\pi}{N}+v_2\sin\frac{2\pi}{N}+\ldots+v_{2N-1}\sin\frac{(2N-1)\pi}{N}\right). \tag{18b}$$

Procedure. The importance of the method lies in its simplicity and accuracy. From the observed radial velocities at different epochs, one has to determine first period by the well-known procedures. The mean velocity

$$V_m = \tfrac{1}{2}(V_{\max}+V_{\min}) \tag{19}$$

is obtained and by plotting the radial velocity curve (velocities at different phase), the half amplitude of the curve, K is obtained. As a next step the curve is plotted again as $(V - V_m)/K$ at different phases. In this presentation the ordinate will be from $+1$ K to -1 K (as shown in Figure 2). This radial velocity curve, is applicable to any set of observed radial velocities, provided, e and ω are the same. Now divide the period into any even number $2N$ of equal parts, beginning at the epoch to. Let v_0, $v_1, \ldots, v_{2N-1}$ be the values of $(V - V_m)/K$ at different epoch (v_0 corresponding to t_0). Then

$$(V-V_m)/K \quad \text{at} \quad t_0 = v_0 = A_0 + \sum_{p=1}(A_p\cos pM - B_p\sin pM), \tag{20}$$

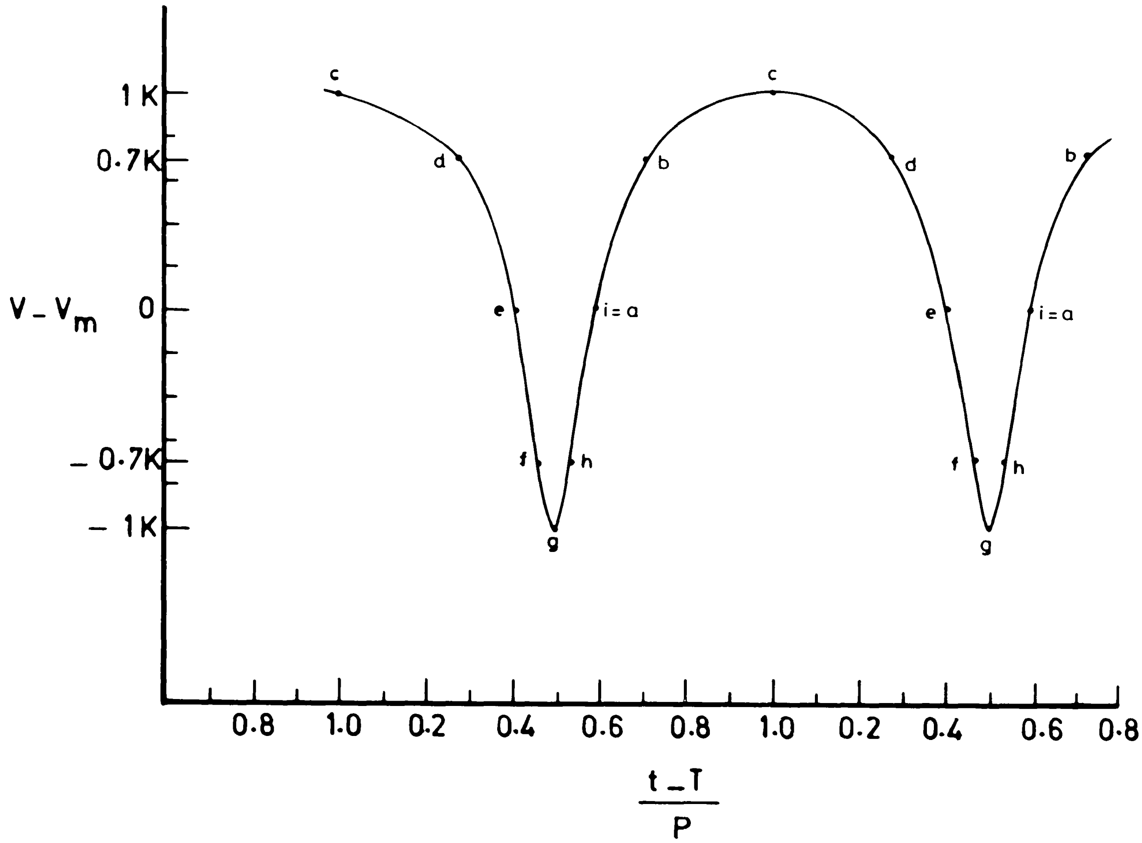

Fig. 2. Radial velocity curve for $e = 0.5$, $\omega = 180°$.

where

$$M = (2\pi/P)(t - t_0). \tag{21}$$

By comparing this series with (5) we get A_0, A_p, B_p as given by Equations (8), (9), and (10). For calculating e and ω only A_0 and A_1 are necessary. The expressions for A_0 and A_1 in terms of $v_0, v_1, \ldots, v_{2N-1}$ are given by (17) and (18). From (15) and (16) one gets the value of e and then from (8) value of ω.

Now to check at the values of e and ω, use expression (5) at $M/2\pi = 0, \ldots, 1$ to calculate $(V - V_m)/K$ at different $M/2\pi$ and plot the curve $(V - V_m/K$ vs $M/2\pi$. If the values of e and ω are correct, this curve should be exactly the same as given in Figure 2. Some improvement can be made in the values of e and therefore of ω, to arrive at the correct curve.

At the time of passing through periastron, $(V - V_m)/K = \cos\omega$, where V = radial velocity at the time of periastron passage T. The velocity of the system V_0 is now obtained from

$$V_0 = V_m - Ke\cos\omega. \tag{22}$$

3. Application to o And

Figures 3 and 4 give velocity curve as radial velocity observed vs phase and as suggested above, respectively, for the determination of the orbit of the spectroscopic binary o And. The observations used are as given in Table I. These were taken from 15 October, 1981 to 5 October, 1982 using a 100 cm reflecting telescope of Kavalur Observatory on 09802 Kodak Plates at Cassegrain focus at a reciprocal dispersion of 17.2 Å mm^{-1} at λ6562.817 Å. The phase has been calculated from JD 2 444 948.147 and a period of 1.674 58 days. We have arrived at the following orbital elements:

$$K = 34.165 \text{ km},$$
$$V_0 = 13.50 \text{ km s}^{-1},$$
$$e = 0.539\,43,$$
$$\omega = 177.046°,$$
$$T_0 = \text{JD } 2\,444\,894.0905,$$
$$P = 1.674\,58 \text{ d},$$
$$a \sin i = 662\,437.8 \text{ km},$$
$$f(m) = 0.004\,13 \text{ solar masses}.$$

