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Ion-sound turbulence due to shock gradients in collisionless plasmas

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Abstract. The dispersion relation for ion sound waves generated in a perpendicular shock is derived and the energy density of ion-sound turbulence is obtained using quasilinear theory. The result is compared with the lower hybrid turbulence generated under similar conditions. It is shown that ion-sound turbulence is a better candidate for the generation of type-I radio bursts in the solar corona.

Keywords. Ion-sound turbulence; quasilinear effects; lower hybrid turbulence; type-I radio bursts.

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1. Introduction

The generation of ion sound turbulence is an interesting problem both in laboratory and in the astrophysical contexts (Kaplan and Tsytovich 1973). The ion-sound instability is generated because of a relative velocity between the ions and electrons in a plasma when this velocity exceeds the ion-sound velocity in the plasma. The growth of ion-sound waves, therefore, depends upon the drift current present in the plasma. When shock waves propagate through a magnetised plasma the shock gradients generate currents perpendicular to the magnetic field. These currents generate ion-sound waves propagating perpendicular to the magnetic field. In non-isothermal plasmas, the generation is very efficient. Even in isothermal plasmas, ion-sound waves can be generated with a high energy density, provided the Buneman instability generated in the front of the shock heats the electrons differentially so that non-isothermality is generated (Galeev 1976; Kaplan and Tsytovich 1973).

In the present paper we derive the expression for the growth rate of the ion sound waves generated by shock gradients and the energy density of the ion-sound (IS) turbulence saturated by quasilinear effect. We compare the energy density of the lower hybrid (LH) turbulence with that of IS turbulence generated under similar conditions and show that the IS turbulence grows to higher levels. IS turbulence could be a better candidate for the low frequency turbulence needed to generate type-I solar radio bursts.

Presently, it is widely accepted that type-I radio bursts are generated by weak collisionless perpendicular shock waves caused by the newly emerging magnetic flux in

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the solar corona (Spicer *et al* 1981; Wentzel 1981, 1982). In this theory, the upper hybrid (UH) waves provide the adequate brightness temperature and the LH waves provide the adequate opacity. The wave numbers of both of these waves are normal to the magnetic field and satisfy the resonance condition for the coalescence of UH and LH waves. The LH waves are generated by the perpendicular current due to the shock gradients. The LH waves stochastically accelerate the electrons so that they are trapped in the magnetic mirrors, develop loss cone anisotropy and hence generate the UH waves.

In Benz and Wentzel's (1981) ion-acoustic wave model of type-I bursts, the ion-sound waves are generated by parallel currents due to coronal evolution. In the emerging flux theory, perpendicular currents feed the LH waves. We propose here that ion-sound waves can be generated by the perpendicular currents and these IS waves can interact with the UH waves to generate the type-I radiation.

2. Dispersion relation for ion-sound waves generated by shock waves

We consider a collisionless shock wave propagating perpendicular (in the x-direction) to a magnetic field in the z direction. Since the ion Larmor radius exceeds the shock thickness, ions are treated as unmagnetised. The electrons gyrate many times in the magnetic field in the shock transit time and hence they are magnetised. In the shock frame, the density and magnetic field gradients are in the x-direction. As the electrons are decelerated in the x direction due to induction and the ions are unaffected, there is a net charge separation in the shock-front so that an electric field E_x arises. It can be calculated from the steady state Maxwell's equations as

$$E_x = -\varepsilon_B B_0^2/4\pi ne, \tag{1}$$

where

$$\varepsilon_B = \mathrm{d}\ln B_0/\mathrm{d}x, \tag{2}$$

is the magnetic field gradient. B_0 is the magnetic field at $x = 0$. n and $-e$ are the density and charge of the electrons. This electric field gives rise to a cross-field drift given by

$$V_E = -cE_x/B_0 = \frac{cB_0}{4\pi ne}\varepsilon_B. \tag{3}$$

The gradients in density and magnetic field give rise to the drifts

$$v_n = \varepsilon_n v_e^2/\omega_{ce};\quad \varepsilon_n = \mathrm{d}\ln n/\mathrm{d}x, \tag{4}$$

and

$$v_B = \varepsilon_B v_\perp^2/\omega_{ce}, \tag{5}$$

where $\omega_{ce} = eB_0/m_e c$ is the electron cyclotron frequency, v_e is the electron thermal velocity, m_e is the electron mass and c is the velocity of light in free space.

We are interested in solving the Vlasov-Poisson equations to obtain the linear dispersion relation. The equilibrium distribution function can be obtained in terms of the following integrals of motion:

$$W_T = \tfrac{1}{2}mv^2 - Fx,\quad \text{(total energy)},$$

$$P_y = mv_y - \frac{e}{c}\int B_0(x)\,\mathrm{d}x,\quad (y\text{-momentum}),$$

$$P_z = mv_z,\quad (z\text{-momentum}).$$

Here F is the force due to the charge separation electric field:

$$F = -eE_x. \tag{6}$$

The simplest equilibrium which reduces to a Maxwellian in the absence of inhomogeneities may be given as,

$$f_{0e}(x, \mathbf{v}) = \left(\frac{m_e}{2\pi T_e}\right)^{3/2}\left[1+\alpha\left(x-\frac{v_y}{c}\right)\right]\exp\left[-\frac{m_e v^2}{2T_e}-Fx\right], \tag{7}$$

where T_e is the electron temperature and α is the parameter characterizing the spatial variation. It is related to the density gradient by the relation,

$$\varepsilon_n = \alpha + (F/T_e). \tag{8}$$

The perturbation of such an equilibrium can be described by the linearized Vlasov equation:

$$\frac{\partial f_1}{\partial t}+\mathbf{v}\cdot\frac{\partial f_1}{\partial \mathbf{r}}-\frac{e}{m}\left[\frac{\mathbf{v}\times\mathbf{B}_0(x)}{c}-\frac{F}{e}\hat{e}_x\right]\cdot\frac{\partial f_1}{\partial \mathbf{v}} = -\frac{e}{m}\nabla\varphi(\mathbf{r}, t)\cdot\frac{\partial f_{0e}}{\partial \mathbf{v}} \tag{9}$$

where

$$f_1(\mathbf{r}, \mathbf{v}, t) = f_e(\mathbf{r}, \mathbf{v}, t) - f_{0e}(x, \mathbf{v})$$

is the perturbed distribution function and $\varphi(\mathbf{r}, t)$ is the perturbed potential. Equation (9) can be solved using the method of characteristics (see e.g. Krall and Trivelpiece 1973). The characteristics are

$$\mathrm{d}\mathbf{r}/\mathrm{d}t = \mathbf{v},$$

$$\mathrm{d}\mathbf{v}/\mathrm{d}t = -\omega_{ce}(1-\varepsilon_B x)\,\mathbf{v}\times\hat{e}_z+\frac{F}{m_e}\hat{e}_z, \tag{10}$$

where we have assumed the magnetic field variation in the form,

$$B_0(x) = B_0(1-\varepsilon_B x). \tag{11}$$

The particle trajectories $\mathbf{r}'$, $\mathbf{v}'$ which arrive at $\mathbf{r}$, $\mathbf{v}$ when $t' \to t$ can be obtained from (10) as,

$$\begin{aligned} v_x' &= v_\perp \cos[\theta+\omega_{ce}(t'-t)] + v_E \sin\omega_{ce}(t'-t), \\ v_y' &= v_\perp \sin[\theta+\omega_{ce}(t'-t)] - v_E[\cos\omega_{ce}(t'-t)-1] - v_B, \\ v_z' &= v_z, \end{aligned} \tag{12}$$

and

$$\begin{aligned} x' &= x+\frac{v_\perp}{\omega_{ce}}\{\sin[\theta+\omega_{ce}(t'-t)]-\sin\theta\}+\frac{v_E}{c}[1-\cos\omega_{ce}(t'-t)] \\ y' &= y-\frac{v_\perp}{\omega_{ce}}[\cos\{\theta+\omega_{ce}(t'-t)]-\cos\theta\}-\frac{v_E}{\omega_{ce}}\sin\omega_{ce}(t'-t) \\ &\quad + v_E(t'-t) - v_B(t'-t) \\ z' &= z+v_z(t'-t). \end{aligned} \tag{13}$$