The radial velocity curve indicates the presence of gas stream and effect of rotation. This is indicated by the large difference between the computed and observed radial velocities just before and after the primary eclipse. This has been observed by Struve (1949) to be between phase 0.8 and phase 1.0 in U Cephei, SX Cassiopeiae, RX Cassiopeiae, and U Coronae Borealis. The position of the primary

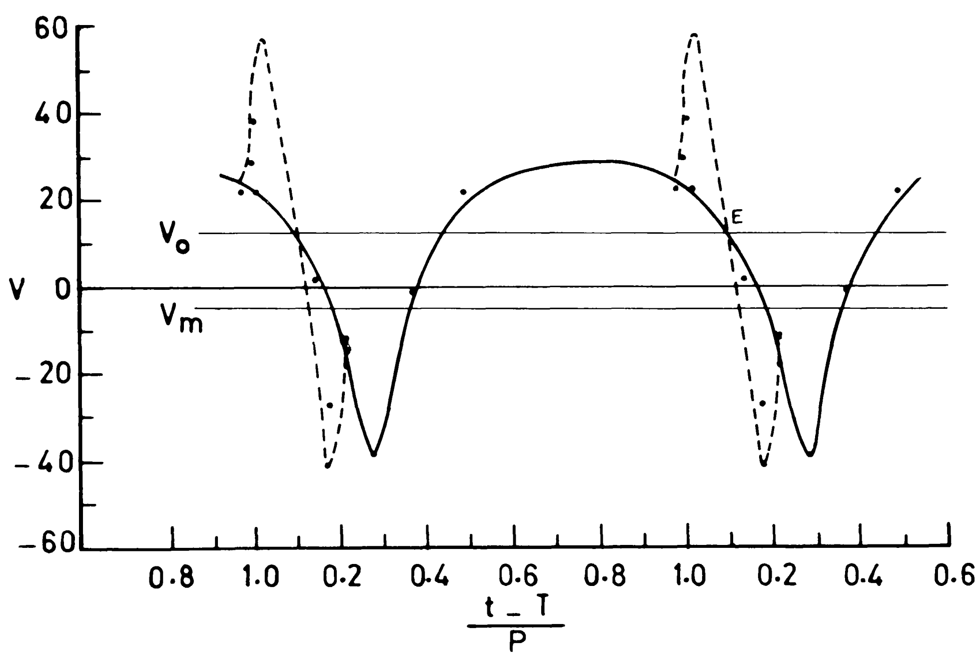

Fig. 3. Radial velocity curve of o And.

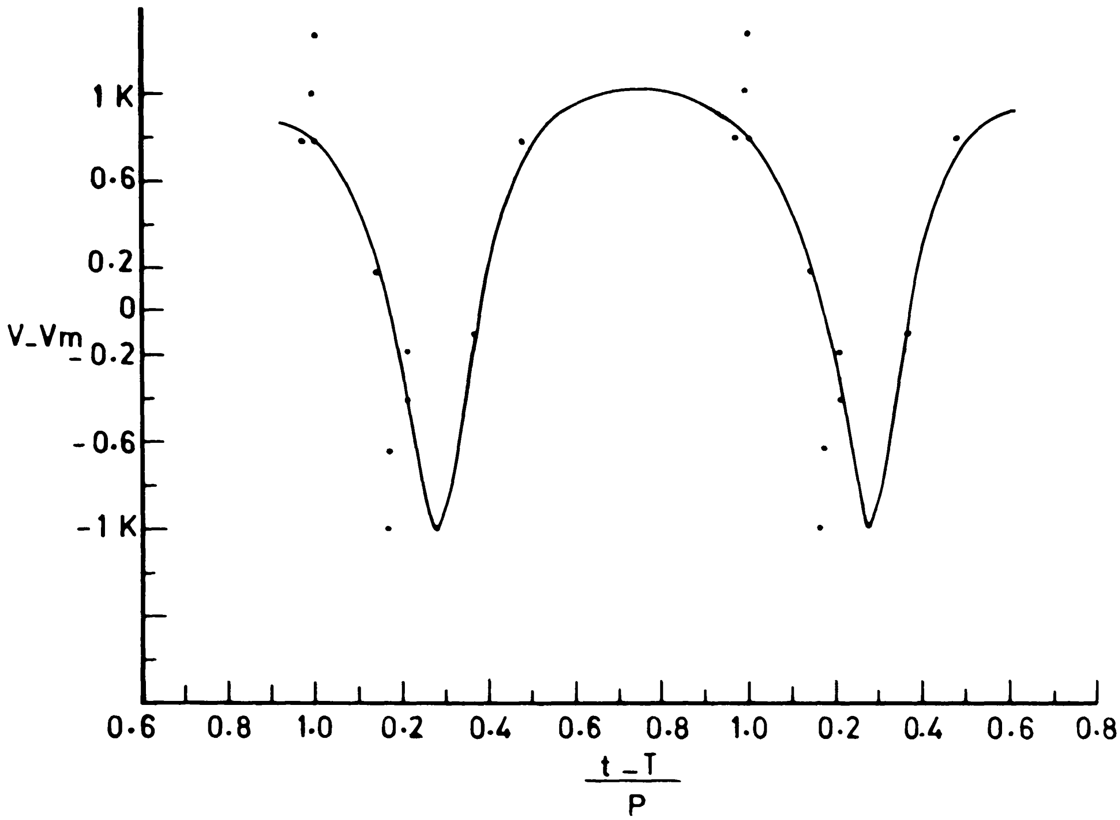

Fig. 4. Radial velocity curve of o And.

TABLE I

Date	JD	Phase	Radial velocity observed (km s^{-1})	Radial velocity calculated (km s^{-1})	O–C
15 Oct. 1981	4893.166	0.167	−41.13	–	–
16 Oct. 1981	4894.094	0.278	−39.07	−39.06	−0.01
8 Dec. 1981	4947.092	0.369	− 0.66	− 5.95	+5.29
8 Dec. 1981	4948.109	0.977	22.00	21.98	+0.02
9 Dec. 1981	4948.132	0.991	29.26	–	–
9 Dec. 1981	4948.147	0.000	38.26	–	–
9 Dec. 1981	4948.157	0.006	22.00	23.71	1.71
11 Dec. 1981	4950.063	0.144	1.12	6.97	5.85
9 Jan. 1982	4979.102	0.485	21.59	22.34	−0.75
23 Jan. 1982	5144.430	0.213	−11.85	−14.74	+2.89
5 Oct. 1982	5248.188	0.174	−27.06	–	–
5 Oct. 1982	5248.261	0.217	−19.11	−15.97	−3.14

eclipse is, where V_0 axis cuts the steeper side of the velocity curve. This behaviour is very clear in Figure 3, (o And). This shows that o And is a very close binary so that interaction like tidal waves or more violent phenomena is taking place. In the orbit determination we have left four observed velocities from taking into calculation. These velocities differ from the assumed radial velocity curve.