Here, $v_x = v_\perp \cos\theta$, $v_y = v_\perp \sin\theta$. We have neglected terms of order ε_B^2, under the assumption of weak spatial gradient. Equation (9) can be formally integrated as

$$f_1(\mathbf{r}, \mathbf{v}, t) = -\frac{e}{m}\int_{-\infty}^{t} \mathrm{d}t' \nabla\varphi(\mathbf{r}', t')\cdot\frac{\partial f_{0e}(x', \mathbf{v}')}{\partial \mathbf{v}'}. \tag{14}$$

Taking

$$\varphi(\mathbf{r}, t) = \varphi(x)\exp(ik_y y + ik_z z - i\omega t), \tag{15}$$

and assuming that the local approximation (Krall 1968) is valid (i.e., $\varphi(x) = \varphi(o)$) one can perform the integration in (14) with the aid of (12) and (13). The result is

$$f_1^{(e)} = \varphi(0)\frac{e}{T_e} f_{0e}(x, \mathbf{v})\left[1 - (\omega - \omega^*)\cdot\sum_{p,q}\frac{J_p\left(\frac{k_y v_\perp}{\omega_{ce}}\right) J_q\left(\frac{k_y v_\perp}{\omega_{ce}}\right)\exp(i(p-q)\theta)}{(\omega - k_y v_E + k_y v_B - p\omega_{ce})}\right], \tag{16}$$

where

$$\omega^* = k_y v_E - k_y v_n.$$

Since the ions are unmagnetised, their distribution is simple:

$$f_1^{(i)} = -\frac{e}{m_i}\varphi(0)\frac{\mathbf{k}\cdot\frac{\partial f_{0i}}{\partial \mathbf{v}}}{(\omega - \mathbf{k}\cdot\mathbf{v})}, \tag{17}$$

where

$$f_{0i} = n_0\left(\frac{m_i}{2\pi T_i}\right)^{3/2}\exp\left[-\frac{m_i \mathbf{v}^2}{2T_i}\right], \tag{18}$$

T_i, m_i being ion temperature and ion mass respectively. Substituting (17) and (16) in the Poisson's equation,

$$-\nabla^2\phi = 4\pi e\int \mathrm{d}^3\mathbf{v}[f_1^{(i)} - f_1^{(e)}], \tag{19}$$

one can eliminate $\varphi(0)$ to get the dispersion relation,

$$D(x = 0, k, \omega) = 1 + \chi_i + \chi_e = 0, \tag{20}$$

where

$$\chi_i = \frac{k_i^2}{k^2} W\left(\frac{\omega}{k v_i}\right) \tag{21}$$

and

$$\chi_e = \frac{k_e^2}{k^2}\left\{1 + \frac{m_e}{T_e}\left(\frac{m_e}{2\pi T_e}\right)^{1/2}\int_0^\infty v_\perp \mathrm{d}v_\perp \int_{-\infty}^{\infty}\mathrm{d}v_z\cdot\right.$$

$$\left.\sum_p \frac{(\omega - \omega^*) J_p^2\left(\frac{k_y v_\perp}{\omega_{ce}}\right)}{(\omega - k_y v_E + k_y v_B - k_z v_z - p\omega_{ce})}\exp\left[-\frac{v_\perp^2 + v_z^2}{2 v_e^2}\right]\right\}. \tag{22}$$

Where k_α and v_α are the Debye wave number and thermal velocity of species $\alpha (= e, i)$. The W-function is defined as

$$W(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{x\exp(-x^2/2)\,\mathrm{d}x}{x - z}. \tag{23}$$

We consider the range of frequencies $k_y v_i < \omega < \omega_{ce}$. In this range, one can neglect terms of order $\omega/p\omega_{ce}$, $p = \pm 1, \pm 2 \ldots$ and expand $W(\omega/k_y v_i)$ asymptotically. From the Rankine-Hugoniot relations, we note that the jump in magnetic field and density across a laminar shock are of the same order. Defining an average gradient-B drift, we find that $\bar{v}_B \sim v_n$, Hence we get

$$\frac{\omega - k_y v_E + k_y v_n}{\omega - k_y v_E + k_y \bar{v}_B} \simeq 1. \tag{24}$$

With further assumption that $k_y v_e > \omega_{ce}$ and $(k_z/k_y > (v_E/v_e)$, we can solve the dispersion relation (20) to get the frequency ω_r and growth rate γ $(\omega = \omega_r + i\gamma; |\gamma/\omega| \ll 1)$ as,

$$\omega_r = \pm \frac{\omega_{pi}(1 + 3T_i/T_e)}{\left[1 + \frac{k_e^2}{k^2}(1 + \Lambda_0)\right]^{1/2}} \tag{25}$$

and

$$\begin{aligned} \frac{\gamma}{\omega_r} = & -\left(\frac{\pi}{8}\right)^{1/2} \frac{k_i^2}{k^2}\left(\frac{\omega_r}{k_y v_i}\right) \exp\left(-\frac{\omega_r^2}{2k^2 v_i^2}\right) \\ & -\left(\frac{\pi}{2}\right)\frac{k_e^2}{k^2}\left(\frac{v_E}{\bar{v}_B} - \frac{\omega_r}{k_y \bar{v}_B} - 1\right) J_0^2\left[\frac{k_y v_e}{\omega_{ce}}\left\{2\left(\frac{v_E}{\bar{v}_B} - \frac{\omega_r}{k_y \bar{v}_B}\right)\right\}^{1/2}\right] \\ & \times \left[1 + \frac{k_z^2 v_e^2}{k_y^2 v_E^2}\right] \exp\left[-\frac{v_E}{\bar{v}_B} - \frac{\omega_r}{k_y \bar{v}_B} + \frac{k_z^2 v_e^2}{k_y^2 v_E^2}\right] \end{aligned} \tag{26}$$

where

$$k^2 = k_y^2 + k_z^2. \tag{26a}$$

Here,

$$\Lambda_0 = I_0\left(\frac{k_y^2 v_e^2}{\omega_{ce}^2}\right)\exp\left[-\frac{k_y^2 v_e^2}{\omega_{ce}^2}\right] \simeq \frac{1}{\sqrt{2\pi}}\frac{\omega_{ce}}{k_y v_e} \ll 1.$$

The frequency of the waves given by (25) is nothing but the ion-sound frequency because $\Lambda_0 \ll 1$ and $T_e \gg T_i$. The frequency can be written as

$$\omega_r = \pm \frac{k c_s (1 + 3T_i/T_e)}{\left[1 + \frac{k^2}{k_e^2}(1 - \Lambda_0)\right]^{1/2}}$$

where $c_s = \sqrt{T_e/m_i}$ is the ion sound velocity.

The first term in (26) is the ion-Landau damping. The second term is due to electron Landau damping and drifts. The mode with lower sign in (25) grows when

$$(v_E/\bar{v}_B) + (|\omega_r|/k_y \bar{v}_B) > 1. \tag{27}$$

Now, $v_E/\bar{v}_B = 2/\beta > 1$, and hence the instability criterion is easily satisfied provided the ion Landau damping is negligible. This is in fact so because of the condition $T_e \gg T_i$. Differentiating (26) with respect to k, we find that γ reaches a maximum value of $(m_e/m_i)^{1/2}\omega_{ce}$ when $k \sim k_e$. When the wavenumber decreases, γ also decreases and $\gamma = \frac{1}{2}(m_e/m_i)^{1/2}\omega_{ce}$ when $k \sim k_e(\omega_{ce}/\omega_{pe})$. Hence one can conclude that the maximum growing modes lie in the range, $k_e(\omega_{ce}/\omega_{pe})$ to k_e.