4. Application to δ Orionis

The observations used are as given in Table II (Singh, 1982). The observations were taken from 19 February, 1981 to 26 February, 1981; 3 April, 1981 to 8 April, 1981; using a 50 cm reflecting telescope, and 13 October, 1981 to 15 October, 1981; 10 November, 1981 to 11 November, 1981; and 8 December, 1981 to 11 December, 1981, using a 100 cm reflecting telescope of Kavalur Observatory on 09802 Kodak plates, at Cassegrain focus, at a reciprocal dispersion of 17.2 Å mm^{-1} at λ6562.817 Å. For radial velocity measurements 23 spectra have been used (Singh, 1982). The phase has been calculated from JD 2428382.263. By applying the present method we have arrived at $e = 0.25$ and $\omega = 136.88°$, where as our previously determined values were $e = 0.254$, $\omega = 139.07°$.

5. Radial Velocity Curve Sets

Table II represents a set of radial velocity curves for different e and ω. These have been computed from expression (5). These curves can be used in routine fashion for the determination of preliminary elements preparatory to a least-squares correction, in the following way. First of all one has to determine the period of the orbit by known methods. As a second step one has to plot observed radial velocity curve. From this curve one has to find out K, and V_m, and then again plot the observed radial velocity curve as described earlier, and then match this curve with the radial velocity curve given in the set, taking a guide from these values of e and ω.

6. Conclusions and Discussion

The present method appears upon trial to be somewhat less expeditious in practice than that of Lehmann-Filhés, which is in general use. It should be more accurate as the elements are deduced from twelve or more points of the velocity curve, instead of four. There are some cases, to which the geometrical methods are inapplicable, and in these cases the present one may be found useful.

One such case would occur when observations are incomplete because certain phases could not have been observed. Another case in which this method is useful is that of a star attended by two dark companions with commensurable periods. The analytical method enables us to separate the resultant motion into the two components orbital motions.

The advantages of this method are: (1) its accuracy; (2) its rapidity; (3) the possibility of making adjustments in the elements before the final presentation. The standard set of curves for (e, ω) given by this method can be used in routine fashion for determination of preliminary elements preparatory to a least-squares correction.

This method presented here is based on techniques somewhat similar to those to be found by Russell's (1902) method and Irwin's method (1952).

This method can be used for synthesising radial velocity curves for (e, ω) sets, for studying different effects such as gas streaming, rotational etc in binaries, by adding additional terms to the expression (5).

Appendix 1

In this appendix we will prove that radial velocity curves for different sets of (e, ω) with amplitude 1 K to -1 K can be drawn by using tables given in Irwin's (1952) paper on 'The Determination of a Spectroscopic Binary Orbit'. The procedure is as follows.

With reference to Figure 2 (Figure 1, Irwin, 1952), the amplitude and phase at points c, d, e, f, g, h, $i = a$, b, c are as given below

	Amplitude	Phase
c	1 K	0
d	0.7 K	$P_2 - P_1$
e	0	$S_2 + P_2 - P_1$
f	-0.7 K	$S_3 + S_2 + P_2 - P_1$
g	$-$ K	$S_3 + S_2 + P_3 + P_2 - P_1$
h	-0.7 K	$S_3 + S_2 + P_4 + P_2 - P_1$
$i = a$	0	$S_4 + S_3 + S_2 + P_4 + P_2 - P_1$
b	0.7 K	$S_4 + S_3 + S_2 + S_1 + P_4 + P_2 - P_1$
c	1 K	1

where

$$S_1 = \frac{t_b - t_a}{P}, \qquad S_2 = \frac{t_e - t_d}{P}, \qquad S_3 = \frac{t_f - t_e}{P}, \qquad S_4 = \frac{t_i - t_h}{P}$$

$$p_1 = \frac{t_c - t_b}{P}, \qquad p_2 = \frac{t_d - t_b}{P}, \qquad p_3 = \frac{t_g - t_f}{P}, \qquad p_4 = \frac{t_h - t_f}{P}$$

Here t_a, t_e and t_i refer to points on the V_m axis, t_b and t_d refer to points where $(V - V_m)/K = 0.7$ and t_f and t_h refer to points where $(V - V_m)/K = -0.7$ and t_c and t_g refer to the times of maximum and minimum velocities, respectively. The points are shown in Figure 2 (Figure 1 by Irwin, 1952). These parameters defined here are tabulated by Irwin (1952) with an accuracy of better than 0.0006. In these tables values are given at each 15° of ω from 0° to 360°, and e at 0.1 intervals from 0 to 0.95.