3. Quasilinear saturation of ion-sound instability

Once the ion-sound waves start growing, the electrons in the resonance region start diffusing in velocity space due to quasilinear interaction. Therefore, the equilibrium distribution function of electrons changes slowly with time. Because of this, the energy density of the ion-sound waves saturates. The change in equilibrium distribution function can be found as

$$\partial f_0/\partial t = \left\langle \frac{c}{B_0^2}[\nabla\phi \times \mathbf{B}_0]\cdot\nabla f_1 + \frac{e}{m_e}\frac{\partial\phi}{\partial z}\frac{\partial f_1}{\partial v_z}\right\rangle, \tag{28}$$

where $\langle \ldots \rangle$ denotes average over fast time scale. f_1 is the rapidly fluctuating part of the distribution and $f_0 \gg |f_1|$. Calculating as in the linear case, we get

$$\frac{\partial f_0}{\partial t} = \frac{e^2}{m^2}\sum_k\left[k_z\frac{\partial}{\partial v_z} + 2\omega\frac{\partial}{\partial v_\perp^2} - k_y\frac{\partial}{\partial(v_y-\omega_{ce}x)}\right]J_0^2\left(\frac{k_y v_\perp}{\omega_{ce}}\right)$$
$$\times\, k_z\frac{\partial}{\partial v_z} + 2\omega\frac{\partial}{\partial v_\perp^2} - k_y\frac{\partial}{\partial(v_y-\omega_{ce}x)}\bigg] f_0\cdot|\phi|^2\,\delta(\omega - k_y v_E + k_y v_B - k_z v_z). \tag{29}$$

The calculation is similar to that of Krall and Book (1969). But in our case the resonance region is determined by v_E rather than $\bar{v}_B$. Since most of the waves propagate in a narrow cone around the direction of drift, we assume $k_z = 0$ and we get the effective collision frequency due to ion-sound turbulence as

$$\nu_{\text{eff}} = \frac{\int v_y(\partial f_0/\partial t)\,\mathrm{d}^3 v}{\int v_y f_0\,\mathrm{d}^3 v}. \tag{30}$$

The change in the equilibrium distribution function due to quasilinear effect, viz, $\partial f_0/\partial t$ is obtained from (29). This change is proportional to the energy density of ion-sound (IS) waves which enters through the quasilinear diffusion coefficient. Evaluation of the integral (30) yields,

$$\nu_{\text{eff}} = \left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{\beta}{2}\right)\left(\frac{\omega_{ce}\,\omega_{pe}^2}{c^2\,\varepsilon_B^2}\right)\left(1 + \ln\frac{\omega_{pe}^2}{\omega_{ce}^2}\right)\frac{W_s}{nT_e} \tag{31}$$

where $W_s = \sum_k k^2|\phi|^2$ is the energy density of the ion-sound waves.

We can take the scale length of magnetic field variation in the shock as

$$\varepsilon_B \simeq (\Delta B/B_0)\,L_s^{-1}, \tag{32}$$

where $\Delta B = (B_2 - B_1)$ is the jump in magnetic field (see figure 1) across the shock and L_s is the shock thickness. Putting this in the marginal stability case of (27) viz,

$$\frac{2}{\beta} - \frac{c_s\,\omega_{ce}}{v_e^2\,\varepsilon_B} = 1, \tag{33}$$

we get an expression for the shock thickness L_s:

$$L_s = \left(\frac{\Delta B}{B}\right)\left(\frac{2}{\beta} - 1\right)\frac{v_e^2}{\omega_{ce} c_s}. \tag{34}$$