TABLE II

e	$\omega/2\pi$	$(V = V_m)/K$ $\frac{M}{2\pi} = 0.000$	0.083	0.167	0.250	0.333	0.417	0.500	0.583	0.667	0.750	0.833	0.917	A_0	A_1	B_1
0.083	0.000	1.000	0.817	0.363	−0.165	−0.613	−0.902	−1.000	−0.902	−0.613	−0.165	0.363	0.817	−0.083	0.992	0.000
0.083	0.083	0.866	0.419	−0.151	−0.636	−0.926	−0.997	−0.866	−0.565	−0.136	0.350	0.781	0.996	−0.072	0.859	0.497
0.083	0.167	0.500	−0.091	−0.625	−0.937	−0.991	−0.825	−0.500	−0.076	0.377	0.772	0.989	0.908	−0.042	0.496	0.861
0.083	0.250	0.000	−0.577	−0.932	−0.986	−0.790	−0.432	−0.000	0.432	0.790	0.986	0.932	0.577	−0.000	0.000	0.994
0.083	0.333	−0.500	−0.908	−0.989	−0.772	−0.377	0.076	0.500	0.825	0.991	0.937	0.625	0.091	0.042	−0.496	0.801
0.083	0.417	−0.866	−0.996	−0.781	−0.350	0.136	0.565	0.866	0.997	0.926	0.636	0.151	−0.419	0.072	−0.859	0.497
0.083	0.500	−1.000	−0.817	−0.363	0.165	0.613	0.902	1.000	0.902	0.613	0.165	−0.363	−0.817	0.083	−0.992	0.000
0.083	0.583	−0.866	−0.419	0.151	0.636	0.926	0.997	0.866	0.565	0.136	−0.350	−0.781	−0.996	0.072	−0.859	−0.497
0.083	0.667	−0.500	0.091	0.625	0.937	0.991	0.825	0.500	0.076	−0.377	−0.772	−0.989	−0.908	0.042	−0.496	−0.861
0.083	0.750	−0.000	0.577	0.932	0.986	0.790	0.432	0.000	−0.432	−0.790	−0.986	−0.932	−0.577	0.000	−0.000	−0.994
0.083	0.833	0.500	0908	0.989	0.772	0.377	−0.076	−0.500	−0.825	−0.991	−0.937	−0.625	−0.091	−0.042	0.496	−0.861
0.083	0.917	0.866	0.996	0.781	0.350	−0.136	−0.565	−0.866	−0.997	−0.926	−0.636	−0.151	0.419	−0.072	0.859	−0.497
0.167	0.000	1.000	0.748	0.205	−0.322	−0.705	−0.928	−1.000	−0.928	−0.705	−0.322	0.205	0.748	−0.167	0.969	0.000
0.167	0.083	0.866	0.317	0.311	−0.752	−0.965	−0.990	−0.866	−0.616	−0.256	0.195	0.667	0.980	−0.144	0.839	0.488
0.167	0.167	0.500	−0.200	−0.745	−0.981	−0.967	−0.787	−0.500	−0.140	0.262	0.659	0.950	0.949	−0.083	0.484	0.845
0.167	0.250	0.000	−0.663	−0.979	−0.947	−0.709	−0.374	−0.000	0.374	0.709	0.947	0.979	0.663	−0.000	0.000	0.976
0.167	0.333	−0.500	−0.949	−0.950	−0.659	−0.262	0.140	0.500	0.787	0.967	0.981	0.745	0.200	0.083	−0.484	0.845
0.167	0.417	−0.866	−0.980	−0.667	−0.195	0.256	0.616	0.866	0.990	0.965	0.752	0.311	−0.316	0.144	−0.839	0.488
0.167	0.500	−1.000	−0.748	−0.206	0.322	0.705	0.928	1.000	0.928	0.705	0.322	−0.205	−0.748	0.167	−0.969	0.000
0.167	0.583	−0.866	−0.317	0.311	0.752	0.965	0.990	0.866	0.616	0.256	−0.195	−0.667	−0.980	0.144	−0.839	−0.488
0.167	0.667	−0.500	0.200	0.745	0.981	0.967	0.787	0.500	0.140	−0.262	0.659	−0.950	−0.949	0.083	−0.484	−0.845
0.167	0.750	−0.000	0.663	0.979	0.947	0.709	0.374	0.000	−0.374	−0.709	−0.947	−0.979	−0.663	0.000	−0.000	−0.926
0.167	0.833	0.500	0.949	0.950	0.659	0.262	−0.140	−0.500	−0.787	−0.967	−0.981	−0.979	−0.663	0.000	−0.000	−0.926
0.167	0.917	0.866	0.980	0.667	0.195	−0.256	−0.616	−0.866	−0.990	−0.965	−0.752	−0.311	0.316	−0.144	0.839	−0.488
0.250	0.000	1.000	0.654	0.032	−0.463	−0.777	−0.947	−1.000	−0.947	−0.777	−0.463	0.032	0.654	−0.250	0.930	0.000
0.250	0.083	0.866	0.188	−0.472	−0.844	−0.988	−0.981	−0.866	−0.659	−0.358	0.043	0.527	0.945	−0.217	0.806	0.473
0.250	0.167	0.500	−0.328	0.850	−0.999	−0.934	−0.753	−0.500	0.157	0.157	0.537	0.882	0.982	−0.125	0.465	0.819
0.250	0.250	0.000	−0.757	−0.999	−0.887	−0.630	−0.322	−0.000	0.322	0.630	0.887	0.999	0.757	−0.000	0.000	0.946
0.250	0.333	−0.500	−0.982	−0.882	−0.537	−0.157	0.194	0.500	0.753	0.934	0.999	0.850	0.328	0.125	−0.465	0.819
0.250	0.417	−0.866	−0.945	−0.527	−0.043	0.358	0.659	0.866	0.981	0.988	0.844	0.472	−0.188	0.217	−0.806	0.473

0.250	0.500	−1.000	−0.654	−0.032	0.463	0.777	0.947	1.000	0.947	0.777	0.463	−0.032	−0.654	0.250	−0.930	0.000
0.250	0.583	−0.866	−0.188	0.472	0.844	0.988	0.981	0.866	0.659	0.358	−0.043	−0.527	−0.945	0.217	−0.806	−0.473
0.250	0.667	−0.500	0.328	0.850	0.999	0.934	0.753	0.500	0.194	−0.157	−0.537	−0.882	−0.982	0.125	−0.465	−0.819
0.250	0.750	−0.000	0.757	0.999	0.887	0.630	0.322	0.000	−0.322	−0.630	−0.887	−0.999	−0.757	0.000	−0.000	−0.946
0.250	0.833	0.500	0.982	0.882	0.537	0.157	−0.914	−0.500	−0.753	−0.934	−0.999	−0.850	−0.328	−0.125	0.465	−0.819
0.250	0.917	0.866	0.945	0.527	0.43	−0.358	−0.659	−0.866	−0.981	−0.988	−0.844	−0.472	0.188	−0.217	0.806	−0.473
0.333	0.000	1.000	0.526	−0.148	−0.584	−0.833	−0.961	−1.000	−0.961	−0.833	−0.584	−0.148	0.526	−0.333	0.877	0.000
0.333	0.083	0.866	0.030	−0.622	−0.912	−0.998	−0.971	−0.866	−0.693	−0.445	−0.100	0.367	0.881	−0.289	0.759	0.452
0.333	0.167	0.500	−0.474	−0.930	−0.995	−0.896	−0.721	−0.500	−0.240	0.062	0.411	0.783	1.000	−0.167	0.438	0.783
0.333	0.250	0.000	−0.851	−0.989	−0.812	−0.553	−0.278	−0.000	0.278	0.553	0.812	0.989	0.851	−0.000	0.000	0.904
0.333	0.333	−0.500	−1.000	−0.783	−0.141	−0.062	0.240	0.500	0.721	0.896	0.995	0.930	0.474	0.167	−0.438	0.783
0.333	0.417	−0.866	−0.881	−0.367	0.100	0.445	0.693	0.866	0.971	0.998	0.912	0.622	−0.030	0.289	−0.759	0.452
0.333	0.500	−1.000	−0.526	0.148	0.584	0.833	0.961	1.000	0.961	0.833	0.584	0.148	−0.526	0.333	−0.877	0.000
0.333	0.583	−0.866	−0.030	0.622	0.912	0.998	0.971	0.866	0.693	0.445	0.100	−0.367	−0.881	0.289	−0.759	−0.452
0.333	0.667	−0.500	0.474	0.930	0.995	0.896	0.721	0.500	0.240	−0.062	−0.411	−0.783	−1.000	0.167	−0.438	−0.783
0.333	0.750	−0.000	0.851	0.989	0.812	0.553	0.278	0.000	−0.278	−0.553	−0.812	−0.989	−0.851	0.000	−0.000	−0.904
0.333	0.833	0.500	1.000	0.783	0.411	0.062	−0.240	−0.500	−0.721	−0.896	−0.995	−0.930	−0.474	−0.167	0.438	−0.783
0.333	0.917	0.866	0.881	0.367	−0.100	−0.445	−0.693	−0.866	−0.971	−0.998	−0.912	−0.622	0.030	−0.289	0.759	−0.452
0.417	0.000	1.000	0.357	−0.322	−0.686	−0.877	−0.971	−1.000	−0.971	−0.877	−0.686	−0.322	0.357	−0.417	0.809	0.000
0.417	0.083	0.866	−0.157	−0.752	−0.958	−1.000	−0.960	−0.866	−0.722	−0.519	−0.230	0.194	0.776	−0.361	0.700	0.425
0.417	0.167	0.500	−0.630	−0.981	−0.973	−0.855	−0.692	−0.500	−0.279	−0.022	0.288	0.659	0.987	−0.208	0.404	0.737
0.417	0.250	0.000	−0.934	−0.947	−0.728	−0.481	−0.238	−0.000	0.238	0.481	0.728	0.947	0.934	−0.000	0.000	0.251
0.417	0.333	−0.500	−0.987	−0.659	−0.288	0.022	0.279	0.500	0.692	0.855	0.973	0.981	0.630	0.208	−0.404	0.737
0.417	0.417	−0.866	−0.776	−0.194	0.230	0.519	0.722	0.866	0.960	1.000	0.958	0.752	0.157	0.361	−0.700	0.425
0.417	0.500	−1.000	−0.357	0.322	0.686	0.877	0.971	1.000	0.971	0.877	0.686	0.322	−0.357	0.417	−0.809	0.000
0.417	0.583	−0.866	0.157	0.752	0.958	1.000	0.960	0.866	0.722	0.519	0.230	0.322	−0.357	0.417	−0.809	0.000
0.417	0.667	−0.500	0.630	0.981	0.973	0.855	0.692	0.500	0.279	0.022	−0.288	−0.659	−0.987	0.208	−0.404	−0.737
0.417	0.750	−0.000	0.934	0.947	0.728	0.481	0.238	0.000	−0.238	−0.481	−0.728	−0.947	−0.934	0.000	−0.000	−0.851
0.417	0.833	−0.500	−0.987	0.659	0.288	−0.022	−0.279	−0.500	−0.692	−0.855	−0.973	−0.981	−0.630	−0.208	0.404	−0.737
0.417	0.917	0.866	0.776	0.194	−0.230	−0.159	−0.722	−0.866	−0.960	−1.000	−0.958	−0.752	−0.157	−0.361	0.700	−0.425
0.500	0.000	0.997	0.149	−0.482	−0.768	−0.910	−0.979	−0.999	−0.979	−0.910	−0.768	−0.482	0.149	−0.500	0.727	0.000
0.500	0.083	0.864	−0.365	−0.855	−0.985	−0.995	−0.949	−0.865	−0.746	−0.581	−0.345	0.020	0.622	−0.433	0.629	0.393
0.500	0.167	0.499	−0.781	−0.999	−0.938	−0.813	−0.665	−0.500	−0.314	−0.097	0.170	0.517	0.929	−0.250	0.363	0.681
0.500	0.250	0.000	−0.987	−0.875	−0.640	−0.414	−0.203	−0.000	0.203	0.414	0.640	0.875	0.987	−0.000	0.000	0.787
0.500	0.333	−0.499	−0.929	−0.517	−0.170	0.097	0.314	0.500	0.665	0.813	0.938	0.999	0.781	0.250	−0.363	0.681
0.500	0.417	−0.864	−0.622	−0.020	0.345	0.581	0.746	0.865	0.949	0.995	0.985	0.855	0.365	0.433	−0.629	0.393

Table II (continued)