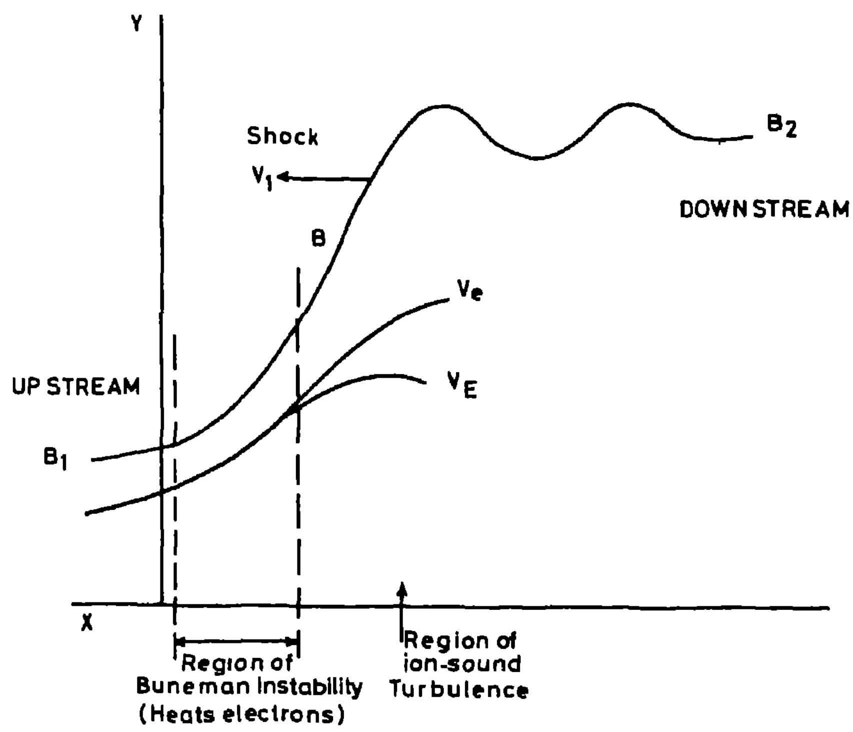

Figure 1. The profiles of B, v_e and V_E at various portions of the shock; the regions of ion-sound and Buneman instabilities are also indicated.

The shock thickness is also related to the effective collision frequency due to the instability generated in the shock by (Spicer *et al* 1981)

$$L_s = \frac{\nu_{\text{eff}} c^2}{\omega_{pe}^2 v_A} \frac{M_A}{M_A^2 - 1}, \tag{35}$$

where $M_A = (v_A/v_1)^{-1}$ is the Alfvenic Mach number, $v_A = B_0/(4\pi n m_i)^{1/2}$ is the downstream Alfven velocity and v_1 is the velocity of the shock. Relations (34) and (35) can be used along with (31) to find

$$\frac{W_s}{nT_e} = \left(\frac{m_i}{m_e}\right)^{1/2} \left(\frac{\Delta B}{B_0}\right) \left(\frac{c_s v_A}{v_e^2}\right) \left(\frac{M_A^2 - 1}{M_A}\right) \left(\frac{2}{2-\beta}\right) \left[1 + \ln \frac{\omega_{pe}^2}{\omega_{ce}^2}\right]^{-1} \tag{36}$$

The above expression indicates that the energy density of the ion-sound turbulence depends upon the jump in magnetic field ΔB, the electron temperature, the Alfvenic Mach number, the coronal plasma β and the ratio of the plasma frequency to the cyclotron frequency.

4. Comparison with lower hybrid turbulence

For weak shock waves, the energy density of the lower hybrid (LH) turbulence has been calculated by Spicer *et al* (1981) as

$$\frac{W_{\text{LH}}}{nT_e} = \frac{\omega_{ce}^2}{\omega_{pe}^2} \left(\frac{v_A}{c_s}\right) \frac{\Delta B}{B_0} \left(\frac{M_A^2 - 1}{M_A}\right) \left(\frac{m_e}{m_i}\right)^{1/2}. \tag{37}$$

Comparing the expressions (37) and (36) one gets

$$\frac{W_{\rm LH}}{W_s} = \frac{\omega_{ce}^2}{\omega_{pe}^2}\left(1 - \frac{1}{2}\beta\right)\left(1 + \ln\frac{\omega_{pe}^2}{\omega_{ce}^2}\right). \tag{38}$$

In most of the plasmas of interest, $\omega_{ce}^2/\omega_{pe}^2 \ll 1$ and hence the LH turbulence saturates at a lower level compared to the IS turbulence. The other factors in (38) are not important. In the coronal region where type-I bursts occur, say 100 MHz level, the magnetic field is ~ 1 gauss (Gopalswamy *et al* 1984), which corresponds to a cyclotron frequency of 2·8 MHz. Then the ratio $\omega_{ce}^2/\omega_{pe}^2 \sim 2{\cdot}8 \times 10^{-2}$ and hence $W_{\rm LH}/W_s \sim 4{\cdot}5 \times 10^{-3}$. Under such circumstances, clearly, IS turbulence is important.