e	$\omega/2\pi$	$(V = V_m)/K$ $\frac{M}{2\pi} = 0.000$	0.083	0.167	0.250	0.333	0.417	0.500	0.583	0.667	0.750	0.833	0.917	A_0	A_1	B_1
0.500	0.500	-0.997	-0.149	0.482	0.768	0.910	0.979	0.000	0.979	0.910	0.768	0.482	-0.149	0.500	-0.727	0.000
0.500	0.583	-0.864	0.365	0.855	0.985	0.995	0.949	0.865	0.746	0.581	0.345	-0.020	-0.622	0.433	-0.629	-0.393
0.500	0.667	-0.499	0.781	0.999	0.938	0.813	0.665	0.500	0.314	0.097	-0.170	-0.517	-0.929	0.250	-0.363	-0.681
0.500	0.750	-0.000	0.987	0.875	0.640	0.414	0.203	0.000	-0.203	-0.414	-0.640	-0.875	-0.987	0.000	-0.000	-0.787
0.500	0.833	0.499	0.929	0.517	0.170	-0.097	-0.314	-0.500	-0.665	-0.813	-0.938	-0.999	-0.781	-0.250	0.363	-0.681
0.500	0.917	0.864	0.622	0.020	-0.345	-0.581	-0.746	-0.865	-0.849	-0.895	-0.985	-0.855	-0.365	-0.433	0.629	-0.393
0.583	0.000	0.986	-0.090	-0.620	-0.832	-0.934	-0.983	-0.998	-0.983	-0.934	-0.832	-0.620	-0.090	-0.583	0.632	0.000
0.583	0.083	0.854	-0.572	-0.926	-0.995	-0.984	-0.936	-0.864	-0.766	-0.634	-0.446	-0.147	-0.416	-0.505	0.547	0.356
0.583	0.167	0.493	-0.901	-0.985	-0.892	-0.770	-0.639	-0.499	-0.344	-0.164	0.060	0.365	0.811	-0.292	0.316	0.616
0.583	0.250	0.000	-0.988	-0.779	-0.549	-0.350	-0.170	-0.000	0.170	0.350	0.549	0.779	0.988	-0.000	0.000	0.711
0.583	0.333	-0.493	-0.811	-0.365	-0.060	0.164	0.344	0.499	0.639	0.770	0.892	0.985	0.901	0.292	-0.316	0.616
0.583	0.417	-0.854	-0.416	0.147	0.446	0.634	0.766	0.864	0.936	0.984	0.995	0.926	0.572	0.505	-0.547	0.356
0.583	0.500	-0.986	0.090	0.620	0.832	0.934	0.93	0.998	0.983	0.934	0.832	0.620	0.090	0.583	-0.632	0.000
0.583	0.583	-0.854	0.572	0.926	0.995	0.984	0.936	0.864	0.766	0.634	0.446	0.147	-0.416	0.505	-0.547	-0.356
0.583	0.667	-0.493	0.901	0.985	0.892	0.770	0.639	0.499	0.344	0.164	-0.060	-0.365	-0.811	0.292	-0.316	-0.616
0.583	0.750	-0.000	0.988	0.779	0.549	0.350	0.170	0.000	-0.170	-0.350	-0.549	-0.779	-0.988	0.000	0.000	-0.711
0.583	0.833	0.493	0.811	0.365	0.060	-0.146	-0.344	-0.499	-0.639	-0.770	-0.892	-0.983	-0.901	-0.292	0.316	-0.616
0.583	0.917	0.854	0.416	-0.147	-0.446	-0.634	-0.766	-0.864	-0.936	-0.984	-0.995	-0.926	-0.572	-0.505	0.547	-0.356
0.667	0.000	0.943	-0.335	-0.732	-0.879	-0.950	-0.983	-0.993	-0.983	-0.950	-0.879	-0.732	-0.335	-0.667	0.525	0.000
0.667	0.083	0.817	-0.749	-0.966	-0.990	-0.966	-0.921	-0.860	-0.781	-0.678	-0.533	-0.302	0.169	-0.577	0.455	0.313
0.667	0.167	0.472	-0.963	-0.940	-0.835	-0.724	-0.612	-0.497	-0.370	-0.225	-0.044	0.208	0.628	-0.333	0.263	0.542
0.667	0.250	0.000	-0.919	-0.663	-0.457	-0.288	-0.140	-0.000	0.140	0.288	0.457	0.663	0.919	-0.000	0.000	0.625
0.667	0.333	-0.472	-0.628	-0.209	0.444	0.225	0.370	0.497	0.612	0.724	0.835	0.940	0.963	0.333	-0.263	0.542
0.667	0.417	-0.817	-0.169	0.302	0.533	0.678	0.781	0.860	0.921	0.966	0.990	0.966	0.749	0.577	-0.455	0.313
0.667	0.500	-0.934	0.335	0.732	0.879	0.950	0.983	0.993	0.983	0.950	0.880	0.732	0.335	0.667	-0.525	0.000
0.667	0.583	-0.817	0.749	0.966	0.990	0.966	0.921	0.860	0.781	0.678	0.533	0.302	-0.169	0.577	-0.455	-0.313
0.667	0.667	-0.472	0.963	0.940	0.835	0.724	0.612	0.497	0.370	0.225	0.044	-0.208	-0.628	0.333	-0.263	-0.542
0.667	0.750	-0.000	0.919	0.663	0.457	0.288	0.140	0.000	-0.140	-0.288	-0.457	-0.663	-0.919	0.000	-0.000	-0.625
0.667	0.833	0.472	0.628	0.209	-0.044	-0.225	-0.370	-0.496	-0.612	-0.724	-0.835	-0.940	-0.963	-0.333	0.263	-0.542
0.667	0.917	0.817	0.169	-0.302	-0.533	-0.678	-0.781	-0.860	-0.921	-0.966	-0.990	-0.966	-0.749	-0.577	0.453	-0.313

0.750	0.000	0.818	−0.558	−0.818	−0.913	−0.957	−0.979	−0.985	−0.979	−0.957	−0.913	−0.818	−0.558	−0.750	0.407	0.000
0.750	0.083	0.709	−0.870	−0.974	−0.971	−0.943	−0.903	−0.853	−0.793	−0.716	−0.610	−0.443	−0.095	−0.650	0.353	0.264
0.750	0.167	0.409	−0.950	−0.869	−0.769	−0.675	−0.585	−0.493	−0.394	−0.282	−0.144	0.051	0.393	−0.375	0.204	0.457
0.750	0.250	0.000	−0.775	−0.531	−0.361	−0.227	−0.110	−0.000	0.110	0.227	0.361	0.531	0.775	−0.000	0.000	0.528
0.750	0.333	−0.409	−0.393	−0.051	0.144	0.282	0.394	0.493	0.585	0.675	0.769	0.869	0.950	0.375	−0.204	0.457
0.750	0.417	−0.709	0.095	0.443	0.610	0.716	0.793	0.853	0.903	0.943	0.971	0.97	0.870	0.650	−0.353	0.264
0.750	0.500	−0.818	0.558	0.818	0.913	0.957	0.979	0.985	0.979	0.957	0.913	0.818	0.558	0.750	−0.407	0.000
0.750	0.583	−0.709	0.870	0.974	0.971	0.943	0.903	0.853	0.793	0.716	0.610	0.443	0.95	0.650	−0.353	−0.264
0.750	0.667	−0.409	0.950	0.869	0.769	0.675	0.585	0.493	0.394	0.282	0.144	−0.051	−0.393	0.375	−0.204	−0.457
0.750	0.750	−0.000	0.775	0.531	0.361	0.227	0.110	0.000	−0.110	−0.227	−0.361	−0.531	−0.775	0.000	−0.000	−0.528
0.750	0.833	0.409	0.393	0.051	−0.144	−0.282	−0.394	−0.493	−0.585	−0.675	−0.769	−0.869	−0.950	−0.375	0.204	−0.457
0.750	0.917	0.709	−0.095	−0.443	−0.610	−0.716	−0.793	−0.853	−0.903	−0.943	−0.971	−0.974	−0.870	−0.650	0.353	−0.264
0.833	0.000	0.524	−0.738	−0.883	−0.936	−0.962	−0.974	−0.978	−0.974	−0.962	−0.936	−0.883	−0.738	−0.833	0.280	0.000
0.833	0.083	0.454	−0.927	−0.959	−0.943	−0.916	−0.884	−0.847	−0.804	−0.750	−0.679	−0.571	−0.351	−0.722	0.242	0.208
0.833	0.167	0.262	−0.868	−0.778	−0.697	−0.624	−0.557	−0.489	−0.418	−0.338	−0.240	−0.105	0.130	−0.417	0.140	0.360
0.833	0.250	0.000	−0.576	−0.388	−0.264	−0.166	−0.080	−0.000	0.080	0.166	0.264	0.388	0.576	−0.000	0.000	0.416
0.833	0.333	−0.262	−0.130	0.105	0.240	0.338	0.418	0.489	0.557	0.624	0.697	0.778	0.868	0.417	−0.140	0.360
0.833	0.417	−0.454	0.351	0.571	0.679	0.750	0.804	0.847	0.884	0.916	0.943	0.959	0.927	0.722	−0.242	0.208
0.833	0.500	−0.524	0.738	0.883	0.936	0.962	0.974	0.978	0.974	0.962	0.936	0.883	0.738	0.833	−0.280	0.000
0.833	0.583	−0.454	0.927	0.959	0.943	0.916	0.884	0.847	0.804	0.750	0.679	0.571	0.351	0.722	−0.242	−0.208
0.833	0.667	−0.262	0.868	0.778	0.697	0.624	0.557	0.489	0.418	0.338	0.240	0.105	−0.130	0.417	−0.140	−0.360
0.833	0.750	−0.000	0.576	0.388	0.264	0.166	0.080	0.000	−0.080	−0.166	−0.264	−0.388	−0.576	0.000	−0.000	−0.416
0.833	0.833	0.262	0.130	−0.105	−0.240	−0.338	−0.418	−0.489	−0.557	−0.624	−0.697	−0.778	−0.868	−0.417	0.140	−0.360
0.833	0.917	0.454	−0.351	−0.571	−0.679	−0.750	−0.804	0.847	−0.884	−0.916	−0.943	−0.959	−0.927	−0.722	0.242	−0.208
0.917	0.000	−0.053	−0.879	−0.939	−0.961	−0.972	−0.978	−0.979	−0.978	−0.972	−0.961	−0.939	−0.879	−0.917	0.144	0.000
0.917	0.083	−0.046	0.938	−0.934	−0.915	−0.894	−0.872	−0.848	−0.822	−0.790	−0.750	−0.692	−0.584	−0.794	0.124	0.141
0.917	0.167	−0.027	−0.746	−0.679	−0.623	−0.576	−0.532	−0.490	−0.445	−0.396	−0.338	−0.260	−0.132	−0.458	0.072	0.244
0.917	0.250	−0.000	−0.355	−0.242	−0.165	−0.104	−0.050	−0.000	0.50	0.104	0.165	0.242	0.355	0.000	0.000	0.282
0.917	0.333	0.027	0.132	0.260	0.338	0.396	0.445	0.490	0.532	0.576	0.623	0.679	0.746	0.458	−0.072	0.244
0.917	0.417	0.046	0.584	0.692	0.750	0.790	0.822	0.848	0.872	0.894	0.915	0.934	0.938	0.794	−0.124	0.141
0.917	0.500	0.053	0.879	0.939	0.961	0.972	0.978	0.979	0.978	0.972	0.961	0.939	0.879	0.917	−0.144	0.000
0.917	0.583	0.46	0.938	0.934	0.915	0.894	0.872	0.848	0.822	0.790	0.750	0.692	0.584	0.794	−0.124	−0.141
0.917	0.667	0.027	0.746	0.679	0.623	0.576	0.532	0.490	0.445	0.396	0.338	0.260	0.132	0.458	−0.072	−0.244
0.917	0.750	0.000	0.355	0.242	0.165	0.104	0.050	0.000	−0.050	−0.104	−0.165	−0.242	−0.355	0.000	−0.000	−0.282
0.917	0.833	−0.027	−0.132	−0.260	−0.338	−0.396	−0.445	−0.490	−0.532	−0.576	−0.623	−0.679	−0.746	−0.458	0.072	−0.244
0.917	0.917	−0.046	−0.584	−0.692	−0.750	−0.790	−0.822	−0.848	−0.872	−0.894	−0.915	−0.934	−0.938	−0.794	0.124	−0.141

For a given (e, ω)-set one can determine amplitude and phase as shown above, and draw radial velocity curve similar to Figure 2, this can be matched with the observed radial velocity curve drawn +1 K to −1 K amplitude, in order to determine preliminary elements preparatory to a least squares correction. But the accuracy of e and ω values will be limited to 0.1 and 15°, respectively.

As an example for $e = 0.5$, $\omega = 180°$, we have obtained:

	Amplitude	Phase
c	1 K	0
d	0.7 K	0.276
e	0	0.402
f	−0.7 K	0.460
g	− K	0.499
H	−0.7 K	0.539
$i = a$	0	0.597
b	0.7 K	0.723
c	1 K	1.00

The radial velocity curve drawn from these values is shown in Figure 2. For 0 And we have obtained $e = 0.539\,43$, $\omega = 177.046°$ which are very near to these values. A comparison of this curve and for 0 And, Figure 4 reveals the technique. This modified procedure for determining e and ω from Irwin's tables will be better than graphical method of finding out e and ω, by using the Irwin's tables.

Appendix 2

The coefficients of the Fourier series

$$f(x) = a_0 + \sum_{p=1}^{\infty} (a_b \cos px + b_p \sin px), \tag{1}$$

are given by

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)\,\mathrm{d}x, \tag{2}$$

$$a_p = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos px\,\mathrm{d}x, \tag{3}$$

$$b_p = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin px\,\mathrm{d}x. \tag{4}$$

In particular problems (as the present one) functions which have to be expanded in Fourier series are given as tables or graphs. In this case to obtain the coefficients, one has to use the technique of approximate integration. The rectangular rule reduces to the following procedure.