Though we have compared the LH and IS turbulences under identical conditions, one should note that we have assumed $T_e \gg T_i$. The solar corona is usually isothermal ($T_e \approx T_i$) under which case the Landau damping for the IS waves will be dominant. But it has been pointed out (Galeev 1976; Tidman and Krall 1971) that a variety of instabilities are excited at the shock front and most of them essentially heat the electrons and quenched at the initial portion of the shock front itself. Deeper into the shock, the condition $T_e > T_i$ is established and IS waves grow and these waves determine the structure of the shock wave by limiting the perpendicular current, quasilinearly. The situation is explained in figure 1. The Buneman instability with $v_E \sim v_e$ first occurs near the front of the shock which heats the electrons and increases T_e/T_i. Deeper into the shock $v_E < v_e$ and the Buneman instability is quenched whereas IS instability is excited. In fact, the magnetic field gradient $\varepsilon_{\rm Bin}$ needed for generating the Buneman instability can be obtained as

$$v_E \simeq v_e$$

i.e, $$\frac{cB_0}{4\pi ne}\quad \varepsilon_{\rm Bin} = v_e,$$

or $$\varepsilon_{\rm Bin} = 7{\cdot}8 \times 10^{-3}\,{\rm cm}^{-1}.$$

For IS waves,

$$\varepsilon_B \simeq 1{\cdot}8 \times 10^{-4}\,{\rm cm}^{-1}.$$

This means the density gradient at the initial portion should be 43 times more than that in the interior. This will be satisfied in the shock configuration assumed by Vlahos *et al* (1982). Moreover, Moller-Pederson *et al* (1978) have concluded that whatever be the initial value of T_e/T_i, ultimately IS turbulence will be generated.

Now, let us consider the generation of type-I radio bursts. Basically we need high frequency waves to coalesce with low frequency waves to produce electromagnetic radiation as type-I radio bursts. The generation of these waves must be due to some agency moving through the corona. The small drift rate of type-I chains indicates that the agency must be a weak shock. The weak shock, therefore, should generate both high and low frequency turbulences. In the emerging flux theory of Spicer *et al* (1981), the high frequency waves are UH waves generated by energetic electrons stochastically accelerated by LH waves. We argue that the IS turbulence is a plausible candidate for the low frequency waves because of their higher level of saturation energy and the larger width in wave number space so that the resonance condition for the interaction with high frequency turbulence is easily satisfied. We consider these points in detail below.

4.1 *Energy requirement of low frequency waves*

For the UH waves to provide adequate brightness temperature, the low frequency waves should have an energy density,

$$\frac{W^{\sigma}}{nT_e} \gtrsim \frac{6\sqrt{3}}{\pi} \frac{v_e c}{\omega_{pe} L_N v_{\varphi}^{\sigma}}, \tag{39}$$

where σ represents any suitable low frequency waves that interact with the UH waves to produce the radiation; L_N is the scale height of coronal electron density variation and v_{φ}^{σ} is the phase velocity of the low frequency waves. For a coronal plasma frequency of 100 MHz, $L_N \approx 10^{10}$ cm. For typical phase velocities of the low frequency waves in the million degree corona, one gets from (39),

$$W^{\sigma}/nT_e \gtrsim 1{\cdot}3 \times 10^{-6}. \tag{40}$$

This condition is satisfied only marginally by the LH waves whereas the energy density of IS waves is much larger than the limit (40) as is evident from the discussion in §4.

4.2 *Acceleration of electrons to generate high frequency waves*

Now, the low frequency turbulence should generate energetic electrons. The energetic electrons should develop a loss cone distribution to generate the necessary high frequency waves. The LH waves can stochastically accelerate the electrons to high energies while the IS waves cannot (Lampe and Papadopoulos 1977; Kaplan and Tsytovich 1973). But there is another efficient process by which the IS waves can produce energetic particles with a loss cone distribution in order to generate UH waves. Whistler waves and IS waves have the same range of frequencies. Hence, the IS waves can get converted into Whistler waves through non-linear scattering from the ions and electrons. The characteristic time of conversion is

$$\tau_{\text{scatter}} = \frac{20}{\beta}\left(\frac{W_s}{nT_e}\right)^{-1} \omega_{ce}^{-1}$$

where $\beta = (8\pi nT/B_0^2)$ and ω_{ce} is the electron cyclotron frequency. These Whistlers have electric field normal to the magnetic field and hence increase the transverse energy of the electrons as they are absorbed by the electrons. Once the transverse velocity of the particle exceeds certain threshold value determined by the mirror ratio, the electrons are trapped. The characteristic time over which this trapping occurs is

$$\tau_{\text{heat}} = \frac{6}{\pi}\left(\frac{\omega_{pe}}{\omega_{ce}}\right)\left(\frac{v_h}{v_e}\right)^2\left(\frac{W_w}{nT_e}\right)^{-1} \omega_{ce}^{-1};$$

where W_w is the energy density of Whistler waves. For a v_h (velocity of the heated electron due to Whistler absorption) of $\sim 7\ v_e$ and $W_w \simeq 0{\cdot}5\ W_s$ one gets $\tau_{\text{scatter}} \sim 0{\cdot}01$ sec and $\tau_{\text{heat}} \sim 0{\cdot}5$ sec. Both these time scales are well within the collisional damping time (Kaplan and Tsytovich 1973).

$$\tau_{\text{coll}} = (T_i/T_e)^{3/2}\,(N_D/\omega_{pi}),$$

where T_i, ω_{pi}-ion temperature and ion plasma frequency, N_D-Debye number $= n\lambda_e^3$, λ_e being electron Debye radius. Hence the ion sound turbulence provides an alternative mechanism to produce loss cone distribution of energetic electrons.

4.3 *Overlap in wavenumber space*

For the efficient interaction of the high and low frequency waves, the following resonance conditions should be satisfied:

$$\mathbf{k}_L + \mathbf{k}_\sigma = \mathbf{k}_t, \tag{41}$$

$$\omega_t + \omega_\sigma = \omega_t, \tag{42}$$

where (k, ω) are the wave number and frequency, and L, σ, t represent the UH, low frequency and transverse waves respectively. Since $k_t \ll k_L$, k_σ one needs $k_L \sim k_\sigma$. For maximum growing modes (Wentzel 1981),

$$k_L \simeq 2k_e(v_e/v_h), \tag{43}$$

$$k_{LH} \lesssim k_e(\omega_{ce}/\omega_{pe}), \tag{44}$$

and for the ion sound waves,

$$k_s = k_e\left(\frac{\omega_{ce}}{\omega_{pe}}\right) t_0\ k_e. \tag{45}$$

Since $\omega_{ce}/\omega_{pe} \ll 1$ for the coronal plasma level at 100 MHz, we see that there is a better overlap in the k-space in the case of IS waves compared to the LH waves. (44) demands that v_h must be at least 20 v_e to satisfy the resonance condition while moderate electrons heating is sufficient in the case of ion sound wave because of its wider range of maximum growth.

It is clear from the above discussions that the IS turbulence is a more likely candidate for the generation of type-I bursts in the solar corona.

5. Conclusions

The dispersion relation for the perpendicular IS waves is derived and the growth rate is evaluated. The saturation level of ion-sound turbulence is obtained from a quasilinear analysis and compared with that of lower hybrid waves. It is found that the ion-sound turbulence saturates at a higher level compared to the lower hybrid turbulence and therefore, the former is a better candidate compared to the latter because of the higher energy density and the fact that there is a better overlap in the wave number space. The shock waves produced by the newly emerging flux give rise to the ion-sound turbulence which, according to our analysis, is the most favourable candidate to interact with the high frequency waves.

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