Let the interval $(0, 2\pi)$ be divided into m equal parts by the points

$$0, \frac{2\pi}{m}, 2\frac{2\pi}{m}, \ldots, (m-1)\frac{2\pi}{m}, 2\pi, \tag{5}$$

and suppose that the values of the function $f(x)$ at these points are known to be

$$v_0, v_1, v_2, \ldots, v_{m-1}, v_m. \tag{6}$$

Then we have

$$a_0 = \frac{1}{m}\sum_{k=0}^{m-1} v_k, \tag{7}$$

$$a_p = \frac{2}{m}\sum_{k=0}^{m-1} v_k \cos\frac{2\pi k}{m} p, \tag{8}$$

$$b_p = \frac{2}{m}\sum_{k=0}^{m-1} v_k \sin\frac{2\pi k}{m} p. \tag{9}$$

Calculation of A_0, A_1, B_1, for 12 points program

$12\,A_0$		$6A_1 =$	$6B_1 =$
v_0	$v_0 - v_6$	$[v_0 - v_6]1$	$[(v_1 - v_7) + (v_5 - v_{11})]0.5$
$+v_1$	$v_1 - v_7$	$+[(v_1 - v_7) - (v_5 - v_{11})]0.866$	$[(v_2 - v_8) + (v_4 - v_{10})]0.866$
		$+[(v_2 - v_8) - (v_4 - v_{10})]0.5$	$+[(v_3 - v_9)]1$
$+v_2$	$v_2 - v_8$		
$+v_3$	$v_3 - v_9$		
$+v_4$	$v_4 - v_{10}$		
$+v_5$	$v_5 - v_{11}$		
$+v_6$			
$+v_7$			
$+v_8$			
$+v_9$			
$+v_{10}$			
$+v_{11}$			

Appendix 3

This is the program (Fortran IV) used for calculating the series given by (5) and coefficients A_0 and A_1. In Table II we have presented the numerical values, for 12 points. Further accurate values can be obtained by this program, by taking more small steps, in e and ω. The present table gives values at $\Delta e = 0.083$ and $\Delta\omega = 30°$. The table presenting values at $\Delta e = 0.01$, $\Delta\omega/2\pi = 0.01$ can be supplied on request.

```
          PROGRAM ORBIT METHOD
C         ORBITAL SOLUTION: SPECTROSCOPIC BINARY
          DIMENSION XX(13, 13, 13), E(13), Ø(13), A(13),
                  AO(13, 13), AP(13, 13, 13)
          D = 1.D-5
          E(1) = 0-0
          Ø(1) = 0.0
          A(1) = 0.0
          N = 12
          DØ 10 I = 1, N
          FI = DSQRT(1.0-E(I)*E(I))
          DØ 11 J = 1, N
          DØ 12 K = 1, N
          CF = 0.0
          SF = 0.0
          DØ 13 IP = 1, N
          P = IP
          Z1 = DCØS(P*6.28318*A(K))
          Z2 = DSIN(P*6.28318*A(K))
          JP = IP-1
          KP = IP+1
          X = P*E(I)
          CALL BESJ(X, JP, B1, D, IER)
          CALL BESJ(X, KP, B2, D, IER)
          CF = CF+(B1+B2)*Z1
          SF = SF+(B1-B2)*Z2
          Z3 = DCØS(6.28318*Ø(J))
          Z4 = DSIN(6.28318*Ø(J))
          AØ(I, J) = -E(I)*Z3
          AP(I, J, IP) = (B1+B2)*Z3*FI*FI
13        CONTINUE
          XX(I, J, K) = -E(I)*Z3+Z3*FI*FI*CF-Z4*FI*SF
          A(K+1) = A(K)+1./12.
12        CONTINUE
          PRINT 201, E(I), Ø(J), (XX(I, J, K), K = 1, N), AØ(I, J),
                  AP(I, J, 1)
201       FORMAT (1X, F4, 3, 1X, F5.3, 1X, 12(F6.3, 1X). 1X, F6.3,
          1X, F7.3)
```

```
11      Ø(J+1) = Ø(J)+1./12.
10      E(I+1) = E(I)+1./12.
        PRINT 202, (A(K), K = 1, N)
202     FORMAT (10X, 'M = ', 12(F6.3, 1X))
        STOP
        END

        SUBROUTINE BESJ(X, IP, BJ, D, IER)
        BJ = 0.0
        IF(IP)10, 20, 20
10      IER = 1
        RETURN
20      IF(X)30, 30, 31
30      IER = 2
        RETURN
31      IF(X-15.)32, 32, 34
32      IPTEST = 20.+10.*X-X**2/3
        GØ TØ 36
34      IPTEST = 90.+X/2.
36      IF(IP-IPTEST)40, 38, 38
38      IER = 4
        RETURN
40      IER = 0
        IP1 = IP+1
        BPREV = 0.0
C
C       COMPUTE STARTING VALUE OF M
C
        IF(X-5.)50, 60, 60
50      MA = X+6.
        GØ TØ 70
60      MA = 1.4*X+60./X
70      MB = IP+IDINT(X)/4+2
        MZERO = AMAXO(MA, MB)
C
C       SET UPPER LIMIT OF M
C
        MMAX = IPTEST
100     DØ 190 M = MZERO, MMAX,3
C
C       SET F(M), F(M-1)
C
        FM1= 1.OE-28
```

```
          FM = 0.0
          ALPHA = 0.0
          IF(M-(M/2)*2)120, 110, 120
110       JT = -1
          GØ TØ 130
120       JT = 1
130       M2 = M-2
          DØ 160 K = 1, M2
          MK = M-K
          BMK = 2.*FLOAT(MK)*FM1/X-FM
          FM = FM1
          FM1 = BMK
          IF(MK-IP-1)150, 140, 150
140       BJ = BMK
150       JT = -JT
          S = 1+JT
160       ALPHA = ALPHA+BMK*S
          BMK = 2.*FM1/X-FM
          IF(IP)180, 170, 180
170       BJ = BMK
180       ALPHA = ALPHA+BMK
          BJ = BJ/ALPHA
          IF(ABS(BJ-BPREV)-ABS(D*BJ))200, 200, 190
190       BPREV = BJ
          IER = 3
200       RETURN
          END
```

References

Curtis, H. D.: 1908, *Public. Astron. Soc. Pacific* **20,** 133.
Irwin, J. B.: 1952, Publications of the Goethe Link Observatory, Indiana University, No. 7.
Lehmann-Filhés, R.: 1894, *Astron. Nachr.* **136,** 17.
Plummer, H. C.: 1908, *Astrophys. J.* **28,** 212.
Russell, H. N.: 1902, *Astrophys. J.* **15,** 252.
Schlesinger, F.: 1908, *Publ. Allegheny Obs.* **1,** 33.
Schwarzschild, K.: 1900, *Astron. Nachr.* **152,** 65.
Singe, M.: 1982, *Astrophys. Space Sci.* **87,** 269.
Struve, O.: 1949, *Monthly Notices Roy. Astron. Soc.* **109,** 487.
Zurhellen, W.: 1907a, *Astron Nachr.* **173,** 353.
Zurhellen, W.: 1907b, *Astron Nachr.* **175,** 245.